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ON MINIMAL NON-MSP-GROUPS*
ПРО МІНІМАЛЬНІ НЕ MSP-ГРУПИ

A finite group G is called an MSP-group if all maximal subgroups of the Sylow subgroups of G are S-quasinormal in G. In this paper, wc give a complete classification of those groups which are not MSP-groups but whose proper subgroups are all MSP-groups.

Скінченну групу G називають MSP-групою, якщо всі максимальні підгрупи силовських підгруп G є S-квазінормальними в G. Наведено повну класифікацію груп, які не є MSP-групами, але всі їх власні підгрупи є MSP-групами.

1. Introduction. In this paper, only finite groups are considered and our notation is standard.

Two subgroups H and K of a group G are said to permute if $HK = KH$. A subgroup H of G is called quasinormal in G if it permutes with every subgroup of G. Kegel [9] called a subgroup H of G S-quasinormal in G if it permutes with every Sylow subgroup of G. Srinivasan [14] studied groups in which all maximal subgroups of the Sylow subgroups are S-quasinormal subgroups and we call such groups MSP-groups.

The study of the structure of groups which have some kind of property has attracted much attention in group theory and many meaningful results about this topic have been obtained. For example, Schmidt [13] determined the structure of minimal non-nilpotent groups, and Doerk [6] [determined the structure of minimal non-supersolvable groups. These achievements have indeed pushed forward the developments of group theory. The further results can consult [2, 12, 15].

In this paper, we call a group G a minimal non-MSP-group if every proper subgroup of G is an MSP-group but G itself is not and the minimal non-MSP-groups are classified completely.

2. Preliminary results. We collect some lemmas which will be frequently used in the sequel.

Lemma 1 ([14], Theorem 2). *If a group G is an MSP-group, then G is supersolvable.*

Lemma 2. *Let G be a minimal non-MSP-group. Then there exists a normal Sylow p-subgroup P of G and a non-normal Sylow q-subgroup Q of G with $p \neq q$ such that $|G| = p^a q^b$ and at least one of a and b is more than 1.*

Proof. Since every proper subgroup of G is an MSP-group, G is supersolvable or minimal non-supersolvable by Lemma 1. So G is solvable.

Let $\{P_1, P_2, \ldots, P_s\}$ is a Sylow basis of G. Since G is a minimal non-MSP-group, there exists i and a maximal subgroup P^* of P_i such that P^* is not S-quasinormal in G.

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If $s \geq 3$, then P_iP_j $(j = 1, 2, \ldots, i-1,\ i+1, \ldots, s)$ is a proper subgroup of G. Since every proper subgroup of G is an MSP-group, it follows that P^* is S-quasinormal in P_iP_j. Applying [3] (Lemma 2.6), P^* is normal in P_iP_j. Therefore, P^* is normal in G, a contradiction. Hence $|G| = p^a q^b$ and at least one of a and b is more than 1.

The following fact follows from [31] (Lemma 2.6) and [16] (Theorem 6).

Lemma 3. *A group G is an MSP-group if and only if $G = H \rtimes \langle x \rangle$ where*:

(i) *H is a normal nilpotent Hall subgroup of G*;

(ii) *every generator of every Sylow subgroup of $\langle x \rangle$ induces a power automorphism of order dividing a prime in $H/\Phi(H)$.*

Lemma 4 ([10], 13.4.3)**.** *Let α be a power automorphism of an abelian group A. If A is a p-group of finite exponent, then there is a positive integer l such that $a^\alpha = a^l$ for all a in A. If α is nontrivial and has order prime to p, then α is fixed-point-free.*

Lemma 5 [8]**.** *Let $\{P_1, P_2, \ldots, P_r\}$ is a Sylow basis of a solvable group G. Then the following statements are equivalent*:

(a) *Every subgroup of P_i permutes with every subgroup of P_j for $i \neq j$.*

(b) *The nilpotent residual N of G is an abelian Hall subgroup of G and every element of G induces a power automorphism in N.*

Lemma 6 ([7], Lemma 2.9)**.** *If a p-group G of order p^{n+1} has a unique non-cyclic maximal subgroup, then G is isomorphic to one of the following groups*:

(I) $C_{p^n} \times C_p = \langle a, b \mid a^{p^n} = b^p = 1,\ [a,b] = 1 \rangle$, *where* $n \geq 2$;

(II) $M_{p^{n+1}} = \langle a, b \mid a^{p^n} = b^p = 1,\ b^{-1}ab = a^{1+p^{n-1}} \rangle$, *where* $n \geq 2$ *and* $n \geq 3$ *if* $p = 2$.

Lemma 7 [6]**.** *Let G be a minimal non-supersolvable group. Then*:

(1) *G has only one normal Sylow p-subgroup P*;

(2) *$P/\Phi(P)$ is a minimal normal subgroup of $G/\Phi(P)$ and $P/\Phi(P)$ is non-cyclic*;

(3) *if $p \neq 2$, then the exponent of P is p*;

(4) *if P is non-abelian and $p = 2$, then the exponent of P is 4*;

(5) *if P is abelian, then the exponent of P is p.*

3. Main results. In this section, we give the complete classification of minimal non-MSP-groups.

Theorem 1. *Let p and q are distinct prime divisors of the order of a group G. Then G is a minimal non-MSP-group if and only if G is of one of the following types*:

(I) $G = \langle x, y \mid x^p = y^{q^n} = 1,\ y^{-1}xy = x^r \rangle$, *where* $r^q \not\equiv 1 \pmod{p}$, $r^{q^2} \equiv$ $\equiv 1 \pmod{p}$, $q \mid p-1$, $n \geq 2$ *and* $0 < r < p$;

(II) *$G = P \rtimes Q$, where $P = \langle a, b \rangle$ is an elementary abelian p-group of order p^2 and $Q = \langle y \rangle$ is cyclic of order q^r; define $a^y = a^i$, $b^y = b^{i^j}$, $p \equiv 1 \pmod{q}$ and $r \geq 1$, where i is the least positive primitive q-th root of unity modulo p, $j = 1 + k$ and $0 < k < q$*;

(III) $G = \langle x, y \mid x^{4p} = 1,\ y^2 = x^{2p},\ y^{-1}xy = x^{-1} \rangle$;

(IV) $G = \langle x, y, z \mid x^p = y^{q^{n-1}} = z^q = 1,\ y^{-1}xy = x^r,\ [x,z] = 1,\ [y,z] = 1 \rangle$, *where* $r \not\equiv 1 \pmod{p}$, $r^q \equiv 1 \pmod{p}$ *and* $n \geq 3$;

(V) $G = \langle x, y, z \mid x^p = y^{q^{n-1}} = z^q = 1,\ y^{-1}xy = x^r,\ [x,z] = 1,\ z^{-1}yz =$ $= y^{1+q^{n-2}} \rangle$, *where* $r \not\equiv 1 \pmod{p}$, $r^q \equiv 1 \pmod{p}$, $n \geq 3$ *and* $n \geq 4$ *if* $q = 2$;

(VI) *$G = P \rtimes Q$, where $Q = \langle y \rangle$ is cyclic of order $q^r > 1$, with $q \nmid p-1$, and P is an irreducible Q-module over the field of p elements with kernel $\langle y^q \rangle$ in Q*;

(VII) *$G = P \rtimes Q$, where P is a non-abelian special p-group of* $\operatorname{rank} 2m$, *the order of p modulo q being $2m$, $Q = \langle y \rangle$ is cyclic of order $q^r > 1$, y induces an automorphism* *in P such that $P/\Phi(P)$ is a faithful and irreducible Q-module, and y centralizes $\Phi(P)$; furthermore, $|P/\Phi(P)| = p^{2m}$ and $|P'| \leq p^m$;*

(VIII) *$G = P \rtimes Q$, where $P = \langle a_0, a_1, \ldots, a_{q-1}\rangle$ is an elementary abelian p-group of order p^q, $Q = \langle y\rangle$ is cyclic of order q^r, q is the highest power of q dividing $p-1$ and $r > 1$. Define $a_j^y = a_{j+i}$ for $0 \leq j < q-1$ and $a_{q-1}^y = a_0^i$, where i is a primitive q-th root of unity modulo p.*

Proof. If G is a minimal non-MSP-group, then we may assume $G = PQ$ with $P \trianglelefteq G$ and $Q \ntrianglelefteq G$ by Lemma 2, where $P \in \mathrm{Syl}_p(G)$ and $Q \in \mathrm{Syl}_q(G)$. Since all the Sylow q-subgroups are conjugate in G, we only consider the case that Q acts on P.

Assume that neither P nor Q is cyclic. For two maximal subgroups Q_1 and Q_2 of Q and every maximal subgroup P_1 of P, since PQ_1 and PQ_2 are MSP-groups, it follows from Lemma 3 that P_1 is normal in not only PQ_1 but also PQ_2. Hence P_1 is normal in G. For two maximal subgroups A and B of P, we have that AQ and BQ are MSP-groups. By Lemma 3, Q is normal in $G = \langle A, B, Q\rangle$, a contradiction. Hence either P or Q is cyclic.

Now we divide into the following four cases to discuss.

(1) Assume that P and Q are cyclic and let $P = \langle x\rangle$ and $Q = \langle y\rangle$ with $|x| = p^m$ and $|y| = q^n$. In this case, $y^{-1}xy = x^r$ with $r^{q^n} \equiv 1 \ (\mathrm{mod}\, p^m)$, $q \mid p-1$, $0 < r < p^m$ and $(p^m, q^n(r-1)) = 1$. Since $\langle y^q\rangle$ is not S-quasinormal in G, we have $(y^q)^{-1}xy^q = x^{r^q} \neq x$. So $r^q \not\equiv 1 \, (\mathrm{mod}\, p^m)$. Since $\langle y^{q^2}\rangle$ is S-quasinormal in $\langle x\rangle\langle y^q\rangle$, it follows from Lemma 3 that $\langle y^{q^2}\rangle$ is normal in $\langle x\rangle\langle y^q\rangle$. So $(y^{q^2})^{-1}xy^{q^2} = x^{r^{q^2}} = x$. Hence $r^{q^2} \equiv 1 \, (\mathrm{mod}\, p^m)$ and y induces a power automorphism of order q^2 in P. Surely, y^q induces a power automorphism of order q in P and every proper subgroup of $\langle y^q\rangle$ is normal in G. If $x^p \neq 1$, then by Lemma 4, $\langle x^p\rangle\langle y^q\rangle \neq \langle x^p\rangle \times \langle y^q\rangle$. Lemma 3 implies that $\langle y^q\rangle$ is normal in $\langle x^p\rangle\langle y\rangle$. Thus, $\langle x^p\rangle\langle y^q\rangle = \langle x^p\rangle \times \langle y^q\rangle$, a contradiction. So G is of type (I).

(2) Assume that G is supersolvable and P is non-cyclic with $d(P) = k$ and $Q = \langle y\rangle$, where $d(P)$ is the rank of P.

We can assume that

$$1 \trianglelefteq \ldots \trianglelefteq R \trianglelefteq P \trianglelefteq \ldots \trianglelefteq G$$

is a chief series of G. By Maschke's Theorem [10] (Theorem 8.1.2), there exists a subgroup N of P such that $P/\Phi(P) = R/\Phi(P) \times N/\Phi(P)$, where $|N/\Phi(P)| = p$ and $N/\Phi(P) \trianglelefteq G/\Phi(P)$. Clearly, $N \trianglelefteq G$, $N \ntrianglelefteq R$ and $1 \trianglelefteq N \trianglelefteq P \trianglelefteq G$ is a normal series of G. Applying Schreier's Refinement Theorem [10] (Theorem 3.1.2), P has a maximal subgroup K such that K is normal in G and $K \neq R$. Therefore, P has at least two maximal subgroups R and K which are normal in G.

Now we prove $k = 2$. If $k \geq 3$, then we can let $P/\Phi(P) = \langle \bar{a}_1\rangle \times \langle \bar{a}_2\rangle \times \ldots \times \langle \bar{a}_k\rangle$, where $a_1, a_2, \ldots, a_{k-1} \in R$, $a_2, a_3, \ldots, a_k \in K$. Since $R\langle y\rangle$ is an MSP-group, it follows from Lemma 3 that $(r\Phi(R))^y = r^l\Phi(R)$ for every $r \in R$, where l is a positive integer. Thus $(r\Phi(P))^y = r^l\Phi(P)$ for every $r \in R$. Similarly, $(k\Phi(P))^y = k^m\Phi(P)$ for every $k \in K$, where m is a positive integer. It follows that $a_2^l\Phi(P) = (a_2\Phi(P))^y = a_2^m\Phi(P)$ and therefore $l \equiv m \ (\mathrm{mod}\, p)$. Hence $(a_i\Phi(P))^y = a_i^l\Phi(P)$ for $i = 1, 2, \ldots, k$. It is easy to see that y induces a power automorphism in $P/\Phi(P)$. By Lemma 3, G is an MSP-group. This contradiction implies $k = 2$.

Now we let $P/\Phi(P) = R/\Phi(P) \times K/\Phi(P) = \langle \bar{a}_1\rangle \times \langle \bar{a}_2\rangle$ where $a_1 \in R$, $a_2 \in K$, $\bar{a}_1^y = \bar{a}_1^{k_1}$ and $\bar{a}_2^y = \bar{a}_2^{k_2}$. If $k_1 = k_2$, then G is an MSP-group by Lemma 3, a contradiction. Hence $k_1 \neq k_2$. Furthermore, we have that P has only two maximal subgroups which are normal in G. Clearly, at least one action of which y acts on R and K is nontrivial. Without loss of generality, we may assume that y induces an automorphism α of order q in R. Since every subgroup of $R\langle y\rangle$ is an MSP-group and by induction, it follows from Lemma 3 that every subgroup of R permutes with every subgroup of $\langle y\rangle$. By Lemma 5, R is abelian and α is a power automorphism of order q in R. By Lemma 4, α is fixed-point-free. So we have either $K\cap R = 1$ if $K\langle y\rangle = K\times\langle y\rangle$ or $K\langle y\rangle \neq K\times\langle y\rangle$. If $K\cap R = 1$ and $K\langle y\rangle = K\times\langle y\rangle$, then P is an elementary abelian group of order p^2. If $K\langle y\rangle \neq K\times\langle y\rangle$, similar arguments as above, K is abelian and y induces a power automorphism of order q in K. Thus, $\Phi(P) = R\cap K \leqslant Z(P)$, $[R, \langle y^q\rangle] = 1$ and $[K, \langle y^q\rangle] = 1$.

Hence $\langle y^q\rangle \trianglelefteq G$. If $|P : Z(P)| = p$, then P is abelian. If $|P : Z(P)| = p^2$, then $\Phi(P) = R\cap K = Z(P)$ and so P is minimal non-abelian. Clearly, all maximal subgroups of G are only RQ^u, KQ^u and $P\langle y^q\rangle$ where $u \in G$. Applying Agrawal's Theorem in [1], G is a minimal non-PST-group. An examination of the list of minimal non-PST-groups in [11] (Theorem 1) , we have that G is only of type (II).

(3) Assume that P is cyclic and Q is non-cyclic. If Q has two non-cyclic maximal subgroups Q_1 and Q_2, then Lemma 3, $PQ_1 = P\times Q_1$ and $PQ_2 = P\times Q_2$. Hence $Q = Q_1Q_2$ is normal in G, a contradiction. Therefore, every maximal subgroup of Q is cyclic or Q has a unique non-cyclic maximal subgroup.

Case 1. Every maximal subgroup of Q is cyclic. It is easy to see that Q is the quaternion group Q_8. Let $Q = \langle a, b \mid a^4 = 1,\ b^2 = a^2,\ b^{-1}ab = a^{-1}\rangle$ and $P = \langle z\rangle$. If $z^p \neq 1$, then $\langle z^p\rangle Q = \langle z^p\rangle\times Q$ by Lemma 3. On the other hand, if $\langle z\rangle\langle a\rangle = \langle z\rangle\times\langle a\rangle$ and $\langle z\rangle\langle b\rangle = \langle z\rangle\times\langle b\rangle$, then G is nilpotent, a contradiction. Therefore, there exists a nontrivial power automorphism that a or b acts on P by conjugation. Without loss of generality, we assume that a acts on P by conjugation is nontrivial. By Lemma 4, $\langle z^p\rangle\langle a\rangle \neq \langle z^p\rangle\times\langle a\rangle$. This contradicts $\langle z^p\rangle Q = \langle z^p\rangle\times Q$. Thus, $z^p = 1$. It follows that $\langle a^2\rangle \leq C_G(P)$ from $P\langle a\rangle$ is an MSP-group. If $C_G(P) = P\times\langle a^2\rangle$, then $G/C_G(P)$ is an elementary abelian group of order 4. However, $G/C_G(P) \lesssim \mathrm{Aut}(P)$ and $\mathrm{Aut}(P)$ is cyclic, a contradiction. So there exists an element of order 4 contained in $C_G(P)$ and $C_G(P) = \langle x\rangle$ is a cyclic group of order $4p$ where x is a generator of $C_G(P)$. Surely, G has an element y of order 4 such that $y \neq x^p$. Now we let $y^{-1}xy = x^r$ where $r \not\equiv 1 \pmod{4p}$. Since $(y^2)^{-1}xy^2 = x^{r^2} = x$, we have $r^2 \equiv 1 \pmod{4p}$. By computations, we have $G = \langle x, y \mid x^{4p} = 1,\ y^2 = x^{2p},\ y^{-1}xy = x^{-1}\rangle$. So G is of type (III).

Case 2. Let $P = \langle x\rangle$ and Q be Lemma 6 (I) with $|Q| = q^n$. Namely, $Q = \langle y, z \mid y^{q^{n-1}} = z^q = 1,\ [y, z] = 1\rangle$ where $n \geq 3$. Then Q has maximal subgroups $H = \langle y\rangle$, $K_0 = \langle y^q, z\rangle$ and $K_s = \langle y^q, zy^s\rangle = \langle zy^s\rangle$ with $s = 1, \ldots, q-1$, where K_0 is the unique non-cyclic maximal subgroup of Q. By Lemma 3, $PK_0 = P\times K_0$ and $PH \neq P\times H$. For an MSP-group PH of G, we have that y induces an automorphism of order q in P. Surely, $z \in Z(G)$. Furthermore, similar arguments in Case 1, we can prove $x^p = 1$. Hence $G = \langle x, y, z \mid x^p = y^{q^{n-1}} = z^q = 1,\ y^{-1}xy = x^r,\ [x, z] = 1,\ [y, z] = 1\rangle$, where $r \not\equiv 1 \pmod{p}$, $r^q \equiv 1 \pmod{p}$ and $n \geq 3$. So G is of type (IV).

Case 3. Let $P = \langle x\rangle$ and Q be Lemma 6 (II) with $|Q| = q^n$. Namely, $Q = \langle y, z \mid y^{q^{n-1}} = z^q = 1,\ z^{-1}yz = y^{1+p^{n-2}}\rangle$ where $n \geq 3$ and $n \geq 4$ if $p = 2$. In the similar way as above, y induces a power automorphism of order q in P and $\langle z\rangle \leq$

$\leq C_G(P)$. Furthermore, we can also prove $x^p = 1$, $y^{-1}xy = x^r$ where $r \not\equiv 1 \pmod p$ and $r^q \equiv 1 \pmod p$. So G is of type (V).

(4) Assume that G is minimal non-supersolvable and $Q = \langle y \rangle$ is cyclic.

Now we prove that: if $Q \leq M \lessdot G$, then $\Phi(P)$ is a Sylow p-subgroup of M.

Denote $M = P_3Q$, where P_3 is a Sylow p-subgroup of M. By $[P_3, Q] \leq P \cap P_3Q = P_3$, we have $N_G(P_3) \geq P_3Q = M$. And since $N_P(P_3) > P_3$, it follows that P_3 is normal in G. By Lemma 7 and the maximality of M, $P_3 = \Phi(P)$ is the Sylow p-subgroup.

Case 1. If G is also a minimal non-nilpotent group, then by [17] (Theorem 2.8), P is non-cyclic. Applying [5] (Theorem 3), G is of either type (VI) or type (VII).

Case 2. If G is not a minimal non-nilpotent group and P is abelian, applying [4] (Theorems 9, 10), we assume that: $G = PQ$, where $P = \langle a_0, a_1, \ldots, a_{q-1} \rangle$ is an elementary abelian p-group of order p^q, $Q = \langle y \rangle$ is cyclic of order q^r, q^f is the highest power of q dividing $p - 1$ and $r > f \geq 1$. Define $a_j^y = a_{j+1}$ for $0 \leq j < q - 1$ and $a_{q-1}^y = a_0^i$, where i is a primitive q^f-th root of unity modulo p.

For an MSP-group $P\langle y^q \rangle$ of G, by Lemma 3, y^q induces a power automorphism of order q on P. Hence $a_0^{i^q} = a_0^{y^{q^2}} = a_0$. Thus $i^q \equiv 1 \pmod p$ and $f = 1$. So G is of type (VIII).

Case 3. Assume that G is not a minimal non-nilpotent group and P is non-abelian. Applying [4] (Theorems 9, 10), we may assume that $G = PQ$ such that $P = \langle a_0, a_1 \rangle$ is an extraspecial group of order p^3 with exponent p, $Q = \langle y \rangle$ is a cyclic group of order 2^r with 2^f the largest power of 2 dividing $p - 1$ and $r > f \geq 1$, and $a_0^y = a_1$ and $a_1^y = a_0^i x$, where $x \in \langle [a_0, a_1] \rangle$ and i is a primitive 2^f-th root of unity modulo p.

Since every subgroup of $P\langle y^2 \rangle$ is an MSP-group, by induction, it follows from Lemma 3 that y^2 induces a power automorphism of order dividing 2 in $\langle a_0 \rangle$. However $a_0^{y^2} = a_1^y = a_0^i x \notin \langle a_0 \rangle$, a contradiction. So G is not of the type as above.

Conversely, it is clear that a group of type (I) is a minimal non-MSP-group.

For type (II), we easily have that P has only two maximal subgroups $\langle a \rangle$ and $\langle b \rangle$ which are normal in G and P has a maximal subgroup $H = \langle ab \rangle$ such that $H^G = P$. So G is a minimal non-MSP-group.

For type (III), it is easy to see that $\langle y \rangle$ is not S-quasinormal in G, so G is not an MSP-group. Since $\langle x^{2p} \rangle = Z(G)$ is the unique subgroup of G of order 2 and G has maximal groups of order $4p$ and 8, it follows that G is a minimal non-MSP-group.

For type (IV), G is not an MSP-group since Q has a maximal subgroup $H = \langle y \rangle$ is not S-quasinormal in G. Similar arguments in Case 2 of (3), G has maximal subgroups PH^u, PK_0^u, PK_s^u and Q^u where $u \in G$, $K_0 = \langle y^q, z \rangle$, $K_s = \langle zy^s \rangle$ with $s = 1, \ldots, q - 1$. Since $\langle y^q \rangle$ is normal in G and $z \in Z(G)$, we have that all proper subgroups of G are MSP-groups. So G is a minimal non-MSP-group.

For type (V), in the similar way as type (IV), we have that G is a minimal non-MSP-group easily.

For types (VI) and (VII), it follows that G is non-supersolvable from [17] (Theorem 2.8). By Lemma 1, G is not an MSP-group. So G is a minimal non-MSP-group.

For type (VIII), by Lemma 1, G is not an MSP-group. Similar arguments as above, we have that G has maximal subgroups $P\langle y^q \rangle^u$ and Q^u where $u \in G$. It is easy to see that G is a minimal non-MSP-group.

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