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GROUPS WITH THE SAME PRIME GRAPH AS THE SIMPLE GROUP $D_n(5)$

ГРУПИ З ТИМ САМИМ ПРОСТИМ ГРАФОМ, ЩО Й ПРОСТА ГРУПА $D_n(5)$

Let G be a finite group. The prime graph of G is denoted by $\Gamma(G)$. Let G be a finite group such that $\Gamma(G) = \Gamma(D_n(5))$, where $n \geq 6$. In the paper, as the main result, we show that if n is odd, then G is recognizable by the prime graph and if n is even, then G is quasirecognizable by the prime graph.

Нехай G — скінченна група. Простий граф групи G позначимо через $\Gamma(G)$. Нехай G — скінченна група така, що $\Gamma(G) = \Gamma(D_n(5))$, де $n \geq 6$. Як основний результат роботи доведено, що для непарних n група G розпізнається простим графом, а для парних n група G є такою, що квазірозпізнається простим графом.

1. Introduction. If n is an integer, then we denote by $\pi(n)$ the set of all prime divisors of n. If G is a finite group, then $\pi(|G|)$ is denoted by $\pi(G)$. The *spectrum* of a finite group G which is denoted by $\omega(G)$ is the set of its element orders. We construct the *prime graph* of G which is denoted by $\Gamma(G)$ as follows: the vertex set is $\pi(G)$ and two distinct primes p and q are joined by an edge (we write $p \sim q$) if and only if G contains an element of order pq. Let $s(G)$ be the number of connected components of $\Gamma(G)$ and let $\pi_i(G)$, $i = 1, \ldots, s(G)$, be the connected components of $\Gamma(G)$. If $2 \in \pi(G)$ we always suppose that $2 \in \pi_1(G)$. In graph theory a subset of vertices of a graph is called an independent set if its vertices are pairwise nonadjacent. Denote by $t(G)$ the maximal number of primes in $\pi(G)$ pairwise nonadjacent in $\Gamma(G)$. In other words, if $\rho(G)$ is some independent set with the maximal number of vertices in $\Gamma(G)$, then $t(G) = |\rho(G)|$. Similarly if $p \in \pi(G)$, then let $\rho(p, G)$ be some independent set with the maximal number of vertices in $\Gamma(G)$ containing p and $t(p, G) = |\rho(p, G)|$.

A finite group G is called *recognizable by prime graph* if $\Gamma(H) = \Gamma(G)$ implies that $H \cong G$. A non-Abelian simple group P is called *quasirecognizable by prime graph* if every finite group whose prime graph is $\Gamma(P)$ has a unique non-Abelian composition factor isomorphic to P (see [11]). Obviously recognition (quasirecognition) by prime graph implies recognition (quasirecognition) by spectrum, but the converse is not true in general. Also some methods of recognition by spectrum cannot be used for recognition by prime graph.

Hagie in [8], determined finite groups G satisfying $\Gamma(G) = \Gamma(S)$, where S is a sporadic simple group. It is proved that if $q = 3^{2n+1}$, $n > 0$, then the simple group ${}^2G_2(q)$ is recognizable by prime graph [11, 32]. A group G is called a CIT group if G is of even order and the centralizer in G of any involution is a 2-group. In [14], finite groups with the same prime graph as a CIT simple group are determined. It is proved that the simple group $F_4(q)$, where $q = 2^n > 2$ (see [12]), and ${}^2F_4(q)$ (see [1]), are quasirecognizable by prime graph. Also in [10], it is proved that if p is a prime number which is not a Mersenne or Fermat prime and $p \neq 11, 13, 19$ and $\Gamma(G) = \Gamma(\mathrm{PGL}(2, p))$, then G has a unique non-Abelian composition factor which is isomorphic to $\mathrm{PSL}(2, p)$ and if $p = 13$, then G has a unique non-Abelian composition factor which is isomorphic to $\mathrm{PSL}(2, 13)$ or $\mathrm{PSL}(2, 27)$. Then it is proved that if p and $k > 1$ are odd and $q = p^k$ is a prime power, then $\mathrm{PGL}(2, q)$ is recognizable by prime graph [2].

In [3], it is proved that if $p = 2^n + 1 \geq 5$ is a prime number, then ${}^2D_p(3)$ is quasirecognizable by prime graph. Then in [4], the authors proved that ${}^2D_{2^m+1}(3)$ is recognizable by prime graph.

Let G be a finite group such that $\Gamma(G) = \Gamma(D_n(5))$, where $n \geq 6$. In this paper as the main result, we show that if n is odd, then G is recognizable by prime graph, and if n is even, then G is quasirecognizable by prime graph.

In this paper, all groups are finite and by simple groups we mean non-Abelian simple groups. All further unexplained notations are standard and refer to [5]. Throughout the proof we use the classification of finite simple groups. In [26] (Tables 2 – 9) independent sets also independent numbers for all simple groups are listed and we use these results in this paper.

2. Preliminary results.

Lemma 2.1 ([28], Theorem 1)**.** *Let G be a finite group with $t(G) \geq 3$ and $t(2, G) \geq 2$. Then the following hold*:

(1) *There exists a finite non-Abelian simple group S such that $S \leq \bar{G} = G/K \leq \mathrm{Aut(S)}$ for the maximal normal soluble subgroup K of G.*

(2) *For every independent subset ρ of $\pi(G)$ with $|\rho| \geq 3$ at most one prime in ρ divides the product $|K||\bar{G}/S|$. In particular, $t(S) \geq t(G) - 1$.*

(3) *One of the following holds*:

(a) *every prime $r \in \pi(G)$ nonadjacent to 2 in $\Gamma(G)$ does not divide the product $|K||\bar{G}/S|$; in particular, $t(2, S) \geq t(2, G)$;*

(b) *there exists a prime $r \in \pi(K)$ nonadjacent to 2 in $\Gamma(G)$; in which case $t(G) = 3$, $t(2, G) = 2$, and $S \cong \mathrm{Alt}_7$ or $L_2(q)$ for some odd q.*

***Remark* 2.1.** In Lemma 2.1, for every odd prime $p \in \pi(S)$, we have $t(p, S) \geq t(p, G) - 1$.

Lemma 2.2 ([22], Lemma 1)**.** *Let N be a normal subgroup of G. Assume that G/N is a Frobenius group with Frobenius kernel F and cyclic Frobenius complement C. If $(|N|, |F|) = 1$, and F is not contained in $NC_G(N)/N$, then $p|C| \in \pi_e(G)$, where p is a prime divisor of $|N|$.*

Lemma 2.3 [9]**.** *Let G be a finite simple group $A_{n-1}(q)$.*

(1) *If there exists a primitive prime divisor r of $q^n - 1$, then G contains a Frobenius subgroup with kernel of order r and cyclic complement of order n.*

(2) *G contains a Frobenius subgroup with kernel of order q^{n-1} and cyclic complement of order $(q^{n-1} - 1)/(n, q - 1)$.*

Lemma 2.4 [9]**.** *Let G be a finite simple group.*

(1) *If $G = C_n(q)$, then G contains a Frobenius subgroup with kernel of order q^n and cyclic complement of order $(q^n - 1)/(2, q - 1)$.*

(2) *If $G = {}^2D_n(q)$, and there exists a primitive prime divisor r of $q^{2n-2} - 1$, then G contains a Frobenius subgroup with kernel of order q^{2n-2} and cyclic complement of order r.*

(3) *If $G = B_n(q)$ or $D_n(q)$, and there exists a primitive prime divisor r_m of $q^m - 1$ where $m = n$ or $n - 1$ such that m is odd, then G contains a Frobenius subgroup with kernel of order $q^{m(m-1)/2}$ and cyclic complement of order r_m.*

Lemma 2.5 [33]**.** *Let p be a prime and let n be a positive integer. Then one of the following holds*:

(i) *there is a primitive prime p' for $p^n - 1$, that is, $p' \mid (p^n - 1)$ but $p' \nmid (p^m - 1)$, for every $1 \leq m < n$ (usually p' is denoted by r_n)*;

(ii) *$p = 2$, $n = 1$ or 6*;

(iii) *p is a Mersenne prime and $n = 2$.*

***Remark* 2.2 [24].** Let p be a prime number and $(q,p)=1$. Let $k\geq 1$ be the smallest positive integer such that $q^k\equiv 1 \pmod p$. Then k is called *the order of q with respect to p* and we denote it by $\mathrm{ord}_p(q)$. Obviously by the Fermat's little theorem it follows that $\mathrm{ord}_p(q)|(p-1)$. Also if $q^n\equiv 1 \pmod p$, then $\mathrm{ord}_p(q)|n$. Similarly if $m>1$ is an integer and $(q,m)=1$, we can define $\mathrm{ord}_m(q)$. If a is odd, then $\mathrm{ord}_a(q)$ is denoted by $e(a,q)$, too. If q is odd, let $e(2,q)=1$ if $q\equiv 1 \pmod 4$ and $e(2,q)=2$ if $q\equiv -1 \pmod 4$.

Lemma 2.6 ([27], Proposition 2.5). *Let $G=D_n^{\varepsilon}(q)$ be a finite simple group of Lie type over a field of characteristic p. Define*

$$\eta(m)=\begin{cases} m & \text{if} \quad m \quad \text{is odd,}\\ m/2 & \text{otherwise.}\end{cases}$$

Let r and s be odd primes and $r,s\in\pi(G)\setminus\{p\}$. Put $k=e(r,q)$ and $l=e(s,q)$, and $1\leq\eta(k)\leq$ $\leq\eta(l)$. Then r and s are nonadjacent if and only if $2\eta(k)+2\eta(l)>2n-(1-\varepsilon(-1)^{k+l})$, and k, l satisfy to:

$$l/k \quad \text{is not an odd natural number,}$$

and if $\varepsilon=+$, then the chain of equalities:

$$n=l=2\eta(l)=2\eta(k)=2k$$

is not true.

3. Main results.

Lemma 3.1. *Let G be a group satisfying the conditions of Lemma 2.1, and let the groups K and S be as in the claim of Lemma 2.1. Let there exist $p\in\pi(K)$ and $p'\in\pi(S)$ such that $p\nsim p'$ in $\Gamma(G)$, and S contains a Frobenius subgroup with kernel F and cyclic complement C such that $(|F|,|K|)=1$. Then $p|C|\in\omega(G)$.*

Proof. We claim that $F\nleq KC_G(K)/K$. Since $KC_G(K)/K\trianglelefteq G/K$, so $S\cap KC_G(K)/K\trianglelefteq S$. Let $S\cap KC_G(K)/K=S$. Then $S\leq KC_G(K)/K$. So for every $t'\in\pi(S)$ and $t\in\pi(K)$ we have $t'\sim t$, which is a contradiction. Consequently $S\cap KC_G(K)/K=1$, since S is a simple group. So $F\nleq KC_G(K)/K$, since $F\leq S$. Therefore $p|C|\in\omega(G)$, by Lemma 2.2.

***Remark* 3.1.** Let $G=D_n(5)$, where $n\geq 14$. Throughout the paper, we denote a primitive prime divisor of 5^i-1 by r_i. By [30] (Tables 1a–1c), we have $s(G)=1$ and $\pi(G)=\pi\Big(5(5^n-$ $-1)\prod_{i=1}^{n-1}(5^{2i}-1)\Big)$. Also by [26] (Tables 6, 8) we know that $\rho(2,D_n(5))\subseteq\{2,r_n,r_{2(n-1)}\}$, $t(D_n(5))\geq[(3n+1)/4]$ and $\rho(D_n(5))\subseteq\left\{r_{2i}\;\middle|\;\left[\dfrac{n+1}{2}\right]\leq i<n\right\}\bigcup\left\{r_i\;\middle|\;\left[\dfrac{n}{2}\right]\leq i\leq n,\ i\equiv 1 \pmod 2\right\}$.

Therefore if $n\geq 14$ and $A=\{r_n,r_{n-2},r_{2(n-1)},r_{2(n-2)},r_{2(n-3)}\}$, then A is an independent set in $\Gamma(D_n(5))$.

***Corollary* 3.1.** *If $G=D_n(5)$, where $n\geq 14$, then $t(257,G)\geq 62$, $t(193,G)\geq 40$, $t(1201,G)\geq$ ≥ 142, $t(14281,G)\geq 80$, $t(1129,G)\geq 65$, $t(157,G)\geq 32$ and $t(19,G)\geq 11$.*

***Table* 1. An upper bound for $t(p', G)$**

(p, p')	$A_{n'}(p^\alpha)$ or ${}^2A_{n'}(p^\alpha)$	$B_{n'}(p^\alpha)$ or $C_{n'}(p^\alpha)$	$D_{n'}(p^\alpha)$ or ${}^2D_n(p^\alpha)$
$(2, 257)$	17	14	15
$(3, 193)$	17	13	15
$(7, 1201)$	9	7	9
$(13, 14281)$	9	7	9
$(31, 1129)$	9	7	9
$(313, 157)$	–	–	6

Proof. We know that $e(193, 5) = 192$ and so if $193 \in \pi(G)$, then $n \geq 96$. By [26] (Table 8), $B = \{r_{2(n-1)}, r_{2(n-2)}, \ldots, r_{2(n-47)}\}$ is an independent set of $\Gamma(G)$, since $(n+1)/2 \leq n - 47$. Therefore $|B| = 48$. If $r_{2i} \in B$, then $n - 47 \leq i \leq n - 1$, therefore $2\eta(2i) + 2\eta(192) > 2n$. Hence $r_{2i} \nsim 193$ in $\Gamma(G)$ if and only if $i/96$ and $96/i$ are not odd natural numbers. Easily we can see that $96/i$ is an odd number if and only if $i = 32$ or $i = 96$. Also 96 divides at most one element of $\{n - 47, \ldots, n\}$. Therefore at least 40 elements of B are not adjacent to 193.

Similarly to above since $e(257, 5) = 256$, $e(1201, 5) = 600$, $e(14281, 5) = 340$, $e(1129, 5) = 282$, $e(157, 5) = 156$ and $e(19, 5) = 9$ we have $t(257, G) \geq 62$, $t(1201, G) \geq 142$, $t(14281, G) \geq 80$, $t(1129, G) \geq 65$, $t(157, G) \geq 32$ and $t(19, G) \geq 11$.

Corollary 3.1 is proved.

Lemma 3.2. *Let G be a finite simple group of Lie type over* $\mathrm{GF}(q)$, *where $q = p^\alpha$. Let p' be a prime divisor of $|G|$. In Table* 1, *we give some upper bounds for $t(p', G)$ for some simple groups G and some prime numbers p'.*

Proof. We determine $t(257, G)$ in each case, whenever $q = 2^\alpha$, and the proof of the other cases are similar. Now we consider each case separately.

Case 1. Let $G = A_{n'-1}(q)$, where $q = 2^\alpha$. We know that $e(257, q) \mid 16$, since $e(257, 2) = 16$. If $e(257, q) = 1$, then 257 is adjacent to each prime divisor of $q^i - 1$, where $i \leq n' - 2$, by [26] (Proposition 4.1), so $t(257, G) \leq 3$. Otherwise since $e(257, q) \mid 16$, then 257 is adjacent to each prime divisor of $q^i - 1$, where $i \leq n' - 16$, by [26] (Proposition 2.1), so $|\rho(257, G) \setminus \{257\}| \leq 16$ and so $t(257, G) \leq 17$.

Case 2. Let $G = {}^2A_{n'-1}(q)$, where $q = 2^\alpha$. If $e(257, q) = 2$, then 257 is adjacent to each prime divisor of $q^i - 1$, where $\nu(i) \leq n' - 2$, by [26] (Proposition 4.2), so $t(257, G) \leq 3$. Otherwise since $e(257, q) \mid 16$, then 257 is adjacent to each prime divisor of $q^i - (-1)^i$, where $\nu(i) \leq n' - 16$, by [26] (Proposition 2.2), so $|\rho(257, G) \setminus \{257\}| \leq 16$ and so $t(257, G) \leq 17$.

Case 3. Let $G = B_{n'}(q)$, where $q = 2^\alpha$. We have $e(257, q) \mid 16$, since $e(257, 2) = 16$. Therefore 257 is adjacent to each prime divisor of $q^i - 1$, where $\eta(i) \leq n' - 8$, by [27] (Proposition 2.4), so $|\rho(257, G) \setminus \{257\}| \leq 13$ and so $t(257, G) \leq 14$.

Case 4. Let $G = D^\varepsilon_{n'}(q)$, where $q = 2^\alpha$. Similarly to the above $e(257, q) \mid 16$. Therefore 257 is adjacent to each prime divisor of $q^i - 1$, where $\eta(i) \leq n' - 9$, by Lemma 2.6, so $|\rho(257, G) \setminus \{257\}| \leq 14$ and so $t(257, G) \leq 15$.

Lemma 3.2 is proved.

Theorem 3.1. *Let G be a finite group such that $\Gamma(G) = \Gamma(D_n(5))$, where $n \geq 6$. If n is odd, then G is recognizable by prime graph and if n is even, then G is quasirecognizable by prime graph.*

Proof. If G is a finite group with $\Gamma(G) = \Gamma(D_n(5))$, where $n \geq 6$ and K is the maximal normal soluble subgroup of G, then Lemma 2.1 implies that G has a unique non-Abelian simple group S such that $S \leq \bar{G} = G/K \leq$Aut$(S)$. In order to prove the main theorem we must show that $S \cong D_n(5)$ and if n is odd, then $K = 1$ and $G \cong D_n(5)$. We will prove these statements in the following lemmas.

Lemma 3.3. *The simple group S is not isomorphic to a sporadic group.*

Proof. By Lemma 2.1, $11 \geq t(S) \geq t(G) - 1$, therefore $n \leq 16$, by [26].

On the other hand, we know that $r_{2(n-1)} \in \pi(S)$. If $n = 16$, then $r_{2(n-1)} = 7621$ divides $|S|$, which is a contradiction. Similarly for $6 \leq n \leq 15$, we get a contradiction.

Lemma 3.3 is proved.

Lemma 3.4. *The simple group S is not isomorphic to an alternating group.*

Proof. Let $S \cong A_{n'}$. We get a contradiction in two steps.

Step 1. Let $n \geq 14$. By assumption $t(G) \geq 10$ and so $t(S) \geq 9$, which implies that $n' \geq 19$. If $x \in \pi(S)$, such that $x \nsim 19$ in $\Gamma(S)$, then $n' - 19 < x \leq n'$, by [26] (Proposition 1.1). Also there are $[20/2] + [20/3] - [20/6] = 13$ elements of $[n' - 19, n']$ which are divisible by 2 or 3. Therefore $t(19, S) \leq 8$, which is a contradiction by Corollary 3.1.

Step 2. Let $6 \leq n \leq 13$. If $n = 13$, then $r_{2(n-1)} = 390001$ divides $|S|$. So $n' \geq 390001$, therefore $37 \in \pi(S)$, which is a contradiction, since $37 \notin \pi(D_{13}(5))$. Similarly for $6 \leq n \leq 12$, we get a contradiction.

Lemma 3.4 is proved.

Let G be a finite simple group of Lie type over GF(q), where $q = p^\alpha$. In the sequel we denote a primitive prime divisor of $q^i - 1$ by r'_i.

Lemma 3.5. *The simple group S is not isomorphic to a finite simple group of Lie type over a field of characteristic p, where $p \neq 5$.*

Proof. Let S be isomorphic to a finite simple group of Lie type over a field of characteristic p, where $p \neq 5$. We get a contradiction in two steps.

Step 1. Let $n \geq 14$. By Lemma 2.1, $t(S) \geq t(G) - 1$ so $t(S) \geq 9$. In the sequel we consider each possibility for S, by [30] (Tables 1a–1c).

We denote by $A^+_{n'}(q)$ the simple group $A_{n'}(q)$, and by $A^-_{n'}(q)$ the simple group ${}^2A_{n'}(q)$.

Case 1. Let $S \cong A^\varepsilon_{n'-1}(q)$, where $q = p^\alpha$. By Lemma 2.1, $t(S) \geq t(G) - 1$ so

$$2n' > 3n - 9. \tag{1}$$

We know that $A = \{r_n, r_{n-2}, r_{2(n-1)}, r_{2(n-2)}, r_{2(n-3)}\}$ is an independent set in $\Gamma(G)$, by Remark 3.1.

If $S \cong A_{n'-1}(q)$, then by [26] (Propositions 3.1, 4.1), each r'_i, where $i \notin \{n'-1, n'\}$, is adjacent to 2 and p in $\Gamma(S)$.

If $S \cong {}^2A_{n'-1}(q)$, then by [26] (Propositions 3.1, 4.2), every r'_i, where $\nu(i) \notin \{n' - 1, n'\}$, is adjacent to 2 and p in $\Gamma(S)$.

On the other hand, by Lemma 2.1, $|A \cap \pi(S)| \geq 4$, therefore p is adjacent to at least two elements of $A \cap \pi(S)$ in $\Gamma(S)$. For example, let $r_{2(n-3)} \sim p \sim r_{2(n-2)}$ in $\Gamma(S)$. Therefore $r_{2(n-3)} \sim p \sim$ $\sim r_{2(n-2)}$ in $\Gamma(G)$. Let $a = e(p, 5)$. Since $p \sim r_{2(n-2)}$ it follows that $2(n-2) + 2\eta(a) \leq 2n$ or $2(n-2)/a$ is odd, by Lemma 2.6. Similarly since $p \sim r_{2(n-3)}$ it follows that $2(n-3) + 2\eta(a) \leq 2n$ or $2(n-3)/a$ is odd. So $\eta(a) \leq 3$, which implies that $a \in \{1, 2, 3, 4, 6\}$ and so $p \in \{2, 3, 7, 13, 31\}$. Similarly to the above for every $r_i, r_j \in \{r_n, r_{n-2}, r_{2(n-1)}, r_{2(n-2)}, r_{2(n-3)}\}$, if $r_i \sim p \sim r_j$, then $p \in \{2, 3, 7, 13, 31\}$.

Let $S \cong A_{n'-1}(q)$. If $p = 2$, then since $n' \geq 16$ by (1) and $e(257, 2^\alpha) \mid 16$ it follows that $257 \in \pi(S)$. Hence by Lemma 3.2, $t(257, S) \leq 17$ and by Corollary 3.1, $t(257, G) \geq 62$. Therefore by Remark 2.1, we get a contradiction. Similarly for every $p \in \{3, 7, 13, 31\}$, we get a contradiction.

Let $S \cong {}^2A_{n'-1}(q)$. If $p = 3$, then since $n' \geq 16$ by (1) and $e(193, 3^\alpha) \mid 16$ it follows that $193 \in \pi(S)$. Therefore by Lemma 3.2, $t(193, S) \leq 17$ and by Corollary 3.1, $t(193, G) \geq 40$. Therefore by Remark 2.1, we get a contradiction. Similarly for every $p \in \{2, 7, 13, 31\}$, we get a contradiction.

Case 2. Let $S \cong B_{n'}(q)$ or $C_{n'}(q)$, where $q = p^\alpha$. By Lemma 2.1, $t(S) \geq t(G) - 1$, so

$$3n' > 3n - 12. \tag{2}$$

We know that by [26] (Propositions 3.1, 4.3), every r'_i is adjacent to 2 and p in $\Gamma(S)$, where $\eta(i) \neq$ $\neq n'$. On the other hand, by Lemma 2.1, $|A \cap \pi(S)| \geq 4$. Therefore at least two elements of $A \cap \pi(S)$ are adjacent to p in $\Gamma(S)$. Denote $e(p, 5)$ by a. Similarly to the above case, we get that $p \in \{2, 3, 7, 13, 31\}$.

If $p = 7$, then since $n' \geq 11$ and $e(1201, 7^\alpha) \mid 8$ it follows that $1201 \in \pi(S)$. Hence by Lemma 3.2, $t(1201, S) \leq 7$ and by Corollary 3.1, $t(1201, G) \geq 142$. Therefore by Remark 2.1, we get a contradiction. Similarly for every $p \in \{2, 3, 13, 31\}$, we get a contradiction.

We denote by $D^+_{n'}(q)$ the simple group $D_{n'}(q)$, and by $D^-_{n'}(q)$ the simple group ${}^2D_{n'}(q)$.

Case 3. Let $S \cong D^\varepsilon_{n'}(q)$, where $q = p^\alpha$. By Lemma 2.1, $t(S) \geq t(G) - 1$, so

$$3n' > 3n - 11. \tag{3}$$

Let $B = A \cup \{r_{2(n-4)}\}$. Since $n \geq 14$, then by Remark 3.1, B is an independent set in $\Gamma(G)$. We know that every r'_i, where $\eta(i) \notin \{n', n'-1\}$ is adjacent to 2 and p in $\Gamma(S)$, by [26] (Propositions 3.1, 4.4). On the other hand, by Lemma 2.1, $|B \cap \pi(S)| \geq 5$, therefore at least two elements of $B \cap \pi(S)$ are adjacent to p in $\Gamma(S)$. Let $a = e(p, 5)$. Similarly to the above case, we have $p \in \{2, 3, 7, 13, 31, 313\}$.

If $p = 13$, then since $n' \geq 10$ and $e(14281, 13^\alpha) \mid 8$ it follows that $14281 \in \pi(S)$. Hence by Lemma 3.2, $t(14281, S) \leq 9$ and by Corollary 3.1, $t(14281, G) \geq 80$. Therefore by Remark 2.1, we get a contradiction. Similarly for every $p \in \{2, 3, 7, 31, 313\}$, we get a contradiction.

Case 4. Let $S \cong E_8(q)$, where $q = p^\alpha$. We know that $t(S) = 12$. If $n \geq 19$, then $t(G) \geq 14$, which is a contradiction, by Lemma 2.1. Therefore $n \leq 18$. We know that $p \in \pi(S)$, therefore $p \in \pi(G)$.

Let $n = 18$. For every $p \in \pi(G) \setminus \{5\}$, easily we can see that $\pi(p^{30} - 1) \nsubseteq \pi(D_{18}(5))$ and we get a contradiction. Similarly for each n and each $p \in \pi(G)$, easily we can see that $\pi(S) \nsubseteq \pi(D_n(5))$.

Step 2. Let $6 \leq n \leq 13$. If $S \cong E_8(q)$, where $q = p^\alpha$, then we have $19 \in \pi(S)$, so $9 \leq n \leq 13$. Similarly to Case 4 of Step 1, we get a contradiction.

Let $S \cong B_{n'}(q)$, $C_{n'}(q)$ or $D_{n'}(q)$, where $q = p^\alpha$.

We know that $\pi(S) \subseteq \pi(G)$. Therefore $p \in \pi(G)$. On the other hand, since $n \geq 6$, by (2) and (3), it follows that $n' \geq 3$. Therefore

$$\pi(p^3 - 1) \subseteq \pi(p^{3\alpha} - 1) = \pi(q^3 - 1) \subseteq \pi(S) \subseteq \pi(G).$$

Let $p = 13$, then $61 \in \pi(D_n(5))$, where $6 \leq n \leq 13$, which is a contradiction. Similarly for each $p \in \bigcup_{n=6}^{13} \pi(D_n(5)) \setminus \{2, 3, 7, 11, 29, 67\}$ we get a contradiction. We know that $r_{2(n-1)} \in \rho(2, S)$. Hence there is $r'_i \in \pi(S)$, such that $r_{2(n-1)} = r'_i$.

If $n = 6$, then $r_{2(n-1)} = 521$. Let $p = 2$, we have $e(521, 2) = 260$. So $260 \mid \alpha i$, and $(2^{260} - 1) \mid (2^{\alpha i} - 1)$. Consequently $\pi(2^{260} - 1) \subseteq \pi(S)$, which is a contradiction. Let $p = 3$. Since $e(521, 3) = 520$, then $\pi(3^{520} - 1) \subseteq \pi(S)$, which is a contradiction. Similarly for other cases of n and p, we get a contradiction.

Therefore let $S \cong A^{\varepsilon}_{n'-1}(q)$ or ${}^2D_{n'}(q)$ where $q = p^{\alpha}$. Similarly to the above we have $p \in \pi(G)$. If $n' < 4$, then $S \cong {}^2D_3(q)$ so $G = D_6(5)$. We have $r_{10} = 521 \in \pi(S)$. If $p = 521$, then $\pi(521^2 - 1) \subseteq \pi(S) \subseteq \pi(G)$, which is a contradiction. Let $p = 7$, since $e(521, 7) = 520$, therefore similarly to the above case we get a contradiction. Similarly for other cases of p, we get a contradiction.

Therefore $n' \geq 4$ and consequently

$$\pi(p^4 - 1) \subseteq \pi(p^{4\alpha} - 1) = \pi(q^4 - 1) \subseteq \pi(S) \subseteq \pi(G).$$

Let $p = 13$, then $17 \in \pi(G)$, which is a contradiction. Similarly for each $p \in \bigcup_{n=6}^{13} \pi(D_n(5)) \setminus \{2, 3, 7, 41, 67\}$ we get a contradiction. Similarly to the above for $p \in \{2, 3, 7, 41, 67\}$, we get a contradiction.

Lemma 3.5 is proved.

Lemma 3.6. *If S is isomorphic to a finite simple group of Lie type in characteristic* 5, *then* $S \cong D_n(5)$.

Proof. Throughout the proof, since $t(S) \geq 3$, using [26] (Tables 8, 9), we consider the following possibilities. We show that $S \cong D_n(5)$ in two steps.

Step 1. Let S be isomorphic to a finite simple exceptional group of Lie type.

Case 1. Let $S \cong E_8(5^{\alpha})$. By Lemma 2.1, $t(S) \geq t(G) - 1$ so $n \leq 18$. On the other hand, we have $\pi(S) \subseteq \pi(G)$, therefore $30\alpha \leq 2(n-1)$. Also we know that $r_{2(n-1)} \in \rho(2, S) = \{r'_{15}, r'_{20}, r'_{24}, r'_{30}\}$, so we consider the following cases:

Let $r_{2(n-1)} = r'_{15}$. Let p_0 be a primitive prime divisor of $5^{2(n-1)} - 1$. Since $r_{2(n-1)} = r'_{15}$, it follows that $p_0 \mid (5^{15\alpha} - 1)$. Therefore $2(n-1) \leq 15\alpha$, which is a contradiction. Similarly, when $r_{2(n-1)} = r'_{20}$ and $r_{2(n-1)} = r'_{24}$, we get a contradiction.

Let $r_{2(n-1)} = r'_{30}$. Similarly to the above $2(n-1) \leq 30\alpha$. Consequently, $n - 1 = 15\alpha$ and since $15 \mid (n-1)$ and $n \leq 18$ we have $n = 16$ and $\alpha = 1$. Therefore $S \cong E_8(5)$. We know that $r_{13} \in \pi(G)$ and $r_{13} \notin \pi(S)$. So $r_{13} \in \pi(\bar{G}/S) \cup \pi(K)$. Therefore $r_{13} \in \pi(K)$, since $\mathrm{Out}(S) = 1$. Using [25], we have $D_8(5) \leq E_8(5)$ and $D_8(5)$ contains a Frobenius subgroup $5^{21} : r_7$. Since then $r_{30} \nsim r_{13}$ by Lemma 3.1, we have $r_{13} \sim r_7$ in $\Gamma(G)$, which is a contradiction, by Lemma 2.6.

Case 2. Let $S \cong E_7(5^{\alpha})$. By [26], $t(S) = 8$ and consequently $t(G) \leq 9$, by Lemma 2.1. Therefore $n \leq 12$. Since $\pi(S) \subseteq \pi(G)$, then $18\alpha \leq 2(n-1)$. On the other hand, we know that $r_{2(n-1)} \in \rho(2, S) = \{r'_{14}, r'_{18}\}$. Now we consider the following cases:

Let $r_{2(n-1)} = r'_{14}$. Let p_0 be a primitive prime divisor of $5^{2(n-1)} - 1$. Similarly to the above case $2(n-1) \leq 14\alpha$, which is a contradiction.

Let $r_{2(n-1)} = r'_{18}$. Similarly to the above $2(n-1) \leq 18\alpha$. Consequently $n - 1 = 9\alpha$. Therefore $n = 10$, $\alpha = 1$ and $S \cong E_7(5)$. We know that $r_{14} \sim r_4$ in $\Gamma(G)$, by Lemma 2.6, but $r_{14} \nsim r_4$ in $\Gamma(S)$, by [26] (Proposition 2.5). Therefore r_4 or $r_{14} \in \pi(\bar{G}/S) \cup \pi(K)$. On the other hand, we know that $r_{16} \notin \pi(S)$. Since $\{r_4, r_{14}, r_{16}\}$ is an independent set, we get a contradiction, by Lemma 2.1.

Similarly to the above discussion it follows that S is not isomorphic to $E_6(5^{\alpha})$, ${}^2E_6(5^{\alpha})$ and $F_4(5^{\alpha})$.

Step 2. Let S be isomorphic to a finite simple classical group of Lie type.

Case 1. Let $S \cong A_{n'-1}(5^\alpha)$. We know that $\pi(S) \subseteq \pi(G)$, therefore $n'\alpha \leq 2(n-1)$. Also we know that $r_{2(n-1)} \in \rho(2, S) = \{r'_{n'}, r'_{n'-1}\}$, so we consider the following cases:

Let $r_{2(n-1)} = r'_{n'-1}$. Let p_0 be a primitive prime divisor of $5^{2(n-1)} - 1$. Since $r_{2(n-1)} = r'_{n'-1}$, it follows that $p_0 \mid (5^{(n'-1)\alpha} - 1)$. Therefore $2(n-1) \leq (n'-1)\alpha$, which is a contradiction.

Let $r_{2(n-1)} = r'_{n'}$. Similarly to the above $2(n-1) \leq n'\alpha$. Consequently $2(n-1) = n'\alpha$. If $\alpha = 1$, then $r'_{n'-1} = r_{2n-3} \in \pi(S) \subseteq \pi(G)$, which is a contradiction. Therefore $\alpha \geq 2$ so by (1), we have $2(n-1) = n'\alpha \geq 2n' > 3n - 9$, which implies that $n = 6$. Hence $n' = 5, \alpha = 2$ or $n' = 2, \alpha = 5$. If $n' = 2$, then we get a contradiction by (1). So $S \cong A_4(5^2)$. Then $r_5, r_{10} \in \pi(S)$ and r_5 and r_{10} are primitive prime divisors of $(5^2)^5 - 1$ and so $r_5 \sim r_{10}$ in $\Gamma(S)$, but $r_5 \nsim r_{10}$ in $\Gamma(G)$, which is a contradiction.

Case 2. Let $S \cong {}^2A_{n'-1}(5^\alpha)$. We know that $\pi(S) \subseteq \pi(G)$. Also we know that $r_{2(n-1)} \in \in \rho(2, S)$, so $\nu(e(r_{2(n-1)}, 5^\alpha)) \in \{n', n'-1\}$. Now we consider the following cases:

If n' is odd, then $2n'\alpha \leq 2(n-1)$, since $\pi(S) \subseteq \pi(G)$. By (1), $n = 6$ hence $S \cong {}^2A_4(5)$. We know that $r_5, r_8 \notin \pi(S)$. Therefore $r_5, r_8 \in \pi(\bar{G}/S) \cup \pi(K)$. We know that $\{r_5, r_8, r_{10}\}$ is an independent set, which is a contradiction, by Lemma 2.1.

If n' is even, then $2(n'-1)\alpha \leq 2(n-1)$. We know that $(5^\alpha + 1)_2 = 2$ so by [26] (Table 6), we have $r_{2(n-1)} \in \{r'_{2(n'-1)}, r'_{n'/2}\}$. If $r_{2(n-1)} = r'_{n'/2}$, then similarly to the above $2(n-1) \leq (n'/2)\alpha$, which is a contradiction. Therefore $r_{2(n-1)} = r'_{2(n'-1)}$ so similarly to the above we have $2(n-1) \leq \leq 2(n'-1)\alpha$. Hence by (1), $n \leq 8$. Let $n = 8$, therefore $n' = 8$ and $S \cong {}^2A_7(5)$. We know that $r_5, r_7 \in \pi(G)$ and $r_5, r_7 \notin \pi(S)$. So $r_5, r_7 \in \pi(\bar{G}/S) \cup \pi(K)$. Since $\{r_5, r_7, r_{14}\}$ is an independent set , we get a contradiction, by Lemma 2.1. Similarly $n \neq 6, 7$.

Case 3. Let $S \cong B_{n'}(5^\alpha)$, $C_{n'}(5^\alpha)$ or ${}^2D_{n'}(5^\alpha)$. We have $\pi(S) \subseteq \pi(G)$ so $2n'\alpha \leq 2(n-1)$. Also we know that $r_{2(n-1)} \in \rho(2, S) = \{r'_{2n'}\}$. Similarly to the above we have $2(n-1) \leq 2n'\alpha$, hence $n - 1 = n'\alpha$. Now by (2), $n' > n - 4$, therefore $n'(\alpha - 1) < 3$. If $\alpha = 2, n' = 2$, then $n = 5$, which is a contradiction, since $n \geq 6$. Therefore $\alpha = 1$. Since $n \geq 6$, so $n' \geq 5$. We know that if n is an odd number, then $t(2, D_n(5)) = 3$, which is a contradiction, since $t(2, S) = 2$. Therefore n is even and n' is odd.

Let $S \cong {}^2D_{n-1}(5)$. We have $r_{n-1} \in \pi(G)$ and $r_{n-1} \notin \pi(S)$, therefore $r_{n-1} \in \pi(\bar{G}/S) \cup \pi(K)$. Since $\pi(\mathrm{Out}(S)) = \{2\}$, so $r_{n-1} \in \pi(K)$. By Lemma 2.4, ${}^2D_{n-1}(5)$ contains a Frobenius subgroup of the form $5^{2(n-2)} : r_{2(n-2)}$. We know that $r_{2(n-1)} \in \pi(S)$ and $r_{2(n-1)} \nsim r_{n-1}$ in $\Gamma(G)$. Therefore by Lemma 3.1, $r_{n-1} \sim r_{2(n-2)}$ in $\Gamma(G)$, which is a contradiction.

Let $S \cong B_{n-1}(5)$. We consider two following cases:

1. Let $4 \mid n$. We have $r_{(n-2)/2} \nsim r_{(n+2)/2}$ in $\Gamma(S)$ but $r_{(n-2)/2} \sim r_{(n+2)/2}$ in $\Gamma(G)$. Therefore $r_{(n-2)/2}$ or $r_{(n+2)/2} \in \pi(\bar{G}/S) \cup \pi(K)$. Since $\pi(\mathrm{Out}(S)) = \{2\}$, so $r_{(n-2)/2}$ or $r_{(n+2)/2} \in \pi(K)$. By Lemma 2.4, $B_{n-1}(5)$ contains a Frobenius subgroup of the form $5^{(n-1)(n-2)/2} : r_{n-1}$. We know that $r_{2(n-1)} \in \pi(S)$ and $r_{2(n-1)} \nsim r_{(n-2)/2}, r_{(n+2)/2}$ in $\Gamma(G)$. Therefore by Lemma 3.1, $r_{n-1} \sim r_{(n-2)/2}$ or $r_{n-1} \sim r_{(n+2)/2}$ in $\Gamma(G)$, which is a contradiction.

2. Let $4 \mid (n+2)$. In this case by $r_{(n-4)/2}, r_{(n+4)/2}$ similarly to the above case we get a contradiction.

Similarly for $S \cong C_{n-1}(5)$ we get a contradiction.

Case 4. Let $S \cong D_{n'}(5^\alpha)$. Similarly to the above, we have $2(n'-1)\alpha \leq 2(n-1)$, since $\pi(S) \subseteq \pi(G)$. Also we know that $r_{2(n-1)} \in \rho(2, S) = \{r'_{n'}, r'_{2(n'-1)}\}$. So we consider the following cases:

Let $r_{2(n-1)} = r'_{n'}$. Let p_0 be a primitive prime divisor of $5^{2(n-1)} - 1$. Since $r_{2(n-1)} = r'_{n'}$, it follows that $p_0 \mid (5^{n'\alpha} - 1)$. Therefore $2(n-1) \leq n'\alpha$, which is a contradiction.

Let $r_{2(n-1)} = r'_{2(n'-1)}$. Similarly to the above $2(n-1) \leq 2(n'-1)\alpha$. Consequently $n - 1 = (n'-1)\alpha$. By (3), $n' > n - 3$. If $\alpha \geq 2$, then $n - 1 = (n'-1)\alpha > (n-4)\alpha \geq 2(n-4)$. So $n = 6$ and $n' = 2$, which is a contradiction. Therefore $\alpha = 1$ and $n = n'$, so $S \cong D_n(5)$.

Lemma 3.6 is proved.

Theorem 3.2. *If* $\Gamma(G) = \Gamma(D_n(5))$, *where* $n \geq 6$, *then* $D_n(5) \leq G/K \leq \mathrm{Aut}(D_n(5))$, *where* $K = 1$ *when* n *is odd and* K *is a 2-group when* n *is even.*

Proof. By the above lemmas, it follows that $D_n(5) \leq G/K \leq \mathrm{Aut}(D_n(5))$, where K is the maximal normal soluble subgroup of G. We can assume that K is an elementary Abelian p-group by [16]. Since by [7], we know that $D_n(5)$ acts unisingularly we conclude that $p \neq 5$.

Let n be odd. We claim that for each element $t \in \pi(D_n(5))$, we have $t \nsim r_n$ or $t \nsim r_{2(n-1)}$. Let $e(t,5) = a$. If $t \sim r_n$ and $t \sim r_{2(n-1)}$, then by Lemma 2.6, n/a is odd and $2(n-1) + 2\eta(a) \leq 2n - 2$, since a is odd, which is a contradiction.

We know that $D_n(5)$ contains a Frobenius subgroup with kernel of order $5^{n(n-1)/2}$ and cyclic complement of order r_n, by Lemma 2.4. Also $D_n(5) \leq G/K$, and so G/K contains a Frobenius subgroup T/K of the form $5^{n(n-1)/2} : r_n$. By the above discussion, we know that $p \nsim r_n$ or $p \nsim r_{2(n-1)}$ in $\Gamma(D_n(5))$. Since $p \neq 5$, it follows that $p \sim r_n$, by Lemma 3.1. Also we know that $B_{n-1}(5) \leq D_n(5)$, by [25], and so $B_{n-1}(5) \leq G/K$. Similarly G/K contains a Frobenius subgroup of the form $5^{(n-2)(n-3)/2} : r_{n-2}$, by Lemma 2.4. Since $p \neq 5$ and $p \nsim r_n$ or $p \nsim r_{2(n-1)}$ it follows that $p \sim r_{n-2}$, by Lemma 3.1. Let $e(p,5) = m$. Since $p \sim r_n$ it follows that n/m is odd, by Lemma 2.6. Therefore m is odd. Similarly since $p \sim r_{n-2}$ it follows that $2(n-2) + 2\eta(m) \leq 2n$ or $(n-2)/m$ is odd. Consequently, $m = 1$ and so $p = 2$. So $2 = p \sim r_n$, which is a contradiction. Therefore $K = 1$.

Let n be even. We claim that for each element $t \in \pi(D_n(5))$, we have $t \nsim r_{n-1}$ or $t \nsim r_{2(n-1)}$. Let $e(t,5) = a$. If $t \sim r_{n-1}$ and $t \sim r_{2(n-1)}$, then by Lemma 2.6, $2(n-1) + 2\eta(a) \leq 2n - (1 - (-1)^{n-1+a})$ or $(n-1)/a$ is odd and $2(n-1) + 2\eta(a) \leq 2n - (1 - (-1)^{2(n-1)+a})$ or $2(n-1)/a$ is odd, which is a contradiction.

Similarly to the above we know that G/K contains a Frobenius subgroup of the form $5^{(n-1)(n-2)/2} : r_{n-1}$, by Lemma 2.4. Now since $p \neq 5$ and by the above discussion we have $p \nsim r_{2(n-1)}$ or $p \nsim r_{n-1}$ so by Lemma 3.1, we conclude that $p \sim r_{n-1}$. Also we know that ${}^2D_{n-1}(5) \leq D_n(5)$, by [25]. Similarly G/K contains a Frobenius subgroup of the form $5^{2(n-2)} : r_{2(n-2)}$, by Lemma 2.4. Similarly $p \sim r_{2(n-2)}$, by Lemma 3.1. Let $e(p,5) = m$. Since $p \sim r_{n-1}$, it follows that $2(n-1) + 2\eta(m) \leq 2n - (1 - (-1)^{m+n-1})$ or $(n-1)/m$ is odd, by Lemma 2.6. Similarly since $p \sim r_{2(n-2)}$ it follows that $2(n-2) + 2\eta(m) \leq 2n - (1 - (-1)^{m+2(n-2)})$ or $2(n-2)/m$ is odd. Consequently, $m = 1$, so $p = 2$. Therefore K is a 2-group.

Theorem 3.2 is proved.

Theorem 3.3. *Let* $n \geq 6$ *be odd, if* $D_n(5) \leq G \leq \mathrm{Aut}(D_n(5))$ *and* $\Gamma(G) = \Gamma(D_n(5))$, *then* $G \cong D_n(5)$.

Proof. Suppose that $G \ncong D_n(5)$. We know that $\mathrm{Out}(D_n(5)) = \gamma\delta$, where γ is the graph automorphism of order 2 and δ is the diagonal automorphism of order 4. Consequently we consider the following cases:

1. Let $G \cong D_n(5)\langle\gamma\rangle$. Consider the centralizer $C_{D_n(5)}(\gamma)$, we have $\pi(C_{D_n(5)}(\gamma)) = \pi(B_{n-1}(5))$, by [21]. Therefore $2 \sim r_{2(n-1)}$, which is a contradiction.

2. Let $G \cong D_n(5)\langle\delta\rangle$. So if $\hat{T}$ is a maximal torus of G, then $\hat{T}$ has order $|T||\delta|$, where T is a torus of $D_n(5)$, by [21]. Let $|T| = (5^{n-1}+1)(5+1)/4$. Therefore $2 \sim r_{2(n-1)}$, which is a contradiction.

Similarly for $G \cong D_n(5)\langle\gamma\delta\rangle$ we get a contradiction.

Consequently $G \cong D_n(5)$ and $D_n(5)$ is recognizable by prime graph.

Theorem 3.3 is proved.

Similarly we can prove the following theorem.

Theorem 3.4. *Let $n \geq 6$ be even, if $D_n(5) \leq G/O_2(G) \leq \mathrm{Aut}(D_n(5))$ and $\Gamma(G) = \Gamma(D_n(5))$, then $G \cong D_n(5)/O_2(G)$.*

***Corollary* 3.2.** *Let G be a finite group satisfying $|G| = |D_n(q)|$, where $n \geq 6$. If $\omega(G) = \omega(D_n(q))$, then $G \cong D_n(q)$.*

We note that recently this theorem is proved for each finite simple group (see [31]).

***Corollary* 3.3.** *If $n \geq 6$ is even, then $D_n(q)$ is quasirecognizable by spectrum, i.e., if G is a finite group such that $\omega(G) = \omega(D_n(q))$, then G has a unique non-Abelian composition factor isomorphic to $D_n(q)$.*

If $n \geq 6$ is odd, then $D_n(q)$ is recognizable by spectrum, i.e., if G is a finite group such that $\omega(G) = \omega(D_n(q))$, then $G \cong D_n(q)$.

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