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# ADMISSIBILITY OF ESTIMATED REGRESSION COEFFICIENTS UNDER GENERALIZED BALANCED LOSS *

# ДОПУСТИМІСТЬ ОЦІНЕНИХ КОЕФІЦІЄНТІВ РЕГРЕСІЇ ПРИ УЗАГАЛЬНЕНІЙ ЗБАЛАНСОВАНІЙ ВТРАТІ

There are some discussions about the admissibility of estimated regression coefficients under the balanced loss function in the general linear model. This paper studies this issue for the generalized linear regression model. First, we propose the generalized weighted balance loss function for the generalized linear model. For the proposed loss function, the paper investigates sufficient and necessary conditions for the admissibility of the estimated regression coefficients in two interesting linear estimation classes.

Ведуться деякі дискусії щодо допустимості оцінених коефіцієнтів регресії для збалансованої функції втрат у загальній лінійній моделі. В роботі вивчається ця проблема для узагальненої лінійної моделі регресії. Так, запропоновано узагальнену зважену функцію втрати балансу для узагальненої лінійної моделі. Для вказаної функції втрат ми вивчаємо необхідні та достатні умови допустимості оцінених коефіцієнтів регресії для двох цікавих лінійних випадків оцінювання.

**1. Introduction.** The general linear regression model is

$$y = X\beta + e, \quad \mathrm{E}(e) = 0, \quad \mathrm{Cov}(e) = \sigma^2 I_n, \tag{1.1}$$

where $y \in R^n$ is an observable random vector, $X \in R^{n\times p}$ is a known design matrix with full-column rank, $\beta \in R^p$ and $\sigma^2 > 0$ are unknown parameters, and $e$ is a random error vector. Let $R^n$ and $R^{n\times m}$ stand for the sets of $n$-dimensional column vectors and real $(n \times m)$-matrices, respectively.

Considering goodness of fit, the estimating equation is

$$(y - X\hat{\beta})'(y - X\hat{\beta}) = \min_d (y - Xd)'(y - Xd), \tag{1.2}$$

where $d$ is an any estimator of the regression coefficients $\beta$ in the linear model (1.1). The solution of (1.2), $\hat{\beta} = (X'X)^{-1}X'y$, is called the Least Squares (LS) estimator of regression coefficients $\beta$. The expression of $(y - Xd)'(y - Xd)$ measures a kind of fitting goodness of estimation model. However, statisticians also measure superiorities of parameter estimation with respect to its precision. One of the popular loss functions is the quadratic loss

$$(d - \beta)'(d - \beta), \tag{1.3}$$

which directly depicts the precision of parameter estimation $d$.

Considering both of the above criteria, Zellner [16] proposed the concept of balanced loss function

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$$L_B(d;\beta,\sigma^2|S) = w(y - Xd)'(y - Xd) + (1 - w)(d - \beta)'S(d - \beta), \tag{1.4}$$

where $w \in [0, 1]$ is a real number and $S \in R^{p\times p}$ is some known positive definite matrix. It's obvious that the balanced loss function (1.4) can reflect not only statistical properties of estimator itself, but also goodness of fit, which is more comprehensive when evaluating superiority of estimation, compared with loss function (1.2) or (1.3). Under the balanced loss function (1.4), there has been a number of studies about estimators for regression coefficients $\beta$. In [3], Dey et al. considered estimators $\delta(y) = \hat{\theta} + g(y)$, where $\hat{\theta}$ is the LS estimator, and $y$ is in the action space, under the quadratic loss and the balanced loss. This kind of studies are also can be seen in [10, 11]. In order to estimate the exponential mean time to failure, the authors [13] introduced the weighted balanced loss function, defined as

$$L_{WB}(d;\beta,\sigma^2|S) = wq(\beta)(y - Xd)'(y - Xd) + (1 - w)q(\beta)(d - \beta)'S(d - \beta), \tag{1.5}$$

where $q(\beta)$ is any positive function of $\beta$, called the weight function. The related developments of the weighted loss function were given by [1, 4, 14].

Meanwhile, there are many discussions about the admissibility of estimated regression coefficients $\beta$ in the linear model. First, we introduce the definition of admissible estimator. Let $L(d;\beta,\sigma^2)$ be a given loss function, where $d$ is an any estimator of $\beta$. $R(d;\beta,\sigma^2) = \mathrm{E}[L(d;\beta,\sigma^2)]$, is called the risk function with respect to the loss function $L(d;\beta,\sigma^2)$. An estimator $\delta_1$ is said to dominate an estimator $\delta_2$, if $R(\delta_1;\beta,\sigma^2) \le R(\delta_2;\beta,\sigma^2)$ for all $\beta$, $\sigma^2$, and the above inequality holds strictly for some $\beta_0$, $\sigma_0^2$. An estimator $\delta$ is admissible with respect to the risk function $R(d;\beta,\sigma^2)$ if no other estimator dominates it. Xu and Wu [15] studied the admissibility of estimated regression coefficients for $\beta$ in model (1.1) under the balanced loss function (1.4) and investigated its necessary and sufficient conditions. In [8], Luo and Bai investigated the necessary and sufficient conditions for regression coefficients' unbiasedness, efficiency and admissibility in a general linear model. However, the classic linear regression model (1.1) assumes that the random error is independent, while it is very common to see correlated structure in practice, which can be described by the Gauss – Markov model. The Gauss – Markov model can be expressed as

$$y = X\beta + e, \quad \mathrm{E}(e) = 0, \quad \mathrm{Cov}(e) = \sigma^2 V, \tag{1.6}$$

where $y \in R^n$ is an observable random vector, $X \in R^{n\times p}$ is a known design matrix with full-column rank, $\beta \in R^p$ and $\sigma^2 > 0$ are unknown parameters, $V \in R^n_{\ge}$ is a nonnegative definite matrix, $e$ is a random error vector. The admissibility of estimated regression coefficient in (1.6) was investigated under quadratic loss function and matrix loss in [2]. From then on, there has been considerable interest in this issue. In [12], Rao characterized admissible estimators of a linear parametric function $S\beta$ in the class of all linear estimators under the assumption that $V$ is a positive definite matrix. In [9], the authors extended the results of [12] to the situation where the covariance matrix is singular. Later the problem was developed both in unconstrained case and constrained case for the unknown parameters by [5 – 7] and so on.

In this paper we investigate the admissibility of estimated regression coefficients $\beta$ in (1.6), under the generalized weighted balanced loss function. First, the generalized weighted balance loss function for generalized linear model is proposed in Section 2. In Section 3, we investigate the admissibility of estimated linear coefficients in two linear estimation classes. Finally, concluding remarks are given in Section 4.

**2. Generalized weighted balanced loss function.** Let $R^n_>$ and $R^n_\geq$ stand for the sets of $n$ positive definite matrices and $n$ nonnegative definite matrices, respectively. For a matrix $A$, $A'$, tr$(A)$, rk$(A)$, $A^+$ and $R(A)$ denote transpose, trace, rank, Moore–Penrose inverse, the range space of column vectors of $A$, respectively. $R(A\vdots B)$ represents the range space of column vectors of the merged matrix of $A$ and $B$. $A^\perp$ denotes a matrix of maximum rank that satisfies $A'A^\perp = 0$. The dimension and orthogonal complement subspace of linear space $R(A)$ are denoted by dim$R(A)$ and $N(A)$. For nonnegative definite matrices $A$ and $B$, $A < B$ stands for the positivity of matrix $B - A$. $A \leq B$ stands for the nonnegativity of matrix $B - A$. That is to say, $A \leq B$ denotes there exists the Löwner partial order for matrices $A$ and $B$.

Considering both of goodness of fit and precision of estimation, we propose the generalized weighted balance loss function for generalized linear model (1.6), which is

$$L_{GWB}(d;\beta,\sigma^2|U,S) = wq(\beta)(y - Xd)'T^+(y - Xd) + (1 - w)q(\beta)(d - \beta)'S(d - \beta), \quad (2.1)$$

where $w \in [0,1]$, is a real number, $q(\beta)$ is a positive weight function, $T = V + XUX' \in R^n_\geq$, $U$ is a symmetric $(p \times p)$-matrix such that $R(T) = R(V\vdots X)$, $S \in R^p_>$. It is worthwhile pointing out that the estimator, minimizing the expression $(y - Xd)'T^+(y - Xd)$, is the best linear unbiased estimation of $\beta$, also called the Gauss–Markov estimator. Then, we investigate the admissibility of estimated regression coefficients $\beta$ in generalized linear model (1.6) under the generalized weighted balanced loss function (2.1).

Because $\beta$ is the location parameter of the generalized linear model (1.6), we are interested in the two linear estimation classes of $\beta$,

$\mathfrak{K}_0 = \{Hy:\ H \in R^{p\times n}\}$, the class of all homogeneous linear estimators,

$\mathfrak{K}_1 = \{Hy + a:\ H \in R^{p\times n}, a \in R^p\}$, the class of all linear estimators.

Specific condition of $H$ and $a$ will be discussed in order to guarantee the admissibility of the estimators.

**3. Main results.** Let $R(d;\beta,\sigma^2)$ denote the risk function of estimator $d$ under the generalized weighted balanced loss function (2.1), that is to say, $R(d;\beta,\sigma^2) = \mathrm{E}L_{GWB}(d;\beta,\sigma^2)$.

**Theorem 3.1.** *Under model* (1.6) *and loss function* (2.1), *the necessary and sufficient condition of $Hy$ is a linear admissible estimator of $\beta$ in estimation class $\mathfrak{K}_0$ is that under the quadratic loss function* (1.3), *$Gy$ is an admissible estimator of $C\beta$ in estimation class $\mathfrak{K}_0$, where $G = B^{1/2}(H - wB^{-1}X'T^+)$, $C = (1 - w)B^{-1/2}S$ and $B = wX'T^+X + (1 - w)S$.*

***Proof.*** Suppose $Hy \in \mathfrak{K}_0$. Under loss function (2.1), the risk function is

$$R(Hy;\beta,\sigma^2) = q(\beta)\mathrm{E}[w(y - XHy)'T^+(y - XHy) + (1 - w)(Hy - \beta)'S(Hy - \beta)] =$$

$$= q(\beta)[\beta'(I - X'H')[wX'T^+X + (1 - w)S](I - XH)\beta+$$

$$+w\sigma^2\mathrm{tr}((I - XH)'T^+(I - XH)V) + (1 - w)\sigma^2\mathrm{tr}(SHVH')] =$$

$$= q(\beta)\{\beta'(I - X'H')[wX'T^+X + (1 - w)S](I - XH)\beta-$$

$$-\sigma^2 w^2 \mathrm{tr}\{T^+ X[wX'T^+X + (1-w)S]^{-1} X'T^+V\} + \sigma^2 w \mathrm{tr}(T^+V) +$$

$$+\sigma^2 \mathrm{tr}\{H' - wT^+X[wX'T^+X + (1-w)S]^{-1}\}[wX'T^+X + (1-w)S] \times$$

$$\times \{H - w[wX'T^+X + (1-w)S]^{-1} X'T^+\}V\}.$$

Let $B = kX'T^+X + (1-k)S > 0$, $L = H - kB^{-1}X'T^+$, then $H = L + kB^{-1}X'T^+$. Substituting the expressions of $B$ and $H$ into $R(Hy;\beta,\sigma^2)$, we have

$$R(Hy;\beta,\sigma^2) =$$

$$= q(\beta)\{\beta'[X'L' - (1-w)SB^{-1}]B[LX - (1-w)B^{-1}S]\beta - \sigma^2 w^2 \mathrm{tr}(T^+XB^{-1}X'T^+V) +$$

$$+\sigma^2 w \mathrm{tr}(T^+V) + \sigma^2 \mathrm{tr}\{(H' - wT^+XB^{-1})B(H - wB^{-1}X'T^+)V\}\} =$$

$$= q(\beta)\{\beta'[X'L' - (1-w)SB^{-1}]B[LX - (1-w)B^{-1}S]\beta - \sigma^2 k^2 \mathrm{tr}(T^+XB^{-1}X'T^+V) +$$

$$+\sigma^2 w \mathrm{tr}(T^+V) + \sigma^2 \mathrm{tr}(L'BLV)\}.$$

If there exists $My \in \mathfrak{K}_0$, superior to $Hy$, then

$$R(\beta,\sigma^2,Hy) - R(\beta,\sigma^2,My) \geq 0$$

holds for all $\beta \in R^p$ and $\sigma^2 > 0$, and the equality does not always hold.

Let $\bar{L} = M - wB^{-1}X'T^+$, equivalently $M = \bar{L} + wB^{-1}X'T^+$. Then we obtain

$$R(Hy;\beta,\sigma^2) - R(My;\beta,\sigma^2) =$$

$$= q(\beta)\{\beta'[X'L' - (1-w)SB^{-1}]B[LX - (1-w)B^{-1}S]\beta - \sigma^2 w^2 \mathrm{tr}(T^+XB^{-1}X'T^+V) +$$

$$+\sigma^2 w \mathrm{tr}(T^+V) + \sigma^2 \mathrm{tr}(L'BLV) - \{\beta'[X'\bar{L}' - (1-w)SB^{-1}]B[\bar{L}X - (1-w)B^{-1}S]\beta -$$

$$-\sigma^2 w^2 \mathrm{tr}(T^+XB^{-1}X'T^+V) + \sigma^2 w \mathrm{tr}(T^+V) + \sigma^2 \mathrm{tr}(\bar{L}'B\bar{L}V)\}\} =$$

$$= q(\beta)\{\beta'[X'L' - (1-w)SB^{-1}]B[LX - (1-w)B^{-1}S]\beta + \sigma^2 \mathrm{tr}(L'BLV) -$$

$$-\beta'[X'\bar{L}' - (1-w)SB^{-1}]B[\bar{L}X - (1-w)B^{-1}S]\beta - \sigma^2 \mathrm{tr}(\bar{L}'B\bar{L}V)\}.$$

For simplicity, denote $Q = B^{1/2}$, then

$$R(Hy;\beta,\sigma^2) - R(My;\beta,\sigma^2) =$$

$$= q(\beta)\{\beta'[X'L'Q - (1-w)SQ^{-1}][QLX - (1-w)Q^{-1}S]\beta + \sigma^2 \mathrm{tr}[(QL)'(QL)V] -$$

$$-\beta'[X'\bar{L}'Q - (1-w)SQ^{-1}][Q\bar{L}X - (1-w)Q^{-1}S]\beta - \sigma^2 \mathrm{tr}[(Q\bar{L})'(Q\bar{L})V]\} =$$

$$= q(\beta)\{\mathrm{E}(Gy - C\beta)'(Gy - C\beta) - \mathrm{E}(\bar{G}y - C\beta)'(\bar{G}y - C\beta)\},$$

where $G = QL, \bar{G} = Q\bar{L}$, $C = (1-w)Q^{-1}S$. Thus

$$R(Hy;\beta,\sigma^2) - R(My;\beta,\sigma^2) \geq 0$$

holds for all $\beta \in R^p$ and $\sigma^2 > 0$, and the equality does not always hold, if and only if

$$\mathrm{E}(Gy - C\beta)'(Gy - C\beta) - \mathrm{E}(\bar{G}y - C\beta)'(\bar{G}y - C\beta) \geq 0$$

holds for all $\beta \in R^p$ and $\sigma^2 > 0$, and the equality does not always hold. That is to say, as an estimator of $C\beta$, $\bar{G}y$ is superior to $Gy$ under the quadratic loss function (1.3). Due to the reversibility of $Q = B^{1/2}$, $Hy \to Gy = QLy = Q(H - wB^{-1}X'T^+)y$ is a one-to-one mapping from $\mathfrak{K}_0$ to itself. Thus under the generalized weighted balanced loss function (2.1), $Hy$ is an inadmissible estimation of $\beta$ is equivalent to $Gy$ is an inadmissible estimation of $C\beta$ under quadratic loss function (1.3). Equivalently, under the generalized weighted balanced loss function (2.1), $Hy$ is an admissible estimation of $\beta$ if and only if $Gy$ is an admissible estimation of $C\beta$ under quadratic loss function (1.3).

Theorem 3.1 is proved.

For the admissibility of $Hy + a$ in estimation class $\mathfrak{K}_1$, we can derive the following results.

**Lemma 3.1.** *Under model* (1.6) *and loss function* (2.1), $R(Hy+a;\beta,\sigma^2)$ *and* $R(Hy;\beta,\sigma^2)$ *satisfy the relation expression* $R(Hy+a;\beta,\sigma^2) = R(Hy;\beta,\sigma^2) + q(\beta)[a'Ba - 2a'B(I - HX)\beta]$, *where B is defined as Theorem* 3.1.

***Proof.*** We have

$$R(Hy+a;\beta,\sigma^2) =$$

$$= q(\beta)E[w(y - X(Hy+a))'T^+(y - X(Hy+a)) + (1-w)(Hy+a-\beta)'S(Hy+a-\beta)] =$$

$$= q(\beta)\{E[w(y - XHy)'T^+(y - XHy) + (1-w)(Hy-\beta)'S(Hy-\beta)]+$$

$$+a'wX'T^+Xa + a'(1-w)Sa - 2a'wX'T^+X(I-HX)\beta - 2a'(1-w)S(I-HX)\beta\} =$$

$$= R(Hy;\beta,\sigma^2) + q(\beta)[a'Ba - 2a'B(I-HX)\beta].$$

Now comprehensively considering Theorem 3.1, Lemma 3.1 and the above analysis, we can obtain the result of Theorem 3.2.

**Theorem 3.2.** *Under model* (1.6) *and loss function* (2.1), *the necessary and sufficient condition of* $Hy + a$ *is an admissible estimator of* $\beta$ *in estimation class* $\mathfrak{K}_1$ *is that* $Hy$ *is admissible in class* $\mathfrak{K}_0$ *and* $a \in R(I - HX)$, *where* $R(A)$ *stands for the linear subspace generated by column vectors of A.*

***Proof.*** Firstly, we investigate the expression of $a$. According to Lemma 3.1, if the estimator with the form $Hy + a$ is admissible, $a$ must belong to $R(I - HX)$. Otherwise, $Hy + (I - HX)\gamma$ will be superior to $Hy + a$, where $\gamma \in R^p$. On the other side, if $a \in R(I - HX)$, because $\beta$ is unknown, there is no another vector $b$, such that $b'Bb - 2b'B(I - HX)\beta \leq a'Ba - 2a'B(I - HX)\beta$ for all $\beta$, and the inequality doesn't hold for some $\beta$.

***Remark*** **3.1.** Theorem 2 of [15] investigated the admissibility of $Hy+a$ in general linear model, which is similar to Theorem 3.2. However, our proof simplifies the proving process greatly.

Theorems 3.1 and 3.2 clearly explain the admissibility under the weighted loss function and its relationship with the quadratic loss. In the remaining of this paper, we will discuss the specific situations when the estimated regression coefficient is admissible with respect to $H$ and $a$. Firstly, we rewrite Gauss – Markov model as

$$y = \tilde{X}\tilde{\beta} + e, \quad \mathrm{E}(e) = 0, \quad \mathrm{Cov}(e) = \sigma^2 V, \tag{3.1}$$

where $\tilde{X} = XC^{-1}$, $\tilde{\beta} = C\beta, C$ is defined as Theorem 3.1.

Then we apply Theorem 3.4, Corollary 3.3 in [9] to model (1.6). For the purpose of clarity, we restate the theorem and corollary here, which is referred as Lemmas 3.2 and 3.3 in this paper.

**Lemma 3.2** [9]**.** *Under model* (1.6) *and quadratic loss function* (1.3), $LY$ *is admissible for* $\beta$ *if and only if* $L = M(X'TX)^{-1}X'T$, *where* $M = LX$ *satisfies*

(a) $M((X'TX)^{-1} - I) \geq 0$;

(b) $M((X'TX)^{-1} - I) \geq M((X'TX)^{-1} - I)M'$;

(c) $R[(I - M)(X'TX)(X'TV)^{\perp}] \subset R[(I - M)(X'TX)(X'TV)]$.

**Lemma 3.3** [9]**.** *Consider the setup described in Lemma* 3.2, *where* $X$ *satisfies* $R(X) \subset R(V)$. *Then* $LY$ *is admissible for* $\beta$ *if and only if* $L = M(X'TX)^{-1}X'T$, *where* $M = LX$ *satisfies conditions* (a) *and* (b) *in Lemma* 3.2.

**Theorem 3.3.** *Under model* (1.6) *and loss function* (2.1), $Hy$ *is a linear admissible estimator of* $\beta$ *in estimation class* $\mathfrak{K}_0$ *if and only if* $G = MC(X'\tilde{T}X)^{-1}X'\tilde{T}$, *where* $M = GXC^{-1}$ *satisfies*

(a) $M(C(X'\tilde{T}X)^{-1}C' - I) \geq 0$;

(b) $M(C(X'\tilde{T}X)^{-1}C' - I) \geq M(C(X'\tilde{T}X)^{-1}C' - I)M'$;

(c) $R[(I - M)(C'^{-1}X'\tilde{T}XC^{-1})(C'^{-1}X'\tilde{T}V)^{\perp}] \subset R[(I - M)(C'^{-1}X'\tilde{T}XC^{-1})(C'^{-1}X'\tilde{T}V)]$, *where* $\tilde{T} = V + \tilde{X}U\tilde{X}'$, $G = B^{1/2}(H - wB^{-1}C'^{-1}X'\tilde{T}^{+})$, $C = (1 - w)B^{-1/2}S$ *and* $B = wC'^{-1}X'\tilde{T}^{+}XC^{-1} + (1 - w)S$.

**Theorem 3.4.** *Under model* (1.6) *and loss function* (2.1), *suppose* $R(X) \subset R(V)$. $Hy$ *is a linear admissible estimator of* $\beta$ *in estimation class* $\mathfrak{K}_0$ *if and only if* $G = MC(X'\tilde{T}X)^{-1}X'\tilde{T}$, *where* $M = GXC^{-1}$ *satisfies conditions* (a) *and* (b) *in Theorem* 3.3.

**4. Concluding remarks.** We extend the weighted balanced loss function to generalized weighted balanced loss function and apply it to the Gauss – Markov model. It is worthwhile pointing out that the weighted balanced loss function can not be applied to the Gauss – Markov model. Then we considered the admissible estimator of regression coefficients under the generalized linear regression model. Finally, we give the sufficient and necessary conditions for the admissibility among two linear estimation classes. Theorems 3.1 and 3.2 described the relationship between two admissibility under $L_{GWB}$ and the quadratic loss function (1.3) and Theorems 3.3 and 3.4 enabled us to verify if an estimator is admissible.


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