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CHARACTERIZATION OF WEAKLY BERWALD FOURTH ROOT METRICS

ХАРАКТЕРИЗАЦІЯ СЛАБКИХ ЧОТИРИКОРЕНЕВИХ МЕТРИК БЕРВАЛЬДА

In recent studies, it is shown that the theory of fourth root metrics plays a very important role in physics, theory of space-time structures, gravitation, and general relativity. The class of weakly Berwald metrics contains the class of Berwald metrics as a special case. We establish the necessary and sufficient condition under which the fourth root Finsler space with an (α, β)-metric is a weakly Berwald space.

В останніх дослідженнях було встановлено, що теорія чотирикореневих метрик відіграє важливу роль у фізиці, теорії просторово-часових структур, гравітації та загальній теорії відносності. Клас слабких метрик Бервальда містить клас метрик Бервальда як частинний випадок. Встановлено необхідну та достатню умову, за якої чотирикореневий простір Фінслера з (α, β)-метрикою є слабким простором Бервальда.

1. Introduction. In [15], Shimada developed the theory of m-th root Finsler metrics which applied to biology as an ecological metric [1]. An m-th root metric is regarded as a direct generalization of Riemannian metric in the sense that the second root metric is a Riemannian metric. The third and fourth root metrics are called the cubic metric and quartic metric, respectively.

Recently studies show that the theory of m-th root Finsler metrics plays a very important role in physics, theory of space-time structure, gravitation, general relativity and seismic ray theory [9, 13, 14]. For quartic metrics, a study of the geodesics and of the related geometrical objects are made by S. Lebedev [7]. Also, Einstein equations for some relativistic models relying on such metrics are studied by V. Balan and N. Brinzei in [3].

In [18], Tayebi and Najafi characterized locally dually flat and Antonelli m-th root Finsler metrics. They showed that every m-th root Finsler metric of isotropic mean Berwald curvature reduces to a weakly Berwald metric. In [19], they proved that every m-th root Finsler metric of isotropic Landsberg metric reduces to a Landsberg metric. Then, they showed that every m-th root Finsler metric with almost vanishing H-curvature satisfies $\mathbf{H} = 0$. Recently, Tayebi, Nankali and Peyghan defined some non-Riemannian curvature properties for Cartan spaces and considered Cartan space with the m-th root metric [20]. For other recent papers, see [18 – 22, 26, 27].

Let (M, F) be a Finsler manifold of dimension n, TM its tangent bundle and (x^i, y^i) the coordinates in a local chart on TM. Let F be a scalar function on TM defined by $F = \sqrt[4]{A}$, where A is given by

$$A := a_{ijkl}(x)y^i y^j y^k y^l, \tag{1}$$

with a_{ijkl} symmetric in all its indices [15]. Then F is called an fourth root Finsler metric or an quartic Finsler metric.

Let F be a Finsler metric on a manifold M. The geodesics of F are characterized locally by the equation $\ddot{x}^i(t) + 2G^i(x, \dot{x}(t)) = 0$, where G^i are coefficients of a spray defined on M denoted by $\mathbf{G}(x, y) = y^i \dfrac{\partial}{\partial x^i} - 2G^i \dfrac{\partial}{\partial y^i}$. A Finsler metric F is called a Berwald metric if $G^i = \dfrac{1}{2}\Gamma^i_{jk}(x)y^j y^k$ are quadratic in $y \in T_xM$ for any $x \in M$. Taking a trace of Berwald curvature yields mean Berwald curvature $\mathbf{E}$. A Finsler metric with vanishing mean Berwald curvature is called weakly Berwald

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metric [23]. In [2], Bácsó and Yoshikawa studied some weakly Berwald metrics. Then I. Y. Lee and M. H. Lee studied some weakly-Berwald spaces of special (α,β)-metrics. In [23], Tayebi and Peyghan studied the mean Berwald curvature of R-quadratic Finsler metrics. In [18], Tayebi and Najafi showed that every m-th root metric of isotropic mean Berwald curvature is a weakly Berwald metric. Recently, Najafi and Tayebi have found a condition on (α,β)-metrics under which the notions of isotropic S-curvature, weakly isotropic S-curvature and isotropic mean Berwald curvature are equivalent [10]. Accordingly, it is necessary to study the weakly Berwald fourth root metrics. Let $L^4 = c_1\alpha^4 + c_2\alpha^2\beta^2 + c_3\beta^4$ be a fourth root metric on a manifold M, where $c_1 \neq 0$, $c_2 \neq 0$, $c_3 \neq 0$ are real constants. Suppose that $\alpha = \sqrt{a_{ij}(x)y^iy^j}$ be a Riemannian metric and $\beta = b_i(x)y^i$ a 1-form on M such that $b^2 = b_ib^i \neq 0$; then, we prove the following theorem.

Theorem 1.1. *Let (M, F^n) be a Finsler manifold of dimension $n \geq 3$ equipped with quartic metric $L^4 = c_1\alpha^4 + c_2\alpha^2\beta^2 + c_3\beta^4$, where $\alpha = \sqrt{a_{ij}(x)y^iy^j}$ be a Riemannian metric and $\beta = b_i(x)y^i$ a 1-form on M. Then F^n is a weakly-Berwald metric if and only if there exists a homogeneous polynomial of degree 1, namely $V = v_iy^i$, such that the following hold:*

$$s_i = 0, \qquad r_{ij} = \frac{1}{UW}\Big(b_iv_j + b_jv_i\Big), \tag{2}$$

where

$$U := -2c_1^8c_2 - \frac{1}{2}b^4c_1^6c_2^3 - 2b^2c_1^7c_2^2 \quad \text{and} \quad W := \frac{3}{2}b^4c_1^6c_2^3 - 12b^2c_1^8c_3 + 3b^2c_1^7c_2^2 - 6b^4c_1^7c_2c_3$$

are real constants.

2. Preliminaries. Let M be an n-dimensional manifold C^∞. Denote by T_xM the tangent space at $x \in M$, by $TM = \cup_{x\in M}T_xM$ the tangent bundle of M and by $TM_0 = TM \setminus \{0\}$ the slit tangent bundle of M. A Finsler metric on a manifold M is a function $F\colon TM \to [0,\infty)$ with the following properties: (i) F is C^∞ on TM_0; (ii) $F(x,\lambda y) = \lambda F(x,y)$ $\forall\lambda > 0$, $y \in TM$; (iii) for each $y \in T_xM$, the following quadratic form $\mathbf{g}_y$ on T_xM is positive definite:

$$\mathbf{g}_y(u,v) := \frac{1}{2}\left[F^2(y + su + tv)\right]\Big|_{s,t=0}, \quad u,v \in T_xM.$$

Given an n-dimensional Finsler manifold (M,F), then a global vector field $\mathbf{G}$ is induced by F on TM_0, which in a standard coordinate (x^i,y^i) for TM_0 is given by

$$\mathbf{G} = y^i\frac{\partial}{\partial x^i} - 2G^i(x,y)\frac{\partial}{\partial y^i},$$

where $G^i = G^i(x,y)$ are called spray coefficients and given by following:

$$G^i = \frac{1}{4}g^{il}\left[\frac{\partial^2F^2}{\partial x^k\partial y^l}y^k - \frac{\partial F^2}{\partial x^l}\right]. \tag{3}$$

$\mathbf{G}$ is called the spray associated to F.

Define $\mathbf{B}_y : T_xM \otimes T_xM \otimes T_xM \to T_xM$ by $\mathbf{B}_y(u,v,w) := B^i_{jkl}(y)u^jv^kw^l\dfrac{\partial}{\partial x^i}\Big|_x$, where

$$B^i_{jkl} := \frac{\partial^3G^i}{\partial y^j\partial y^k\partial y^l}.$$

$\mathbf{B}_y(u, v, w)$ is symmetric in u, v and w. Based on the homogeneity of spray coefficients, we have $\mathbf{B}_y(y, v, w) = 0$. $\mathbf{B}$ is called the Berwald curvature. A Finsler metric with vanishing Berwald curvature is called a Berwald metric [11, 16].

Define the mean of Berwald curvature by $\mathbf{E}_y : T_xM \otimes T_xM \to \mathbb{R}$, where

$$\mathbf{E}_y(u, v) := \frac{1}{2} \sum_{i=1}^{n} g^{ij}(y) g_y \left(\mathbf{B}_y(u, v, e_i), e_j \right). \tag{4}$$

The family $\mathbf{E} = \{\mathbf{E}_y\}_{y \in TM \setminus \{0\}}$ is called the *mean Berwald curvature* or *E-curvature*. In local coordinates, $\mathbf{E}_y(u, v) := E_{ij}(y) u^i v^j$, where

$$E_{ij} := \frac{1}{2} B^m_{mij}.$$

By definition, $\mathbf{E}_y(u, v)$ is symmetric in u and v, and we have $\mathbf{E}_y(y, v) = 0$. $\mathbf{E}$ is called the mean Berwald curvature. F is called a weakly Berwald metric if $\mathbf{E} = \mathbf{0}$.

3. Proof of Theorem 1.1. A Finsler metric $L(x, y)$ is called (α, β)-metric if L is a positive-homogeneous function of α and β of degree one, where $\alpha^2 = \alpha_{ij}(x) y^i y^j$ is a Riemannian metric and $\beta = \beta_i(x) y^i$ is a one-form. The functions G^i of a Finsler space with an (α, β)-metric are given by $2G^i = \gamma^i_{00} + 2B^i$, where γ^i_{jk} stands for the Christoffel symbols in the space (M, α) (see [8, 17, 24, 25]). Let $G^i_j = \dot{\partial}_j G^i$ and $G^i_{jk} = \dot{\partial}_k G^i_j$. Then we have

$$G^i_j = \gamma^i_{0j} + B^i_j, \qquad G^i_{jk} = \gamma^i_{jk} + B^i_{jkl},$$

where $\dot{\partial}_j B^i = B^i_j$ and $\dot{\partial}_k B^i_j = B^i_{jkl}$.

Then a Finsler space with an (α, β)-metric is a weakly-Berwald space if and only if $B^m_m = \partial B^m / \partial y^m$ is a one form.

Let us put

$$b^i = a^{ir} b_r, \qquad b^2 = a^{rs} b_r b_s, \tag{5}$$

$$r_{ij} = \frac{1}{2}(b_{i/j} + b_{j/i}), \qquad s_{ij} = \frac{1}{2}(b_{i/j} - b_{j/i}), \tag{6}$$

$$r^i_j = a^{ir} r_{rj}, \qquad s^i_j = a^{ir} s_{rj}, \qquad r_i = b_r r^r_i, \qquad s_i = b_r s^r_i. \tag{7}$$

Here, the symbol "/" denotes the h-covariant derivation with respect to the Riemannian connection of (M, α). For Finslerian connections see [4, 5]. It is remarkable that if $r_{ij} = 0$ then β is a killing 1-form and if $s_{ij} = 0$ then β is a closed 1-from (see [12]).

According to [6], the necessary and sufficient condition for a Finsler space F^n with (α, β)-metric to be a weakly-Berwald space is that $G^m_m = \gamma^m_{0m} + B^m_m$ and B^m_m is a homogeneous polynomial in (y^m) of degree one which is given by following:

$$B^m_m = \frac{1}{2\alpha L (\beta L_\alpha)^2 \Omega^2} \left\{ 2\Omega^2 A C^* + 2\alpha L \Omega^2 B s_0 + \alpha^2 L L_\alpha L_{\alpha\alpha} (C r_{00} + D s_0 + E r_0) \right\}, \tag{8}$$

where

$$A = (n+1)\beta^2 L_\alpha (\beta L_\alpha L_\beta - \alpha L L_{\alpha\alpha}) + \alpha \gamma^2 L \left\{ \alpha (L_{\alpha\alpha})^2 - 2 L_\alpha L_{\alpha\alpha} - \alpha L_\alpha L_{\alpha\alpha\alpha} \right\}, \tag{9}$$

$$B = \alpha^2 L L_{\alpha\alpha}, \tag{10}$$

$$C = \beta\gamma^2 \left\{ -\beta^2 (L_\alpha)^2 + 2b^2\alpha^3 L_\alpha L_{\alpha\alpha} - \alpha^2\gamma^2 (L_{\alpha\alpha})^2 + \alpha^2\gamma^2 L_\alpha L_{\alpha\alpha\alpha} \right\}, \tag{11}$$

$$D = 2\alpha \left\{ \beta^3(\gamma^2 - \beta^2) L_\alpha L_\beta - \alpha^2\beta^2\gamma^2 L_\alpha L_{\alpha\alpha} - 2\alpha\beta\gamma^2(\gamma^2 + 2\beta^2) L_\beta L_{\alpha\alpha} \right\} - \\ -\alpha^3\gamma^4 (L_{\alpha\alpha})^2 - \alpha^2\beta\gamma^4 L_\beta L_{\alpha\alpha\alpha} \tag{12}$$

and

$$E = 2\alpha^2\beta^2 L_\alpha \Omega, \tag{13}$$

$$\gamma^2 = b^2\alpha^2 - \beta^2, \tag{14}$$

$$C^* = \frac{\alpha\beta(r_{00}L_\alpha - 2\alpha s_0 L_\beta)}{2(\beta^2 L_\alpha + \alpha\gamma^2 L_{\alpha\alpha})}, \tag{15}$$

$$\Omega = \beta^2 L_\alpha + \alpha\gamma^2 L_{\alpha\alpha} \tag{16}$$

provided that $\Omega \neq 0$.

In order to prove the Theorem 1.1, we need the following lemma.

Lemma 3.1. *Let F^n be a Finsler space with an fourth root Finsler metric $F(x, y)$. Then*:

(1) *In case of $n = 2$, F^4 is always written in the form*

$$F^4 = d_1\alpha^4 + d_2\alpha^2\beta^2,$$

with constants d_1, d_2 by choosing suitable quadratic form α^2 and one form β, where α^2 may be degenerate.

(2) *In case of $n \geq 3$, if F is a function of a nondegenerate quadratic form $\alpha^2 = \alpha_{ij}(x)y^iy^j$ and a one-form $\beta = b_i(x)y^i$ which is homogeneous in α and β of degree one, then F^4 is written in the form*

$$F^4 = c_1\alpha^4 + c_2\alpha^2\beta^2 + c_3\beta^4$$

with constants c_1, c_2 and c_3.

Proof. (1) First, we consider two-dimensional Finsler space with a quartic metric. Putting $t := \dfrac{y^2}{y^1}$, the quartic metric L is written as follows:

$$\left(\frac{F}{y^1}\right)^4 = a_{1111} + 4a_{1112}t + 6a_{1122}t^2 + 4a_{1222}t^3 + a_{2222}t^4 (= f(t)). \tag{17}$$

In case of $a_{2222} \neq 0$, the algebraic equation $f(t) = 0$ of the degree four with real coefficients has always two real roots or four real roots or four complex roots.

Case A. If $f(t)$ has two complex roots of z_1 and z_2, then it is clear that $\overline{z}_1$ and $\overline{z}_2$ are roots of $f(t)$. Hence, we obtain

$$f(t) = a_{2222}\left(t^2 - \frac{1}{2}\mathrm{Re}\, z_1 t + \gamma\right)\left(t^2 - \frac{1}{2}\mathrm{Re}\, z_2 t + \delta\right),$$

where $\delta := z_2\overline{z}_2$ and $\gamma := z_1\overline{z}_1$ are real numbers. Let us put

$$\alpha_1^2 := (y^2)^2 - \frac{1}{2}\mathrm{Re}\, z_1 y^1 y^2 + \gamma(y^1)^2, \qquad \alpha_2^2 := a_{2222}\left\{(y^2)^2 - \frac{1}{2}\mathrm{Re}\, z_2 y^1 y^2 + \delta(y^1)^2\right\}.$$

Hence, we get $F^4 = \alpha^4$. In the case of $a_{2222} = 0$ and $a_{1111} \neq 0$, we have

$$f(t) = 4a_{1222}t^3 + 6a_{1122}t^2 + 4a_{1112}t + a_{1111}.$$

Indeed, $f(t)$ has always at least one real root γ. Then we obtain

$$f(t) = (t-\gamma)(4a_{1222}t^2 + bt + c),$$

where b and c are suitable real coefficients. Thus, we put

$$\alpha^2 = 4a_{1222}(y^2)^2 + by^2y^1 + c(y^1)^2 \ \text{ and } \quad \delta = y^2 - \gamma y^1,$$

to get $F^4 = \alpha^2\beta^2$.

If $a_{2222} = 0$ and $a_{1111} = 0$, we have $f(t) = 4a_{1222}t^3 + 6a_{1122}t^2 + 4a_{1112}t$. By putting

$$\alpha^2 = 4a_{1222}(y^2)^2 + 6a_{1122}y^2y^1 + 4a_{1112}(y^1)^2 \ \text{ and } \ \delta_1 = y^1 \quad \text{and} \quad \delta_2 = y^2,$$

one can obtain $F^4 = \alpha^2\beta^2$.

Case B. If $f(t)$ has two real roots a_1, a_2 and two complex roots $z, \overline{z}$, then we get

$$f(t) = a_{2222}\left(t^2 - \frac{1}{2}\mathrm{Re}\, zt + \gamma\right)(t-a_1)(t-a_2),$$

where $\gamma = z\overline{z}$ is a real number.

By putting $\alpha^2 = (y^2)^2 - \frac{1}{2}\mathrm{Re}\, z\, y^1y^2 + \gamma(y^1)^2$ and $\delta_1 = a_{2222}\{y^2 - a_1y^1\}$ and $\delta_2 = y^2 - a_2y^1$, we obtain $F^4 = \alpha^2\beta^2$. In this case, if $a_{2222} = 0$, we have $F^4 = \alpha^2\beta^2$, again.

Case C. If $f(t)$ has four real roots, then we have $F^4 = \beta^4$. This contradicts with our assumption where α^2 may be degenerate. Consequently, for $n = 2$, F^4 is always written in the form $F^4 = a\alpha^4 + b\alpha^2\beta^2$.

(2) Now, suppose that F be a homogeneous function of α and β of degree one, where α^2 is a nondegenerate quadratic form and β is an one-form. Then the Jacobian determinant $\partial(F,\alpha,\beta)/\partial(y^{i_1}, y^{i_2}, y^{i_3})$ must be equal to zero for any different three indices of $i_1, i_2, i_3 = i, j, k, l$.

For $i_1 = i$, $i_2 = j$ and $i_3 = k$, we have

$$\begin{vmatrix} a_i & a_j & a_k \\ \alpha_{ia}y^a & \alpha_{ja}y^a & \alpha_{ka}y^a \\ b_i & b_j & b_k \end{vmatrix} = 0$$

which is a homogeneous polynomial of degree four in y^i. Equating every coefficients of $y^ay^hy^cy^d$ to zero, we get

$$\sum_{(ijk)}\left\{\sum_{(ahcd)}\{\alpha_{ia}(b_ja_{khcd} - b_ka_{jhcd})\}\right\} = 0, \tag{18}$$

where the symbol $\sum_{(ijk)}\{...\}$ denotes the cyclic permutation of i, j, k and summation. Similarly, for other indices, we have

$$\sum_{(jkl)}\left\{\sum_{(ahcd)}\{\alpha_{ia}(b_j a_{khcd}-b_k a_{jhcd})\}\right\}=0,$$

$$\sum_{(kli)}\left\{\sum_{(ahcd)}\{\alpha_{ia}(b_j a_{khcd}-b_k a_{jhcd})\}\right\}=0,$$

$$\sum_{(lij)}\left\{\sum_{(ahcd)}\{\alpha_{ia}(b_j a_{khcd}-b_k a_{jhcd})\}\right\}=0.$$

Here we continue with the equation (18). It is similar to other relationships.

Suppose that (α^{ij}) denotes the inverse matrix of (α_{ij}). Put

$$b^r a_{rjkt}=p_{jkt},\quad b^j p_{jkt}=p_{kt},\quad b^k p_{kt}=p_t,\quad b^t p_t=p,\quad \beta^2=b^r b_r,$$

$$\alpha^{rs}a_{itrs}=q_{it},\quad b^i q_{it}=q_t,\quad b^t q_t=q=\alpha^{rs}p_{rs},\quad p_{iah}\alpha^{ah}=q_i.$$

Contracting (18) with $\alpha^{jd}b^k$ implies that

$$(n+1)\beta^2 a_{iahc}+\alpha_{ia}(p_{hc}-\beta^2 q_{hc})+\alpha_{ih}(p_{ca}-\beta^2 q_{ca})+\alpha_{ic}(p_{ah}-\beta^2 q_{ah})+$$
$$+\,nb_i(b_a q_{hc}+b_h q_{ca}+b_c q_{ah})-(n+1)b_i p_{ahc}-n(b_a p_{ihc}+b_b p_{ica}+b_c p_{iah})=0. \qquad (19)$$

Multiplying (19) with b^c yields, we have

$$\beta^2 p_{iah}+\alpha_{ia}(p_h-\beta^2 q_h)+\alpha_{ih}(p_a-\beta^2 q_a)+nb_i(p_{ah}+b_a q_h+b_h q_a)-(n+1)b_i p_{ah}-n(b_a p_{ih}+b_h p_{ia})=0. \qquad (20)$$

Now, by contracting (20) with b^d, we get

$$\alpha_{ia}(p-\beta^2 q)+nb_i(2p_a+b_a q)-(n+1)b_i p_a-np_i b_a-(n-1)\beta^2 p_{ia}=0. \qquad (21)$$

From (21), we obtain

$$p_{ia}=\frac{1}{(n-1)\beta^2}\left\{\alpha_{ia}(p-\beta^2 q)+(n-1)b_i p_a+nqb_i b_a-np_i b_a\right\}. \qquad (22)$$

Due to the symmetry property of p_{ia} and α_{ia}, from (21) one can get

$$p_i=\gamma b_i, \qquad (23)$$

where γ is scalar. Also, contraction of (20) by α^{ah} gives

$$q_i=\eta b_i, \qquad (24)$$

where η is scalar.

Substituting (23) in (22) yields

$$p_{ia}=e_1\alpha_{ia}+e_2 b_i b_a, \qquad (25)$$

where e_1 and e_2 are certain scalars.

By putting $(23), (24)$ and (25) in (20), we obtain

$$p_{iah} = d_1\alpha_{ia}b_h + d_2\alpha_{ih}b_a + d_3b_ib_a\beta_h + e_1b_i\alpha_{ah}. \tag{26}$$

Now, substituting (23), (24), (25) and (26) in (19) yields

$$a_{iahc} = \tau_1\alpha_{ia}b_hb_c + \tau_2\alpha_{ih}b_ab_c + \tau_3\alpha_{ic}b_hb_a + \tau_4\alpha_{ac}b_ib_h + \tau_5\alpha_{ah}b_cb_i + \tau_6\alpha_{hc}b_ib_a+$$
$$+e_1(\alpha_{ia}\alpha_{hc} + \alpha_{ih}\alpha_{ca} + \alpha_{ic}\alpha_{ah}) + \tau_7b_ib_ab_hb_c, \tag{27}$$

where $\tau_i,\ i = 1,\ldots,7$, are scalars.

By (27), we have $F^4 = 3e_1\alpha^4+\pi\alpha^2\beta^2+\tau_7\beta^4$ where π is a linear combination of $\tau_i,\ i = 1,\ldots,6$. Since it was assumed that F is a function of α and β alone, so it is obvious that e_1 and τ_i, $i = 1,\ldots,7$, and then π must be constant.

***Proof of Theorem* 1.1.** The fourth root Finsler metric $L(\alpha,\beta)$ of F^n is given by

$$L^4(\alpha,\beta) = c_1\alpha^4 + c_2\alpha^2\beta^2 + c_3\beta^4, \tag{28}$$

where c_1, c_2 and c_3 are constants. Let us put

$$X := c_1\alpha^2 + \frac{1}{2}c_2\beta^2, \tag{29}$$

$$Y := \frac{1}{2}c_1c_2\alpha^4 + 3\alpha^2\beta^2c_1c_3 - \frac{1}{4}\alpha^2\beta^2c_2^2 + \frac{1}{2}c_3c_2\beta^4. \tag{30}$$

Then we have

$$L^3L_\alpha = \alpha X, \tag{31}$$

$$L^3L_\beta = c_3\beta^3 + \frac{1}{2}c_2\alpha^2\beta, \tag{32}$$

$$L^7L_{\alpha\alpha} = Y\beta^2, \tag{33}$$

$$L^{11}L_{\alpha\alpha\alpha} = -\frac{3}{2}\alpha^7\beta^2c_1^2c_2 - 15\alpha^5\beta^4c_1^2c_3 - 6\alpha^3\beta^6c_1c_3c_2 + \frac{3}{8}\alpha^3\beta^6c_2^3 - \frac{9}{4}\alpha\beta^8c_2^2c_3+$$
$$+6\beta^8\alpha c_1c_3^2 + \frac{3}{2}\beta^4\alpha^5c_1c_2^2. \tag{34}$$

Substituting (31)–(34) in (9)–(16) and using (8), yield

$$\alpha^2\beta\Phi B^m_m + \alpha^2\beta^2\Psi r_{00} + \alpha^2\beta\Lambda r_0 + \beta\Upsilon s_0 = 0, \tag{35}$$

where Φ, Ψ, Λ and Υ are listed in Appendix 1.

It should be noted that

$$4L^4\beta^8\alpha^6X^2(L^{12}X^3 + 3L^8X^2Y\alpha^2b^2 + 3L^4XY^2\alpha^4b^4 + Y^3\alpha^6b^6 - 3L^8X^2Y\beta^2-$$
$$-6L^4XY^2\alpha^2b^2\beta^2 - 3Y^3\alpha^4b^4\beta^2 + 3L^4XY^2\beta^4 + 3Y^3\alpha^2b^2\beta^4 - Y^3\beta^6) \neq 0 \tag{36}$$

and

$$2\alpha\beta^2(XL^4 + \alpha^2 Y b^2 - Y\beta^2) \neq 0 \qquad (37)$$

because it appears in the denominator of (8).

Thus, $L^4 \neq 0$ and $X \neq 0$. Analogously to the above, this implies $c_1 \neq 0$ and $c_2 \neq 0$ and $c_3 \neq 0$. Suppose that F^n be a weakly-Berwald space, that is, B^m_m is $hp(1)$. The term in (35) which seemingly does not contain α^2 is

$$\beta^{23}\left(-6b^2c_1c_2^3c_3^5 + \frac{27}{8}c_2^6c_3^3 + \frac{9}{4}b^2c_2^5c_3^4 + 24c_1^2c_2^2c_3^5 - 18c_1c_2^4c_3^4\right)s_0$$

only, and we must have $hp(22)$ V_{22} such that

$$\beta^{23}cs_0 = \alpha^2 V_{22},$$

where

$$c = -6b^2c_1c_2^3c_3^5 + \frac{27}{8}c_2^6c_3^3 + \frac{9}{4}b^2c_2^5c_3^4 + 24c_1^2c_2^2c_3^5 - 18c_1c_2^4c_3^4.$$

Analogously to the above, it implies that $s_0 = 0$.

Substituting $s_0 = 0$ into (35) implies that

$$\Phi B^m_m + \beta\Psi r_{00} + \Lambda r_0 = 0. \qquad (38)$$

Let us put

$$U := -2c_1^8c_2 - \frac{1}{2}b^4c_1^6c_2^3 - 2b^2c_1^7c_2^2,$$

$$T := \frac{1}{2}b^6c_1^6c_2^3 + 3b^4c_1^7c_2^2 + 6b^2c_1^8c_2 + 4c_1^9.$$

Then only the term $\alpha^{20}(Ur_0 + TB^m_m)$ of (38) seemingly does not contain β, and we must have $hp(20)$ V_{20} such that

$$\alpha^{20}(Ur_0 + TB^m_m) = \beta V_{20}.$$

From $\alpha^2 \neq 0 \pmod{\beta}$ and $b^2 \neq 0$, it follows that there exists a function $k(x)$ such that

$$Ur_0 + TB^m_m = k(x)\beta. \qquad (39)$$

Multiplying (38) with U and using (39) one can obtain

$$\beta U\Psi r_{00} + \Lambda k(x)\beta + (U\Phi - \Lambda T)B^m_m = 0. \qquad (40)$$

Dividing (40) by β implies that

$$U\Psi r_{00} + \Lambda k(x) + \beta S B^m_m = 0, \qquad (41)$$

where

$$S := \frac{U\Phi - \Lambda T}{\beta^2}.$$

Let us put

$$W := \frac{3}{2}b^4c_1^6c_2^3 - 12b^2c_1^8c_3 + 3b^2c_1^7c_2^2 - 6b^4c_1^7c_2c_3.$$

Since only the term $\alpha^{18}UWr_{00} + Uk(x)\alpha^{20}$ of $U\Psi r_{00} + \Lambda k(x)$ in (41) seemingly does not contain β, thus there exists $hp(19)$ V_{19} such that

$$\alpha^{18}(UWr_{00} + Uk(x)\alpha^2) = \beta V_{19}.$$

Then there must exist $hp(1)$ U_1 satisfying

$$UWr_{00} + Uk(x)\alpha^2 = \beta U_1. \tag{42}$$

It is a contradiction and then $k(x) = 0$. Putting it into (41) implies that

$$U\Psi r_{00} + \beta SB^m_m = 0. \tag{43}$$

Then only the term $\alpha^{18}UWr_{00}$ of $U\Psi r_{00}$ in (43) seemingly does not contain β. Then we must have $hp(20)$ V_{20} such that

$$\alpha^{18}UWr_{00} = \beta V_{20}.$$

According to $\alpha^2 \neq 0 \pmod{\beta}$ and $b^2 \neq 0$, there must exists $hp(1)$ $V = v_iy^i$ satisfying

$$UWr_{00} = \beta V.$$

Hence, we have

$$r_{00} = \frac{1}{UW}\beta V. \tag{44}$$

Conversely, let $r_{00} = \dfrac{1}{UW}\beta V$ and $s_0 = 0$ hold. Then we have

$$r_{ij} = \frac{1}{2UW}(b_iv_j + b_jv_i), \qquad s_i = 0, \tag{45}$$

where $\beta = b_i(x)y^i$ and $V = v_iy^i$. Multiplying (45) by b^iy^j implies that

$$r_0 = \frac{1}{2UW}(b^2V + v_b\beta), \qquad r_j = \frac{1}{2UW}(b^2v_j + v_bb_j), \tag{46}$$

where $v_b = v_ib^i$. According to assumptions and by putting (46) into (35), we obtain

$$\Phi B^m_m + \frac{1}{UW}\beta^2\Psi V + \frac{1}{2UW}\Lambda(b^2V + v_b\beta) = 0. \tag{47}$$

It is easy to see that only the term $\alpha^{20}\Big(TB^m_m + \dfrac{1}{2W}b^2V\Big)$ of $\Phi B^m_m + \dfrac{1}{2UW}\Lambda b^2V$ in (47) seemingly does not contain β. Then we must have $hp(20)$ V_{20} such that

$$\alpha^{20}\left(TB^m_m + \frac{1}{2W}b^2V\right) = \beta V_{20}.$$

The above shows the existence of a function $g(x)$ satisfying

$$V_{20} = g(x)\alpha^{20}.$$

Then

$$B^m_m = -\frac{1}{2WT}b^2V + \frac{1}{T}\beta g(x). \tag{48}$$

Therefore, B^m_m is $hp(1)$ and, hence, the Finsler space with (28) is a weakly-Berwald space.

Theorem 1.1 is proved.

4. Appendix 1.

$$
\begin{aligned}
\Phi = {} & \frac{27}{64}\beta^{20}c_2^8c_3 + 4\alpha^{20}c_1^9 + 4L^4X^2Y^3b^6 - \frac{3}{2}b^4\beta^{20}c_1c_2^4c_3^4 + 6b^2\beta^{20}c_1^2c_2^3c_3^4 - \frac{9}{2}b^2\beta^{20}c_1c_2^5c_3^3 - \\
& -\frac{9}{2}b^2\beta^{20}c_1c_2^5c_3^3 + 6\alpha^{18}b^4\beta^2c_1^6c_2^3 + 36\alpha^{18}b^2\beta^2c_1^8c_3 + 21\alpha^{18}b^2\beta^2c_1^7c_2^2 + 108\alpha^{16}b^4\beta^4c_1^7c_3^2 + \\
& +\frac{15}{4}\alpha^{16}b^4\beta^4c_1^5c_2^4 + \frac{69}{2}\alpha^{16}b^2\beta^4c_1^6c_2^3 - \frac{27}{64}\alpha^4b^4\beta^{16}c_2^8c_3 + 56\alpha^4\beta^{16}c_1^4c_2^2c_3^3 - \frac{15}{2}\alpha^4\beta^{16}c_1^3c_2^4c_3^2 - \\
& -\frac{27}{2}\alpha^4\beta^{16}c_1^2c_2^6c_3 + \frac{27}{64}\alpha^2b^2\beta^{18}c_2^8c_3 - 32\alpha^2\beta^{18}c_1^4c_2c_3^4 + 40\alpha^2\beta^{18}c_1^3c_2^3c_3^3 - \frac{27}{2}\alpha^2\beta^{18}c_1^2c_2^5c_3^2 - \\
& -\frac{3}{4}\alpha^{14}b^4\beta^6c_1^4c_2^5 - 108\alpha^{14}b^2\beta^6c_1^7c_3^2 + \frac{129}{4}\alpha^{14}b^2\beta^6c_1^5c_2^4 - 80\alpha^{14}\beta^6c_1^7c_2c_3 - 108\alpha^{12}b^4\beta^8c_1^6c_3^3 - \\
& -\frac{39}{16}\alpha^{12}b^4\beta^8c_1^3c_2^6 + \frac{123}{8}\alpha^{12}b^2\beta^8c_1^4c_2^5 - 152\alpha^{12}\beta^8c_1^6c_2^2c_3 - \frac{3}{4}\alpha^{10}b^4\beta^{10}c_1^2c_2^7 + \frac{3}{16}\alpha^{10}b^2\beta^{10}c_1^3c_2^6 + \\
& +72\alpha^{10}\beta^{10}c_1^6c_2c_3^2 - 176\alpha^{10}\beta^{10}c_1^5c_2^3c_3 - 216\alpha^8b^4\beta^{12}c_1^5c_3^4 + \frac{21}{64}\alpha^8b^4\beta^{12}c_1c_2^8 - \frac{141}{32}\alpha^8b^2\beta^{12}c_1^2c_2^7 + \\
& +78\alpha^8\beta^{12}c_1^5c_2^2c_3^2 - \frac{259}{2}\alpha^8\beta^{12}c_1^4c_2^4c_3 + 144\alpha^6b^2\beta^{14}c_1^5c_3^4 - \frac{153}{64}\alpha^6b^2\beta^{14}c_1c_2^8 + 32\alpha^6\beta^{14}c_1^5c_2c_3^3 + \\
& +36\alpha^6\beta^{14}c_1^4c_2^3c_3^2 - 59\alpha^6\beta^{14}c_1^3c_2^5c_3 + 36\alpha^{18}b^4\beta^2c_1^7c_2c_3 + 93\alpha^{16}b^4\beta^4c_1^6c_2^2c_3 + 132\alpha^{16}b^2\beta^4c_1^7c_2c_3 - \\
& -132\alpha^4b^4\beta^{16}c_1^3c_2^2c_3^4 + \frac{195}{4}\alpha^4b^4\beta^{16}c_1^2c_2^4c_3^3 + \frac{99}{16}\alpha^4b^4\beta^{16}c_1c_2^6c_3^2 + 168\alpha^4b^2\beta^{16}c_1^4c_2c_3^4 - \\
& -132\alpha^4b^2\beta^{16}c_1^3c_2^3c_3^3 - \frac{9}{8}\alpha^4b^2\beta^{16}c_1^2c_2^5c_3^2 + \frac{189}{16}\alpha^4b^2\beta^{16}c_1c_2^7c_3 - 24\alpha^2b^4\beta^{18}c_1^2c_2^3c_3^4 + \\
& +\frac{39}{4}\alpha^2b^4\beta^{18}c_1c_2^5c_3^3 + 60\alpha^2b^2\beta^{18}c_1^3c_2^2c_3^4 - 48\alpha^2b^2\beta^{18}c_1^2c_2^4c_3^3 + \frac{135}{16}\alpha^2b^2\beta^{18}c_1c_2^6c_3^2 + \\
& +324\alpha^{14}b^4\beta^6c_1^6c_2c_3^2 + 102\alpha^{14}b^4\beta^6c_1^5c_2^3c_3 + 258\alpha^{14}b^2\beta^6c_1^6c_2^2c_3 + 477\alpha^{12}b^4\beta^8c_1^5c_2^2c_3^2 + \\
& +\frac{219}{4}\alpha^{12}b^4\beta^8c_1^4c_2^4c_3 - 342\alpha^{12}b^2\beta^8c_1^6c_2c_3^2 + 339\alpha^{12}b^2\beta^8c_1^5c_2^3c_3 - 324\alpha^{10}b^4\beta^{10}c_1^5c_2c_3^3 + \\
& +486\alpha^{10}b^4\beta^{10}c_1^4c_2^3c_3^2 - \frac{15}{4}\alpha^{10}b^4\beta^{10}c_1^3c_2^5c_3 - 459\alpha^{10}b^2\beta^{10}c_1^5c_2^2c_3^2 + 303\alpha^{10}b^2\beta^{10}c_1^4c_2^4c_3 - \\
& -177\alpha^8b^4\beta^{12}c_1^4c_2^2c_3^3 + \frac{1125}{4}\alpha^8b^4\beta^{12}c_1^3c_2^4c_3^2 - \frac{285}{16}\alpha^8b^4\beta^{12}c_1^2c_2^6c_3 + 24\alpha^8b^2\beta^{12}c_1^5c_2c_3^3 - \\
& -\frac{711}{2}\alpha^8b^2\beta^{12}c_1^4c_2^3c_3^2 + \frac{735}{4}\alpha^8b^2\beta^{12}c_1^3c_2^5c_3 - 288\alpha^6b^4\beta^{14}c_1^4c_2c_3^4 + 42\alpha^6b^4\beta^{14}c_1^3c_2^3c_3^3 + \\
& +\frac{297}{4}\alpha^6b^4\beta^{14}c_1^2c_2^5c_3^2 - 6\alpha^6b^4\beta^{14}c_1c_2^7c_3 - 96\alpha^6b^2\beta^{14}c_1^4c_2^2c_3^3 - \frac{513}{4}\alpha^6b^2\beta^{14}c_1^3c_2^4c_3^2 + \\
& +\frac{543}{8}\alpha^6b^2\beta^{14}c_1^2c_2^6c_3 - 20\alpha^{16}\beta^4c_1^8c_3 + 50\alpha^{16}\beta^4c_1^7c_2^2 - \frac{27}{64}\alpha^4b^2\beta^{16}c_2^9 - 32\alpha^4\beta^{16}c_1^5c_3^4 +
\end{aligned}
$$

$$+\frac{243}{64}\alpha^4\beta^{16}c_1c_2^8+3\alpha^{20}b^4c_1^7c_2^2+6\alpha^{20}b^2c_1^8c_2+80\alpha^{14}\beta^6c_1^6c_2^3+24\alpha^{12}\beta^8c_1^7c_3^2+89\alpha^{12}\beta^8c_1^5c_2^4+$$

$$+71\alpha^{10}\beta^{10}c_1^4c_2^5+16\alpha^8\beta^{12}c_1^6c_3^3+\frac{323}{8}\alpha^8\beta^{12}c_1^3c_2^6+\frac{9}{64}\alpha^6b^4\beta^{14}c_2^9+\frac{63}{4}\alpha^6\beta^{14}c_1^2c_2^7+$$

$$+\frac{9}{16}b^4\beta^{20}c_2^6c_3^3+\frac{27}{32}b^2\beta^{20}c_2^7c_3^2-8\beta^{20}c_1^3c_2^2c_3^4+9\beta^{20}c_1^2c_2^4c_3^3-$$

$$-\frac{27}{8}\beta^{20}c_1c_2^6c_3^2+20\alpha^{18}\beta^2c_1^8c_2+\frac{27}{64}\alpha^2\beta^{18}c_2^9,$$

$$\Psi=-L^4X^3Y^2b^4c_2n-3\alpha^{18}c_1^8c_2+189\alpha^8b^6\beta^{10}c_1^4c_2c_3^4-18\alpha^8b^6\beta^{10}c_1^3c_2^3c_3^3+\frac{3}{4}\alpha^8b^6\beta^{10}c_1^2c_2^5c_3^2-$$

$$-\frac{1}{16}\alpha^8b^6\beta^{10}c_1c_2^7c_3+36\alpha^8b^4\beta^{10}c_1^5c_3^4n+\frac{1}{128}\alpha^8b^4\beta^{10}c_1c_2^8n-\frac{249}{2}\alpha^8b^4\beta^{10}c_1^4c_2^2c_3^3+$$

$$+\frac{1167}{8}\alpha^8b^4\beta^{10}c_1^3c_2^4c_3^2-\frac{131}{16}\alpha^8b^4\beta^{10}c_1^2c_2^6c_3+\frac{5}{16}\alpha^8b^2\beta^{10}c_1^2c_2^7n+54\alpha^8b^2\beta^{10}c_1^5c_2c_3^3-$$

$$-\frac{1233}{4}\alpha^8b^2\beta^{10}c_1^4c_2^3c_3^2+\frac{787}{16}\alpha^8b^2\beta^{10}c_1^3c_2^5c_3+6\alpha^8\beta^{10}c_1^6c_3^2c_3n-\frac{255}{2}\alpha^8\beta^{10}c_1^5c_2^2c_3^2n+$$

$$+\frac{71}{2}\alpha^8\beta^{10}c_1^4c_2^4c_3n+\frac{9}{2}\alpha^8\beta^{10}c_1^4c_2^4c_3n-30\alpha^6X^2Y^2b^6c_1^2c_3+3\alpha^6X^2Y^2b^6c_1c_2^2+$$

$$+81\alpha^6b^6\beta^{12}c_1^3c_2^2c_3^4-13\alpha^6b^6\beta^{12}c_1^2c_2^4c_3^3+\frac{15}{16}\alpha^6b^6\beta^{12}c_1c_2^6c_3^2+132\alpha^6b^4\beta^{12}c_1^4c_2c_3^4-$$

$$-98\alpha^6b^4\beta^{12}c_1^3c_2^3c_3^3+\frac{885}{16}\alpha^6b^4\beta^{12}c_1^2c_2^5c_3^2-\frac{109}{32}\alpha^6b^4\beta^{12}c_1c_2^7c_3-48\alpha^6b^2\beta^{12}c_1^5c_3^4n+$$

$$+\frac{13}{32}\alpha^6b^2\beta^{12}c_1c_2^8n+15\alpha^6b^2\beta^{12}c_1^4c_2^2c_3^3-\frac{273}{2}\alpha^6b^2\beta^{12}c_1^3c_2^4c_3^2+\frac{713}{32}\alpha^6b^2\beta^{12}c_1^2c_2^6c_3-$$

$$-21\alpha^6\beta^{12}c_1^5c_2c_3^3n-\frac{435}{4}\alpha^6\beta^{12}c_1^4c_2^3c_3^2n+22\alpha^6\beta^{12}c_1^3c_2^5c_3n+9\alpha^6X^2Y^2b^4c_1^2c_2+$$

$$+\frac{33}{2}\alpha^4b^6\beta^{14}c_1^2c_2^3c_3^4-\frac{9}{4}\alpha^4b^6\beta^{14}c_1c_2^5c_3^3+\frac{3}{128}\alpha^4b^4\beta^{14}c_2^8c_3n+52\alpha^4b^4\beta^{14}c_1^3c_2^2c_3^4-$$

$$-\frac{251}{8}\alpha^4b^4\beta^{14}c_1^2c_2^4c_3^3+\frac{351}{32}\alpha^4b^4\beta^{14}c_1c_2^6c_3^2+76\alpha^4b^2\beta^{14}c_1^4c_2c_3^4-37\alpha^4b^2\beta^{14}c_1^3c_2^3c_3^3-$$

$$-\frac{339}{16}\alpha^4b^2\beta^{14}c_1^2c_2^5c_3^2+\frac{225}{64}\alpha^4b^2\beta^{14}c_1c_2^7c_3-6\alpha^4\beta^{14}c_1^5c_3^3c_3n-52\alpha^4\beta^{14}c_1^4c_2^2c_3^3n-$$

$$-\frac{195}{4}\alpha^4\beta^{14}c_1^3c_2^4c_3^2n+\frac{243}{32}\alpha^4\beta^{14}c_1^2c_2^6c_3n-\frac{39}{32}\alpha^4\beta^{14}c_1^2c_2^6c_3n+\frac{3}{4}\alpha^4X^2Y^2b^6\beta^2c_2^3+$$

$$+3\alpha^2X^4c_1c_2^2b^2L^8-30\alpha^2X^4c_1^2c_3b^2L^8+\frac{13}{8}\alpha^2\beta^{16}b^6c_1c_3^4c_2^4+7\alpha^2\beta^{16}b^4c_1^2c_3^4c_2^3-$$

$$-\frac{49}{16}\alpha^2\beta^{16}b^4c_1c_2^5c_3^3-\frac{3}{32}\alpha^2\beta^{16}b^2nc_2^8c_3+22\alpha^2\beta^{16}c_1^3c_3^4c_2^2b^2-\frac{125}{8}\alpha^2\beta^{16}c_1^2c_2^4c_3^3b^2+$$

$$+\frac{15}{16}\alpha^2\beta^{16}c_1c_2^6c_3^2b^2+23\alpha^2\beta^{16}nc_1^4c_3^4c_2-\frac{95}{2}\alpha^2\beta^{16}nc_1^3c_2^3c_3^3-\frac{75}{8}\alpha^2\beta^{16}nc_1^2c_2^5c_3^2+$$

$$+\frac{59}{64}\alpha^2\beta^{16}nc_1c_2^7c_3-\frac{35}{64}\alpha^2\beta^{16}c_3nc_2^7c_1-\frac{9}{4}\alpha^2X^2Y^2b^4\beta^4c_2^3+3\alpha^2X^2Y^2\beta^4c_1^2c_2+$$

$$+\frac{111}{8}\beta^{18}c_3nc_1^2c_2^4c_3^2+\frac{57}{32}\beta^{18}c_3nc_1c_2^6c_3-36X^2Y^2b^4\beta^6c_1c_3^2+\frac{27}{2}X^2Yb^4\beta^6c_2^2c_3-$$

$$-3L^4X^3Yb^2\beta^4c_2^3-60L^4X^3Y\beta^4c_1^2c_3+6L^4X^3Y\beta^4c_1c_2^2+\frac{3}{2}X^2Y^2b^2\beta^4c_2^3\gamma^2+$$

$$+30X^2Y^2\beta^4c_1^2c_3\gamma^2-3X^2Y^2\beta^4c_1c_2^2\gamma^2+3\alpha^{16}b^6\beta^2c_1^5c_2^3c_3+\frac{1}{8}\alpha^{16}b^4\beta^2c_1^5c_2^4n+$$

$$+16\alpha^{16}b^4\beta^2c_1^6c_2^2c_3-\alpha^{16}b^2\beta^2c_1^6c_2^3n+37\alpha^{16}b^2\beta^2c_1^7c_2c_3+\frac{65}{2}\alpha^{14}b^2\beta^4c_1^6c_2^2c_3+$$

$$+14\alpha^{14}\beta^4c_1^7c_2c_3n+2\alpha^{14}\beta^4c_1^7c_2c_3n+27\alpha^{14}b^6\beta^4c_1^5c_2^2c_3^2-\frac{5}{2}\alpha^{14}b^6\beta^4c_1^4c_2^4c_3-$$

$$-\frac{1}{4}\alpha^{14}b^4\beta^4c_1^4c_2^5n+141\alpha^{14}b^4\beta^4c_1^6c_2c_3^2+\frac{53}{2}\alpha^{14}b^4\beta^4c_1^5c_2^3c_3+24\alpha^{14}b^2\beta^4c_1^7c_3^2n-$$

$$-\frac{5}{2}\alpha^{14}b^2\beta^4c_1^5c_2^4n+108\alpha^{12}b^6\beta^6c_1^5c_2c_3^3-\frac{9}{2}\alpha^{12}b^6\beta^6c_1^4c_2^3c_3^2-\frac{1}{2}\alpha^{12}b^6\beta^6c_1^3c_2^5c_3+$$

$$+36\alpha^{12}b^4\beta^6c_1^6c_3^3n-\frac{1}{16}\alpha^{12}b^4\beta^6c_1^3c_2^6n+\frac{471}{2}\alpha^{12}b^4\beta^6c_1^5c_2^2c_3^2+\frac{73}{4}\alpha^{12}b^4\beta^6c_1^4c_2^4c_3-$$

$$-\frac{5}{2}\alpha^{12}b^2\beta^6c_1^4c_2^5n-273\alpha^{12}b^2\beta^6c_1^6c_2c_3^2+\frac{175}{4}\alpha^{12}b^2\beta^6c_1^5c_2^3c_3+12\alpha^{12}\beta^6c_1^7c_3c_3n+$$

$$+\frac{55}{2}\alpha^{12}\beta^6c_1^6c_2^2c_3n+\frac{13}{2}\alpha^{12}\beta^6c_1^6c_2^2c_3n+54\alpha^{10}b^6\beta^8c_1^4c_2^2c_3^3-\frac{21}{2}\alpha^{10}b^6\beta^8c_1^3c_2^4c_3^2+$$

$$+\frac{3}{4}\alpha^{10}b^6\beta^8c_1^2c_2^6c_3+\frac{5}{64}\alpha^{10}b^4\beta^8c_1^2c_2^7n-87\alpha^{10}b^4\beta^8c_1^5c_2c_3^3+\frac{945}{4}\alpha^{10}b^4\beta^8c_1^4c_2^3c_3^2-$$

$$-\frac{41}{16}\alpha^{10}b^4\beta^8c_1^3c_2^5c_3-24\alpha^{10}b^2\beta^8c_1^6c_3^3n-\alpha^{10}b^2\beta^8c_1^3c_2^6n-375\alpha^{10}b^2\beta^8c_1^5c_2^2c_3^2+$$

$$+\frac{229}{4}\alpha^{10}b^2\beta^8c_1^4c_2^4c_3-84\alpha^{10}\beta^8c_1^6c_2c_3^2n+\frac{151}{4}\alpha^{10}\beta^8c_1^5c_2^3c_3n+\frac{33}{4}\alpha^{10}\beta^8c_1^5c_2^3c_3n+$$

$$+\frac{1}{4}\beta^{18}b^4c_3^4nc_1c_2^4-2\beta^{18}b^2nc_1^2c_2^3c_3^4+\frac{5}{4}\beta^{18}b^2nc_1c_2^5c_3^3-\beta^{18}c_3nc_1^3c_3^3c_2^2-14\alpha^{16}\beta^2c_1^8c_3-$$

$$-\frac{23}{2}\alpha^{16}\beta^2c_1^7c_2^2-22\alpha^{14}\beta^4c_1^6c_2^3+30\alpha^{12}\beta^6c_1^7c_3^2-\frac{53}{2}\alpha^{12}\beta^6c_1^5c_2^4-\frac{337}{16}\alpha^{10}\beta^8c_1^4c_2^5+$$

$$+\frac{1}{256}\alpha^8b^6\beta^{10}c_2^9+12\alpha^8\beta^{10}c_1^6c_3^3-\frac{353}{32}\alpha^8\beta^{10}c_1^3c_2^6+\frac{1}{16}\alpha^6b^4\beta^{12}c_2^9-\frac{55}{16}\alpha^6\beta^{12}c_1^2c_2^7-$$

$$-\frac{57}{256}\alpha^4b^2\beta^{14}c_2^9-\frac{3}{64}\beta^{18}b^2c_2^7c_3^2-32\alpha^4\beta^{14}c_1^5c_3^4-\frac{15}{32}\alpha^4\beta^{14}c_1c_2^8-\frac{9}{64}\alpha^2\beta^{16}nc_2^9+$$

$$+\frac{1}{16}\beta^{18}b^6c_2^5c_3^4-\frac{1}{32}\beta^{18}b^4c_2^6c_3^3-\frac{1}{16}\beta^{18}nc_2^8c_3-\frac{5}{64}\beta^{18}c_3nc_2^8-8\beta^{18}c_1^3c_2^2c_3^4+5\beta^{18}c_1^2c_2^4c_3^3-$$

$$-\frac{3}{4}\beta^{18}c_1c_2^6c_3^2+\frac{1}{8}\alpha^{18}b^6c_1^5c_2^4-\frac{1}{2}\alpha^{18}b^4c_1^6c_2^3+18\alpha^{18}b^2c_1^8c_3-\frac{9}{2}\alpha^{18}b^2c_1^7c_2^2+108\alpha^8b^4\beta^{10}c_1^5c_3^4+$$

$$+\frac{11}{64}\alpha^8 b^4\beta^{10}c_1c_2^8-\frac{103}{64}\alpha^8 b^2\beta^{10}c_1^2c_2^7-6\alpha^8\beta^{10}c_1^6c_3^3n-\frac{115}{16}\alpha^8\beta^{10}c_1^3c_2^6n+84\alpha^8\beta^{10}c_1^5c_2^2c_3^2-$$

$$-\frac{199}{8}\alpha^8\beta^{10}c_1^4c_2^4c_3-\frac{1}{32}\alpha^6 b^6\beta^{12}c_2^8c_3-\frac{1}{128}\alpha^6 b^4\beta^{12}c_2^9n+72\alpha^6 b^2\beta^{12}c_1^5c_3^4-$$

$$-\frac{137}{128}\alpha^6 b^2\beta^{12}c_1c_2^8-\frac{29}{8}\alpha^6\beta^{12}c_1^2c_2^7n-6\alpha^6\beta^{12}c_1^5c_2c_3^3+60\alpha^6\beta^{12}c_1^4c_2^3c_3^2-14\alpha^6\beta^{12}c_1^3c_2^5c_3+$$

$$+\frac{3}{32}\alpha^4 b^6\beta^{14}c_2^7c_3^2-\frac{15}{32}\alpha^4 b^4\beta^{14}c_2^8c_3+\frac{3}{32}\alpha^4 b^2\beta^{14}c_2^9n+22\alpha^4\beta^{14}c_1^5c_3^4n-\frac{69}{64}\alpha^4\beta^{14}c_1c_2^8n+$$

$$+5\alpha^4\beta^{14}c_1^4c_2^2c_3^3+\frac{201}{18}\alpha^4\beta^{14}c_1^3c_2^4c_3^2-\frac{495}{32}\alpha^4\beta^{14}c_1^2c_2^6c_3+\frac{147}{16}\alpha^4\beta^{14}c_1^2c_2^6c_3+3\alpha^2X^4c_1^2c_2L^8-$$

$$-\frac{1}{8}\alpha^2\beta^{16}b^6c_2^6c_3^3+\frac{45}{64}\alpha^2\beta^{16}b^4c_2^7c_3^2-\frac{21}{128}\alpha^2\beta^{16}c_2^8c_3b^2-32\alpha^2\beta^{16}c_3^4c_1^4c_2+\frac{31}{2}\alpha^2\beta^{16}c_3^3c_1^3c_2^3+$$

$$+\frac{39}{16}\alpha^2\beta^{16}c_3^2c_1^2c_2^5-\frac{39}{32}\alpha^2\beta^{16}c_3c_2^7c_1-2XY^4b^6+48L^4X^3Yb^2\beta^4c_1c_2c_3-$$

$$-24X^2Y^2b^2\beta^4c_1c_2c_3\gamma^2-12\alpha^4X^2Y^2b^6\beta^2c_1c_2c_3-3\alpha^2L^4X^3Y\gamma^2c_1c_2^2b^2+30\alpha^2L^4X^3Y\gamma^2c_1^2c_3b^2+$$

$$+36\alpha^2X^2Y^2b^4\beta^4c_1c_2c_3+120\alpha^2L^4X^3Yb^2\beta^2c_1^2c_3+9\alpha^{18}b^4c_1^7c_2c_3-\frac{3}{16}\alpha^{16}b^6\beta^2c_1^4c_2^5+$$

$$+54\alpha^{16}b^4\beta^2c_1^7c_3^2+\frac{1}{2}\alpha^{16}b^4\beta^2c_1^5c_2^4-\frac{37}{4}\alpha^{16}b^2\beta^2c_1^6c_2^3+4\alpha^{16}\beta^2c_1^8c_3n-\alpha^{16}\beta^2c_1^7c_2^2n-$$

$$-72\alpha^{14}b^2\beta^4c_1^7c_3^2-\frac{47}{8}\alpha^{14}b^2\beta^4c_1^5c_2^4-4\alpha^{14}\beta^4c_1^6c_2^3n-38\alpha^{14}\beta^4c_1^7c_2c_3+\frac{1}{16}\alpha^{14}b^6\beta^4c_1^3c_2^6+$$

$$+\frac{1}{8}\alpha^{14}b^4\beta^4c_1^4c_2^5-\frac{97}{2}\alpha^{12}\beta^6c_1^6c_2^2c_3+\frac{1}{32}\alpha^{12}b^6\beta^6c_1^2c_2^7-54\alpha^{12}b^4\beta^6c_1^6c_3^3-$$

$$-\frac{13}{16}\alpha^{12}b^4\beta^6c_1^3c_2^6-\frac{19}{16}\alpha^{12}b^2\beta^6c_1^4c_2^5-24\alpha^{12}\beta^6c_1^7c_3^2n-\frac{31}{4}\alpha^{12}\beta^6c_1^5c_2^4n+$$

$$+162\alpha^{10}b^6\beta^8c_1^5c_3^4-\frac{3}{128}\alpha^{10}b^6\beta^8c_1c_2^8-\frac{1}{4}\alpha^{10}b^4\beta^8c_1^2c_2^7-$$

$$-18\alpha^{10}b^2\beta^8c_1^6c_3^3-\frac{23}{32}\alpha^{10}b^2\beta^8c_1^3c_2^6-\frac{37}{4}\alpha^{10}\beta^8c_1^4c_2^5n+81\alpha^{10}\beta^8c_1^6c_2c_3^2-41\alpha^{10}\beta^8c_1^5c_2^3c_3-$$

$$-\frac{1}{32}\beta^{18}b^4nc_2^6c_3^3+\frac{1}{4}\beta^{18}b^4c_1c_2^4c_3^4-\frac{3}{16}\beta^{18}b^2nc_2^7c_3^2+\beta^{18}b^2c_1^2c_2^3c_3^4+22\alpha^4b^4\beta^{14}c_1^3c_2^2c_3^4n-$$

$$-60\alpha^4L^4X^3Yb^4c_1^2c_3-\frac{5}{8}\beta^{18}b^2c_1c_2^5c_3^3+5\beta^{18}nc_1^3c_3^4c_2^2-\frac{143}{8}\beta^{18}nc_1^2c_2^4c_3^3-\frac{15}{32}\beta^{18}nc_1c_2^6c_3^2+$$

$$+\frac{3}{4}\beta^2X^4c_2^3b^2L^8c_1^2c_2+30\alpha^4X^2Y^2b^4c_1^2c_3\gamma^2-6\alpha^4X^2Y^2b^2c_1^2c_2\gamma^2-3\alpha^4X^2Y^2b^4c_1c_2^2\gamma^2+$$

$$+12\alpha^4L^4X^3Yb^2+\frac{315}{4}\alpha^6\beta^{12}c_1^4c_2^3c_3c_3n+6\alpha^4L^4X^3Yb^4c_1c_2^2+$$

$$+\frac{99}{2}\alpha^6b^2\beta^{12}c_1^3c_2^4c_3^2n-\frac{25}{2}\alpha^6b^2\beta^{12}c_1^2c_2^6c_3n+21\alpha^6\beta^{12}c_1^5c_2c_3^3n+$$

$$+\frac{19}{64}\alpha^6b^4\beta^{12}c_1c_2^7c_3n-12\alpha^6b^2\beta^{12}c_1^4c_2^2c_3^3n+\frac{165}{2}\alpha^8\beta^{10}c_1^5c_2^2c_3c_3n+48\alpha^6b^4\beta^{12}c_1^4c_2c_3^4n+$$
$$+\frac{47}{2}\alpha^6b^4\beta^{12}c_1^3c_2^3c_3^3n-\frac{75}{16}\alpha^6b^4\beta^{12}c_1^2c_2^5c_3^2n+\frac{1}{16}\alpha^8b^4\beta^{10}c_1^2c_2^6c_3n-60\alpha^8b^2\beta^{10}c_1^5c_2c_3^3n+$$
$$+105\alpha^8b^2\beta^{10}c_1^4c_2^3c_3^2n-\frac{95}{4}\alpha^8b^2\beta^{10}c_1^3c_2^5c_3n-\frac{19}{8}\alpha^{10}b^4\beta^8c_1^3c_2^5c_3n+108\alpha^{10}b^2\beta^8c_1^5c_2^2c_3^2n-$$
$$-\frac{43}{2}\alpha^{10}b^2\beta^8c_1^4c_2^4c_3n+48\alpha^{10}\beta^8c_1^6c_2c_3c_3n+\frac{171}{2}\alpha^8b^4\beta^{10}c_1^4c_2^2c_3^3n-\frac{33}{4}\alpha^8b^4\beta^{10}c_1^3c_2^4c_3^2n-$$
$$-3X^2Y^2\beta^6c_1c_2^2+\frac{1}{4}\alpha^{18}b^4nc_1^6c_2^3-L^4X^3Y^2b^4c_2+30\beta^2X^4c_1^2c_3L^8-3\beta^2X^4c_1c_2^2L^8+$$
$$+\frac{9}{4}X^2Y^2b^2\beta^6c_2^3+30X^2Y^2\beta^6c_1^2c_3+\frac{1}{2}\alpha^4b^4\beta^{14}c_1^2c_2^4c_3^3n-$$
$$-\frac{15}{32}\alpha^4b^4\beta^{14}c_1c_2^6c_3^2n-56\alpha^4b^2\beta^{14}c_1^4c_2c_3^4n+26\alpha^4b^2\beta^{14}c_1^3c_2^3c_3^3n+6\alpha^4b^2\beta^{14}c_1^2c_2^5c_3^2n-$$
$$-\frac{21}{8}\alpha^4b^2\beta^{14}c_1c_2^7c_3n+36\alpha^4\beta^{14}c_1^4c_2^2c_3^3n+\frac{87}{2}\alpha^4\beta^{14}c_1^3c_2^4c_3c_3n+$$
$$+90\alpha^4X^2Y^2b^4\beta^2c_1^2c_3-9\alpha^4X^2Y^2b^4\beta^2c_1c_2^2-9\alpha^4X^2Y^2b^2\beta^2c_1^2c_2-$$
$$-3\alpha^2L^4X^3Y\gamma^2c_1^2c_2+4\alpha^2\beta^{16}b^4c_3^4nc_1^2c_2^3-\frac{7}{16}\alpha^2\beta^{16}b^4c_3^3nc_1c_2^5-20\alpha^2\beta^{16}b^2nc_1^3c_3^4c_2^2+$$
$$+\frac{25}{2}\alpha^2\beta^{16}b^2nc_1^2c_2^4c_3^3-\frac{3}{2}\alpha^2\beta^{16}b^2nc_1c_2^6c_3^2-7\alpha^2\beta^{16}c_3nc_1^4c_3^3c_2+\frac{63}{2}\alpha^2\beta^{16}c_3nc_1^3c_2^3c_3^2+$$
$$+\frac{105}{8}\alpha^2\beta^{16}c_3nc_1^2c_2^5c_3-90\alpha^2X^2Y^2b^2\beta^4c_1^2c_3+9\alpha^2X^2Y^2b^2\beta^4c_1c_2^2+12\alpha^2X^2Y^2b^6\beta^4c_1c_3^2-$$
$$-\frac{9}{2}\alpha^2X^2Y^2b^6\beta^4c_2^2c_3+\frac{3}{2}\alpha^2L^4X^3Yb^4\beta^2c_2^3-\frac{3}{4}\alpha^2X^2Y^2b^4\beta^2c_2^3\gamma^2-6\alpha^2L^4X^3Y\beta^2c_1^2c_2+$$
$$+3\alpha^2X^2Y^2\beta^2c_1^2c_2\gamma^2-12\beta^2X^4c_1c_3c_2b^2L^8-30\beta^2L^4X^3Y\gamma^2c_1^2c_3-\frac{3}{4}\beta^2L^4X^3Y\gamma^2c_2^3b^2+$$
$$+3\beta^2L^4X^3Y\gamma^2c_1c_2^2-36X^2Y^2b^2\beta^6c_1c_2c_3+24L^4X^3Yb^4\beta^4c_1c_3^2-9L^4X^3Yb^4\beta^4c_2^2c_3-$$
$$-12X^2Y^2b^4\beta^4c_1c_3^2\gamma^2+\frac{9}{2}X^2Y^2b^4\beta^4c_2^2c_3\gamma^2+4\alpha^{16}b^4\beta^2c_1^6c_2^2c_3n+4\alpha^{16}b^2\beta^2c_1^7c_2c_3n+$$
$$+21\alpha^{14}b^4\beta^4c_1^6c_2c_3^2n+\frac{19}{4}\alpha^{14}b^4\beta^4c_1^5c_2^3c_3n+4\alpha^{14}b^2\beta^4c_1^6c_2^2c_3n+\frac{81}{2}\alpha^{12}b^4\beta^6c_1^5c_2^2c_3^2n-$$
$$-\frac{13}{8}\alpha^{12}b^4\beta^6c_1^4c_2^4c_3n+72\alpha^{12}b^2\beta^6c_1^6c_2c_3^2n-8\alpha^{12}b^2\beta^6c_1^5c_2^3c_3n+105\alpha^{10}b^4\beta^8c_1^5c_2c_3^3n+$$
$$+\frac{27}{2}\alpha^{10}b^4\beta^8c_1^4c_2^3c_3^2n-12\alpha^2L^4X^3Yb^2\beta^2c_1c_2^2-60\alpha^2X^2Y^2b^2\beta^2c_1^2c_3\gamma^2+$$
$$+6\alpha^2X^2Y^2b^2\beta^2c_1c_2^2\gamma^2-24\alpha^2L^4X^3Yb^4\beta^2c_1c_2c_3+$$
$$+12\beta^2L^4X^3Y\gamma^2c_1c_3c_2b^2+12\alpha^2X^2Y^2b^4\beta^2c_1c_2c_3\gamma^2,$$

$$\Lambda = -\frac{375}{2}\alpha^8 b^2\beta^{12}c_1^3c_2^4c_3^2 + \frac{95}{8}\alpha^8 b^2\beta^{12}c_1^2c_2^6c_3 + 192\alpha^6 b^2\beta^{14}c_1^4c_2c_3^4 -$$

$$-28\alpha^6 b^2\beta^{14}c_1^3c_2^3c_3^3 - \frac{99}{2}\alpha^6 b^2\beta^{14}c_1^2c_2^5c_3^2 + 4\alpha^6 b^2\beta^{14}c_1c_2^7c_3 -$$

$$-24\alpha^{18}b^2\beta^2c_1^7c_2c_3 - 2\alpha^{20}c_1^8c_2 + \frac{9}{64}\alpha^4\beta^{16}c_2^9 - \frac{9}{32}\beta^{20}c_2^7c_3^2 + \frac{1}{2}\alpha^{14}b^2\beta^6c_1^4c_2^5 -$$

$$-86\alpha^{14}\beta^6c_1^6c_2^2c_3 + 72\alpha^{12}b^2\beta^8c_1^6c_3^3 + \frac{13}{8}\alpha^{12}b^2\beta^8c_1^3c_2^6 + 114\alpha^{12}\beta^8c_1^6c_2c_3^2 -$$

$$-113\alpha^{12}\beta^8c_1^5c_2^3c_3 + \frac{1}{2}\alpha^{10}b^2\beta^{10}c_1^2c_2^7 + 153\alpha^{10}\beta^{10}c_1^5c_2^2c_3^2 - 101\alpha^{10}\beta^{10}c_1^4c_2^4c_3 +$$

$$+144\alpha^8b^2\beta^{12}c_1^5c_3^4 - \frac{7}{32}\alpha^8b^2\beta^{12}c_1c_2^8 - 8\alpha^8\beta^{12}c_1^5c_2c_3^3 + \frac{237}{2}\alpha^8\beta^{12}c_1^4c_2^3c_3^2 -$$

$$-\frac{245}{4}\alpha^8\beta^{12}c_1^3c_2^5c_3 + 32\alpha^6\beta^{14}c_1^4c_2^2c_3^3 + \frac{171}{4}\alpha^6\beta^{14}c_1^3c_2^4c_3^2 - \frac{181}{8}\alpha^6\beta^{14}c_1^2c_2^6c_3 -$$

$$-62\alpha^{16}b^2\beta^4c_1^6c_2^2c_3 + 88\alpha^4b^2\beta^{16}c_1^3c_2^2c_3^4 - \frac{65}{2}\alpha^4b^2\beta^{16}c_1^2c_2^4c_3^3 -$$

$$-\frac{33}{8}\alpha^4b^2\beta^{16}c_1c_2^6c_3^2 + 16\alpha^2b^2\beta^{18}c_1^2c_2^3c_3^4 - \frac{13}{2}\alpha^2b^2\beta^{18}c_1c_2^5c_3^3 - 216\alpha^{14}b^2\beta^6c_1^6c_2c_3^2 -$$

$$-\frac{3}{32}\alpha^6b^2\beta^{14}c_2^9 - 48\alpha^6\beta^{14}c_1^5c_3^4 + \frac{51}{64}\alpha^6\beta^{14}c_1c_2^8 - \frac{41}{8}\alpha^{12}\beta^8c_1^4c_2^5 - \frac{1}{16}\alpha^{10}\beta^{10}c_1^3c_2^6 +$$

$$+\frac{47}{32}\alpha^8\beta^{12}c_1^2c_2^7 - \frac{9}{64}\alpha^2\beta^{18}c_2^8c_3 - 2\alpha^{20}b^2c_1^7c_2^2 + 36\alpha^{14}\beta^6c_1^7c_3^2 - \frac{43}{4}\alpha^{14}\beta^6c_1^5c_2^4 -$$

$$-\frac{3}{8}b^2\beta^{20}c_2^6c_3^3 - 2\beta^{20}c_1^2c_2^3c_3^4 + \frac{3}{2}\beta^{20}c_1c_2^5c_3^3 - 12\alpha^{18}\beta^2c_1^8c_3 - 7\alpha^{18}\beta^2c_1^7c_2^2 -$$

$$-\frac{23}{2}\alpha^{16}\beta^4c_1^6c_2^3 - 4L^4X^2Y^3b^4 - 20\alpha^2\beta^{18}c_1^3c_2^2c_3^4 +$$

$$+16\alpha^2\beta^{18}c_1^2c_2^4c_3^3 - \frac{45}{16}\alpha^2\beta^{18}c_1c_2^6c_3^2 + \frac{3}{8}\alpha^4\beta^{16}c_1^2c_2^5c_3^2 - \frac{63}{16}\alpha^4\beta^{16}c_1c_2^7c_3 - 44\alpha^{16}\beta^4c_1^7c_2c_3 +$$

$$+\frac{9}{32}\alpha^4b^2\beta^{16}c_2^8c_3 - 56\alpha^4\beta^{16}c_1^4c_2c_3^4 + 44\alpha^4\beta^{16}c_1^3c_2^3c_3^3 + b^2\beta^{20}c_1c_2^4c_3^4 -$$

$$-4\alpha^{18}b^2\beta^2c_1^6c_2^3 - 72\alpha^{16}b^2\beta^4c_1^7c_3^2 - \frac{5}{2}\alpha^{16}b^2\beta^4c_1^5c_2^4 + 118\alpha^8b^2\beta^{12}c_1^4c_2^2c_3^3 +$$

$$+\frac{5}{2}\alpha^{10}b^2\beta^{10}c_1^3c_2^5c_3 + 216\alpha^{10}b^2\beta^{10}c_1^5c_2c_3^3 - 324\alpha^{10}b^2\beta^{10}c_1^4c_2^3c_3^2 -$$

$$-\frac{73}{2}\alpha^{12}b^2\beta^8c_1^4c_2^4c_3 - 318\alpha^{12}b^2\beta^8c_1^5c_2^2c_3^2 - 68\alpha^{14}b^2\beta^6c_1^5c_2^3c_3,$$

$$\Upsilon = -\beta^2b^2nc_2^3c_1^6\alpha^{20} + \beta^4nc_2^3c_1^6\alpha^{18} - 30\beta^2b^4c_1^6c_2^2c_3\alpha^{20} - 48\beta^2b^2c_3c_1^7c_2\alpha^{20} - 4\beta^4c_3nc_1^7c_2\alpha^{18} -$$

$$-\frac{3}{2}\beta^4b^2nc_2^4c_1^5\alpha^{18} - \frac{67}{2}\beta^4b^4c_3c_1^5c_2^3\alpha^{18} - 198\beta^4b^4c_1^6c_2c_3^2\alpha^{18} - 126\beta^4b^2c_3c_1^6c_2^2\alpha^{18} -$$

$$-10\beta^4b^2nc_3c_1^6c_2^2\alpha^{18}-\frac{1}{2}\beta^4b^4nc_3c_1^5c_2^3\alpha^{18}-32\beta^6b^2c_3^2nc_1^6c_2\alpha^{16}-$$

$$-18\beta^6b^2nc_3c_1^5c_2^3\alpha^{16}-8\beta^6b^4nc_3^2c_1^5c_2^2\alpha^{16}-\frac{25}{2}\beta^2b^2c_1^6c_2^3\alpha^{20}-\frac{3}{4}\beta^2b^4c_1^5c_2^4\alpha^{20}+$$

$$+\frac{3}{4}\beta^4b^4c_1^4c_2^5\alpha^{18}-108\beta^4b^2c_3^2c_1^7\alpha^{18}-\frac{29}{2}\beta^4b^2c_1^5c_2^4\alpha^{18}-$$

$$-24\beta^4c_3c_1^7c_2\alpha^{18}-8\beta^6c_3^2nc_1^7\alpha^{16}+\frac{7}{2}\beta^6nc_2^4c_1^5\alpha^{16}-23\beta^6c_3c_1^6c_2^2\alpha^{16}-9\beta^6b^2c_1^4c_2^5\alpha^{16}-$$

$$-5b^2c_1^7c_2^2\alpha^{22}-\frac{3}{2}b^4c_1^6c_2^3\alpha^{22}-12\beta^2c_1^8c_3\alpha^{20}-4\beta^2c_1^7c_2^2\alpha^{20}-\frac{3}{2}\beta^4c_1^6c_2^3\alpha^{18}-2c_1^8c_2\alpha^{22}-$$

$$-170\beta^6b^2c_3c_1^5c_2^3\alpha^{16}-12\beta^6c_3nc_1^6c_2^2\alpha^{16}-\frac{15}{2}\beta^6b^4c_3c_1^4c_2^4\alpha^{16}-218\beta^6b^2c_3^2c_1^6c_2\alpha^{16}-$$

$$-\frac{1}{2}\beta^6b^2nc_2^5c_1^4\alpha^{16}-\frac{25}{2}\beta^8b^2nc_3c_1^4c_2^4\alpha^{14}-58\beta^8b^2c_3^2nc_1^5c_2^2\alpha^{14}-$$

$$-\frac{11}{2}\beta^8b^4nc_3^2c_1^4c_2^3\alpha^{14}-42\beta^8b^4nc_3^3c_1^5c_2\alpha^{14}+\frac{1}{2}\beta^8b^4nc_3c_1^3c_2^5\alpha^{14}-$$

$$-45\beta^{10}b^2c_3^2nc_1^4c_2^3\alpha^{12}-96\beta^{10}b^2c_3^3nc_1^5c_2\alpha^{12}-\frac{5}{2}\beta^{10}b^2nc_3c_1^3c_2^5\alpha^{12}-$$

$$-432\beta^6b^4c_1^6c_3^3\alpha^{16}+\frac{3}{4}\beta^6b^4c_1^3c_2^6\alpha^{16}+146\beta^8c_3^2c_1^6c_2\alpha^{14}-10\beta^8c_3c_1^5c_2^3\alpha^{14}+\frac{3}{32}\beta^8b^4c_1^2c_2^7\alpha^{14}+$$

$$+216\beta^8b^2c_3^3c_1^6\alpha^{14}-\frac{23}{16}\beta^8b^2c_1^3c_2^6\alpha^{14}+6\beta^8nc_2^5c_1^4\alpha^{14}+\frac{11}{2}\beta^6c_1^5c_2^4\alpha^{16}+64\beta^6c_3^2c_1^7\alpha^{16}+$$

$$+\frac{85}{8}\beta^8c_1^4c_2^5\alpha^{14}-326\beta^6b^4c_3^2c_1^5c_2^2\alpha^{16}+\frac{1}{2}\beta^8b^2nc_2^6c_1^3\alpha^{14}-798\beta^8b^4c_3^3c_1^5c_2\alpha^{14}-$$

$$-268\beta^8b^4c_3^2c_1^4c_2^3\alpha^{14}+\frac{53}{4}\beta^8b^4c_3c_1^3c_2^5\alpha^{14}-48\beta^8b^2c_3^3nc_1^6\alpha^{14}-$$

$$-214\beta^8b^2c_3^2c_1^5c_2^2\alpha^{14}-154\beta^8b^2c_3c_1^4c_2^4\alpha^{14}-16\beta^8c_3^2nc_1^6c_2\alpha^{14}-20\beta^8c_3nc_1^5c_2^3\alpha^{14}-$$

$$-\frac{1}{8}\beta^{10}b^4nc_3c_1^2c_2^6\alpha^{12}+6\beta^{10}b^4nc_3^2c_1^3c_2^4\alpha^{12}-60\beta^{10}b^4nc_3^3c_1^4c_2^2\alpha^{12}-$$

$$-18\beta^2XY^2b^4c_1^2c_2^2\alpha^{10}-6\beta^2XY^2b^6c_1c_2^3\alpha^{10}-174\beta^{12}b^4nc_3^4c_1^4c_2\alpha^{10}+3\beta^{12}b^4nc_3^3c_1^3c_2^3\alpha^{10}+$$

$$+\frac{7}{4}\beta^{12}b^4nc_3^2c_1^2c_2^5\alpha^{10}-\frac{3}{32}\beta^{12}b^4nc_3c_1c_2^7\alpha^{10}-96\beta^{12}b^2c_3^3nc_1^4c_2^2\alpha^{10}+684\beta^{10}b^4c_3^4c_1^5\alpha^{12}+$$

$$+153\beta^{10}c_3^2c_1^5c_2^2\alpha^{12}+24\beta^{10}c_3^3nc_1^6\alpha^{12}+\frac{45}{4}\beta^{10}c_3c_1^4c_2^4\alpha^{12}b^4c_1^2c_2^2L^4\alpha^{10}-84\beta^{14}b^4nc_3^4c_1^3c_2^2\alpha^8+$$

$$+15\beta^{14}b^4nc_3^3c_1^2c_2^4\alpha^8+144\beta^{14}b^2c_3^4nc_1^4c_2\alpha^8-130\beta^{14}b^2c_3^3nc_1^3c_2^3\alpha^8+11\beta^{14}b^2c_3^2nc_1^2c_2^5\alpha^8+$$

$$+\frac{7}{4}\beta^{14}b^2nc_3c_1c_2^7\alpha^8+18\beta^4XY^2b^2c_1^2c_2^2\alpha^8+18\beta^4XY^2b^4c_1c_2^3\alpha^8+120\beta^4XY^2b^6c_1^2c_3^2\alpha^8+$$

$$+468\beta^{12}b^2c_3^4c_1^5\alpha^{10}+\frac{43}{32}\beta^{12}b^2c_1c_2^8\alpha^{10}+\frac{65}{16}\beta^{12}nc_2^7c_1^2\alpha^{10}-$$

$$-104\beta^{12}c_3^3c_1^5c_2\alpha^{10}+\frac{263}{2}\beta^{12}c_3^2c_1^4c_2^3\alpha^{10}+16\beta^{12}c_3c_1^3c_2^5\alpha^{10}-$$

$$-216\beta^4XY^2b^4c_1^2c_2c_3\alpha^8+12\beta^4XY^2b^6c_1c_2^2c_3\alpha^8-18\beta^2L^4X^2Yb^2c_1^2c_2^2\alpha^8-$$

$$-9\beta^2L^4X^2Yb^4c_1c_2^3\alpha^8+\frac{9}{64}\beta^{12}b^4c_2^9\alpha^{10}+\frac{135}{32}\beta^{12}c_1^2c_2^7\alpha^{10}+1068\beta^{12}b^4c_3^4c_1^4c_2\alpha^{10}-$$

$$-993\beta^{12}b^4c_3^3c_1^3c_2^3\alpha^{10}+\frac{701}{8}\beta^{12}b^4c_3^2c_1^2c_2^5\alpha^{10}-\frac{135}{32}\beta^{12}b^4c_3c_1c_2^7\alpha^{10}+48\beta^{12}b^2c_3^4nc_1^5\alpha^{10}-$$

$$-\frac{1}{32}\beta^{12}b^2nc_2^8c_1\alpha^{10}+346\beta^{12}b^2c_3^3c_1^4c_2^2\alpha^{10}-\frac{205}{2}\beta^{12}b^2c_3^2c_1^3c_2^4\alpha^{10}-\frac{143}{4}\beta^{12}b^2c_3c_1^2c_2^6\alpha^{10}+$$

$$+60\beta^{12}c_3^3nc_1^5c_2\alpha^{10}-35\beta^{12}c_3^2nc_1^4c_2^3\alpha^{10}-\frac{45}{4}\beta^{12}c_3nc_1^3c_2^5\alpha^{10}-$$

$$-\beta^{14}b^4nc_3^2c_1c_2^6\alpha^8+144\beta^{14}b^4c_3^5c_1^4\alpha^8-\frac{35}{32}\beta^{14}b^4c_3c_2^8\alpha^8-$$

$$-\frac{3}{32}\beta^{14}b^2nc_2^9\alpha^8+\frac{51}{32}\beta^{14}nc_2^8c_1\alpha^8+44\beta^{14}c_3^3c_1^4c_2^2\alpha^8+68\beta^{14}c_3^2c_1^3c_2^4\alpha^8+$$

$$+\frac{179}{16}\beta^{14}c_3c_1^2c_2^6\alpha^8+108\beta^2L^4X^2Yb^4c_1^2c_2c_3\alpha^8-\frac{3}{2}\beta^4XY^2b^6c_2^4\alpha^8-72\beta^{14}b^4nc_3^5c_1^4\alpha^8+$$

$$+\frac{1}{32}\beta^{14}b^4nc_3c_2^8\alpha^8+336\beta^{14}b^4c_3^4c_1^3c_2^2\alpha^8-330\beta^{14}b^4c_3^3c_1^2c_2^4\alpha^8+$$

$$+\frac{259}{8}\beta^{14}b^4c_3^2c_1c_2^6\alpha^8+318\beta^{14}b^2c_3^4c_1^4c_2\alpha^8+178\beta^{14}b^2c_3^3c_1^3c_2^3\alpha^8-$$

$$-\frac{217}{4}\beta^{14}b^2c_3^2c_1^2c_2^5\alpha^8-\frac{115}{16}\beta^{14}b^2c_3c_1c_2^7\alpha^8+60\beta^{14}c_3^3nc_1^4c_2^2\alpha^8-$$

$$-\frac{65}{2}\beta^{14}c_3^2nc_1^3c_2^4\alpha^8-2\beta^{14}c_3nc_1^2c_2^6\alpha^8+3X^3c_1^2c_2^2b^2L^8\alpha^8+\frac{3}{8}\beta^{14}b^2c_2^9\alpha^8-216\beta^{14}c_3^4c_1^5\alpha^8+$$

$$+\frac{27}{32}\beta^{14}c_1c_2^8\alpha^8-36\beta^6XY^2b^4c_1c_2^2c_3\alpha^6+216\beta^6XY^2b^2c_1^2c_2c_3\alpha^6+18\beta^4L^4X^2Yb^2c_1c_2^3\alpha^6+$$

$$+180\beta^4L^4X^2Yb^4c_1^2c_3^2\alpha^6+36\beta^2X^3c_1^2c_2b^2c_3L^8\alpha^6+24\beta^6XY^2b^6c_1c_2c_3^2\alpha^6-\frac{9}{4}\beta^4L^4X^2Yb^4c_2^4\alpha^6-$$

$$-3\beta^2X^3c_1c_2^3b^2L^8\alpha^6-60\beta^{16}b^4nc_3^5c_1^3c_2\alpha^6-5\beta^{16}b^4nc_3^4c_1^2c_2^3\alpha^6+$$

$$+\frac{15}{8}\beta^{16}b^4nc_3^3c_1c_2^5\alpha^6+8\beta^{16}b^2c_3^4nc_1^3c_2^2\alpha^6-48\beta^{16}b^2c_3^3nc_1^2c_2^4\alpha^6+$$

$$+\frac{81}{8}\beta^{16}b^2c_3^2nc_1c_2^6\alpha^6+6\beta^6XY^2b^6c_2^3c_3\alpha^6-360\beta^6XY^2b^4c_1^2c_3^2\alpha^6-18\beta^6XY^2b^2c_1c_2^3\alpha^6+$$

$$+9\beta^4L^4X^2Yc_1^2c_2^2\alpha^6+\frac{39}{16}\beta^{16}b^4c_3^2c_2^7\alpha^6-1152\beta^{16}b^2c_3^5c_1^4\alpha^6+\frac{9}{32}\beta^{16}b^2c_3c_2^8\alpha^6-$$

$$-336\beta^{16}c_3^4c_1^4c_2\alpha^6+72\beta^{16}c_3^3c_1^3c_2^3\alpha^6+\frac{351}{8}\beta^{16}c_3^2c_1^2c_2^5\alpha^6+\frac{3}{2}\beta^{16}c_3c_1c_2^7\alpha^6+$$

$$+18\beta^4L^4X^2Yb^4cc_1^3c_2\alpha^6-216\beta^4L^4X^2Yb^2c_1^2c_2c_3\alpha^6-6\beta^6XY^2c_1^2c_2^2\alpha^6-3\beta^2X^3c_1^2c_2^2L^8\alpha^6-$$

$$-\frac{3}{32}\beta^{16}b^4nc_3^2c_2^7\alpha^6+120\beta^{16}b^4c_3^5c_1^3c_2\alpha^6-11\beta^{16}b^4c_3^4c_1^2c_2^3\alpha^6-\frac{303}{8}\beta^{16}b^4c_3^3c_1c_2^5\alpha^6+$$

$$+96\beta^{16}b^2c_3^5nc_1^4\alpha^6-\frac{9}{32}\beta^{16}b^2nc_3c_2^8\alpha^6+897\beta^{16}b^2c_3^4c_1^3c_2^2\alpha^6-$$

$$-\frac{203}{2}\beta^{16}b^2c_3^3c_1^2c_2^4\alpha^6-\frac{177}{16}\beta^{16}b^2c_3^2c_1c_2^6\alpha^6-16\beta^{16}c_3^4nc_1^4c_2\alpha^6+46\beta^{16}c_3^3nc_1^3c_2^3\alpha^6-$$

$$-\frac{45}{2}\beta^{16}c_3^2nc_1^2c_2^5\alpha^6+\frac{15}{8}\beta^{16}c_3nc_1c_2^7\alpha^6+\frac{9}{2}\beta^6XY^2b^4c_2^4\alpha^6+\frac{9}{32}\beta^{16}nc_2^9\alpha^6+9\beta^6L^4X^2Yb^4c_2^3c_3\alpha^4+$$

$$+108\beta^6L^4X^2Yc_1^2c_2c_3\alpha^4-360\beta^6L^4X^2Yb^2c_1^2c_3^2\alpha^4+6\beta^4X^3c_1c_2^2b^2c_3L^8\alpha^4+\beta^2L^4X^2Y^2b^4c_2^2n\alpha^4+$$

$$+36\beta^8XY^2b^2c_1c_2^2c_3\alpha^4-72\beta^8XY^2b^4c_1c_2c_3^2\alpha^4+60\beta^4X^3c_1^2c_3^2b^2L^8\alpha^4+\beta^2L^4X^2Y^2b^4c_2^2\alpha^4-$$

$$-14\beta^{18}b^4nc_3^5c_1^2c_2^2\alpha^4+\frac{3}{2}\beta^{18}b^4nc_3^4c_1c_2^4\alpha^4+64\beta^{18}b^2c_3^5nc_1^3c_2\alpha^4-$$

$$-36\beta^{18}b^2c_3^4nc_1^2c_2^3\alpha^4+\frac{5}{2}\beta^{18}b^2c_3^3nc_1c_2^5\alpha^4-18\beta^8XY^2b^4c_2^3c_3\alpha^4-$$

$$-72\beta^8XY^2c_1^2c_2c_3\alpha^4+18\beta^8XY^2b^6c_2^2c_3^2\alpha^4+360\beta^8XY^2b^2c_1^2c_3^2\alpha^4-48\beta^8XY^2b^6c_1c_3^3\alpha^4-$$

$$-9\beta^6L^4X^2Yc_1c_2^3\alpha^4+\frac{9}{2}\beta^6L^4X^2Yb^2c_2^4\alpha^4-36\beta^4X^3c_1^2c_2c_3L^8\alpha^4-\frac{9}{8}\beta^{18}b^4c_3^3c_2^6\alpha^4+$$

$$+\frac{135}{32}\beta^{18}b^2c_3^2c_2^7\alpha^4-32\beta^{18}c_3^5nc_1^4\alpha^4+\frac{27}{32}\beta^{18}c_3nc_2^8\alpha^4-446\beta^{18}c_3^4c_1^3c_2^2\alpha^4+\frac{381}{4}\beta^{18}c_3^3c_1^2c_2^4\alpha^4+$$

$$+\frac{219}{16}\beta^{18}c_3^2c_1c_2^6\alpha^4+2\beta^2Y^4b^6c_2\alpha^4+36\beta^6L^4X^2Yb^4c_1c_2c_3^2\alpha^4-$$

$$-36\beta^6L^4X^2Yb^2c_1c_2^2c_3\alpha^4-\frac{3}{4}\beta^4X^3c_2^4b^2L^8\alpha^4+3\beta^4X^3c_1c_2^3L^8\alpha^4+$$

$$+\frac{1139}{2}\beta^{18}b^2c_3^4c_1^2c_2^3\alpha^4-\frac{403}{4}\beta^{18}b^2c_3^3c_2^5c_1\alpha^4+24\beta^{18}c_3^4nc_1^3c_2^2\alpha^4+$$

$$+\frac{7}{2}\beta^{18}c_3^3nc_1^2c_2^4\alpha^4-\frac{21}{4}\beta^{18}c_3^2nc_1c_2^6\alpha^4-\frac{9}{2}\beta^8XY^2b^2c_2^4\alpha^4+6\beta^8XY^2c_1c_2^3\alpha^4+28\beta^{18}b^4c_3^5c_1^2c_2^2\alpha^4-$$

$$-\frac{51}{4}\beta^{18}b^4c_3^4c_1c_2^4\alpha^4+\frac{9}{16}\beta^{18}b^2c_3^2nc_2^7\alpha^4-824\beta^{18}b^2c_3^5c_1^3c_2\alpha^4+352\beta^{18}c_3^5c_1^4\alpha^4-\frac{27}{32}\beta^{18}c_3c_2^8\alpha^4+$$

$$+4XY^4b^6\alpha^4+12\beta^6X^3c_1c_3^2b^2c_2L^8\alpha^2+4\beta^4L^4X^2Y^2b^4c_2c_3\alpha^2-$$

$$-2\beta^4L^4X^2Y^2b^2c_2^2n\alpha^2+2\beta^2X^3Yb^2c_2^2nL^8\alpha^2-2\beta^2Y^3b^4Xnc_2L^4\alpha^2+$$

$$+72\beta^{10}XY^2b^2c_1c_2c_3^2\alpha^2+18\beta^8X^2Yc_1c_3c_2^2L^4\alpha^2-72\beta^8X^2Yb^4c_1c_3^3L^4\alpha^2-$$

$$-18\beta^8X^2Yb^2c_2^3c_3L^4\alpha^2+27\beta^8X^2Yb^4c_2^2c_3^2L^4\alpha^2+2\beta^2X^3Yb^2c_2^2L^8\alpha^2+$$

$$+10\beta^2XY^3b^4c_2L^4\alpha^2+144\beta^{10}XY^2b^4c_1c_3^3\alpha^2-12\beta^{10}XY^2c_1c_2^2c_3\alpha^2-54\beta^{10}XY^2b^4c_2^2c_3^2\alpha^2+$$

$$+18\beta^{10}XY^2b^2c_2^3c_3\alpha^2+180\beta^8X^2Yc_1^2c_3^2L^4\alpha^2+3\beta^6X^3c_2^3b^2c_3L^8\alpha^2-$$

$$-6\beta^6X^3c_1c_2^2c_3L^8\alpha^2-2\beta^4L^4X^2Y^2b^2c_2^2\alpha^2-\beta^{20}b^4nc_3^5c_1c_2^3\alpha^2+8\beta^{20}b^2c_3^5nc_1^2c_2^2\alpha^2-$$

$$-5\beta^{20}b^2c_3^4nc_1c_2^4\alpha^2+8L^4X^2Y^3b^4\alpha^2-\beta^{20}b^4c_3^4c_2^5\alpha^2-$$

$$-\frac{69}{8}\beta^{20}b^2c_3^3c_2^6\alpha^2+\frac{9}{16}\beta^{20}c_3^2nc_2^7\alpha^2+224\beta^{20}c_3^5c_1^3c_2\alpha^2-202\beta^{20}c_3^4c_1^2c_2^3\alpha^2+\frac{93}{2}\beta^{20}c_3^3c_1c_2^5\alpha^2+$$

$$+\frac{3}{2}\beta^{10}XY^2c_2^4\alpha^2+\frac{3}{4}\beta^6X^3c_2^4L^8\alpha^2-6\beta^4Y^4b^4c_2\alpha^2+4\beta^4Y^4b^6c_3\alpha^2-12\beta^2XY^4b^4\alpha^2-$$

$$-72\beta^8X^2Yb^2c_1c_3^2c_2L^4\alpha^2+4\beta^4L^4X^2Y^2b^4c_2c_3n\alpha^2-120\beta^{10}XY^2c_1^2c_3^2\alpha^2-$$

$$-\frac{9}{4}\beta^8X^2Yc_2^4L^4\alpha^2-60\beta^6X^3c_1^2c_3^2L^8\alpha^2-6L^8X^2Y^2b^4c_2\alpha^2+\frac{1}{8}\beta^{20}b^4nc_3^4c_2^5\alpha^2+$$

$$+2\beta^{20}b^4c_3^5c_1c_2^3\alpha^2+\frac{3}{4}\beta^{20}b^2nc_3^3c_2^6\alpha^2-136\beta^{20}b^2c_3^5c_1^2c_2^2\alpha^2+\frac{151}{2}\beta^{20}b^2c_3^4c_2^4c_1\alpha^2-$$

$$-16\beta^{20}c_3^5nc_1^3c_2\alpha^2+16\beta^{20}c_3^4nc_1^2c_2^3\alpha^2-\frac{21}{4}\beta^{20}c_3^3nc_1c_2^5\alpha^2-\frac{27}{32}\beta^{20}c_3^2c_2^7\alpha^2-4L^4Y^4b^6\alpha^2-$$

$$-6b^2\beta^{22}c_1c_2^3c_3^5+\frac{27}{8}\beta^{22}c_2^6c_3^3+\frac{9}{4}b^2\beta^{22}c_2^5c_3^4+24\beta^{22}c_1^2c_2^2c_3^5-18\beta^{22}c_1c_2^4c_3^4.$$

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