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# INVARIANT SUBMANIFOLDS OF TRANS-SASAKIAN MANIFOLDS
# ІНВАРІАНТНІ ПІДМНОГОВИДИ ТРАНС-МНОГОВИДІВ САСАКЯНА

We show the equivalence of totally geodesicity, recurrence, birecurrence, generalized birecurrence, Ricci-generalized birecurrence, parallelism, biparallelism, pseudoparallelism, bipseudoparallelism of $\sigma$ for the invariant submanifold $M$ of trans-Sasakian manifold $\widetilde{M}$.

Показано еквівалентність повної геодезичності, зворотності, подвійної зворотності, узагальненої подвійної зворотності, узагальненої подвійної зворотності Річчі, паралелізму, подвійного паралелізму, псевдопаралелізму та подвійного псевдопаралелізму $\sigma$ для інваріантного підмноговиду $M$ транс-многовиду Сасакяна $\widetilde{M}$.

**1. Introduction.** Let $M$ be an almost contact Riemannian manifold with a contact form $\eta$, the associated vector field $\xi$, a $(1,1)$-tensor field $\phi$ and the associated Riemannian metric $g$. Further an almost contact metric manifold is a contact metric manifold if $g(X,\phi Y) = d\eta(X,Y)$ for all $X, Y \in TM$. A K-contact manifold is a contact metric manifold while converse is true if the Lie derivative of $\phi$ in the character direction $\xi$ vanishes. A Sasakian manifold is always a K-contact manifold. A 3-dimensional K-contact manifold is a Sasakian manifold. A contact metric manifold is Sasakian if $(\nabla_X \phi)Y = g(X,Y)\xi - \eta(Y)X$. Odd dimensional spheres and $C^\star \times R$ are examples of Sasakian manifolds.

In 1972, K. Kenmotsu [4] studied a class of contact Riemannian manifolds called Kenmotsu manifolds, which is not Sasakian. In fact Kenmotsu proved that a locally Kenmotsu manifold is a warped product $I \times_f N$ of an interval $I$ and a Kahlerian manifold with a warping function $f(t) = se^t$, where $S$ is a non-zero contact. Hyperbolic space is an example of Kenmotsu manifold.

In the Gray – Hervella classification of almost Hermitian manifolds [10], there appears a class $W_4$ of Hermitian manifolds which are closely related to locally conformal Kaehler manifolds. An almost contact metric structure on a manifold $M$ is called a trans-Sasakian structure [11] if the product manifold $M \times R$ belongs to the class $W_4$. The class $C_5 \oplus C_6$ [13] coincides with the class of trans-Sasakian structure of $(\alpha,\beta)$. The monkey saddle is an example of trans-Sasakian manifold. This class consists of both Sasakian and Kenmotsu structures. If $\alpha = 1$, $\beta = 0$, then the class reduces to Sasakian, where as if $\alpha = 0$, $\beta = 1$ their reduces to Kenmotsu. J. C. Marrero [11] has shown that trans-Sasakian manifolds for $n \geq 5$ do not exist. If $\alpha \neq 0$, $\beta = 0$ then it is $\alpha$-Sasakian, if $\alpha = 0$, $\beta \neq 0$ then it is $\beta$-Kenmotsu and if $\alpha = \beta = 0$ then it is cosympletic.

The geometry of invariant submanifolds of trans-Sasakian manifolds is carried out by Aysel Turgut Vanli and Ramazan Sari [3] and they have shown that an invariant submanifold $M$ carries trans-Sasakian structure and established the equivalence of totally geodesicity of $M$, $\sigma$ is parallel, $\sigma$ is 2-parallel, $\sigma$ is semiparallel.

In this paper we extend the study and show that for invariant submanifolds of trans-Sasakian manifolds the equivalence of $M$, totally geodesic, when $\sigma$ is recurrent, 2-recurrent, generalized 2-recurrent, 2-semiparallel, pseudoparallel, 2-pseudoparallel, Ricci-generalized pseudoparallel, 2-Ricci-generalized pseudoparallel their equivalence. Finally it is concluded that the result of Aysel

Turgut Vanli and Ramazan Sari [3] and the above results proved are all equivalent to one another. We provide an example of trans-Sasakian manifold which is not totally geodesic.

**2. Basic concepts.** The covariant differential of the $p^{th}$ order, $p \geq 1$ of a $(0, k)$-tensor field $T$, $k \geq 1$ denoted by $\nabla^p T$, defined on a Riemannian manifold $(M, g)$ with the Levi–Civita connection $\nabla$. The tensor T is said to be recurrent [15], if the following condition holds on $M$:

$$(\nabla T)(X_1, \ldots, X_k; X)T(Y_1, \ldots, Y_k) = (\nabla T)(Y_1, \ldots, Y_k; X)T(X_1, \ldots, X_k) \tag{2.1}$$

and

$$(\nabla^2 T)(X_1, \ldots, X_k; X, Y)T(Y_1, \ldots, Y_k) = (\nabla^2 T)(Y_1, \ldots, Y_k; X, Y)T(X_1, \ldots, X_k)$$

respectively, where $X, Y, X_1, Y_1, \ldots, X_k, Y_k \in TM$. From (2.1) it follows that at a point $x \in M$, if the tensor $T$ is non-zero, then there exists a unique 1-form $\phi$, a $(0, 2)$-tensor $\psi$, defined on a neighborhood $U$ of $x$ such that

$$\nabla T = T \otimes \phi, \quad \phi = d(\log \|T\|) \tag{2.2}$$

and

$$\nabla^2 T = T \otimes \psi \tag{2.3}$$

respectively, hold on $U$, where $\|T\|$ denotes the norm of $T$ and $\|T\|^2 = g(T, T)$. The tensor $T$ is said to be generalized 2-recurrent if

$$((\nabla^2 T)(X_1, \ldots, X_k; X, Y) - (\nabla T \otimes \phi)(X_1, \ldots, X_k; X, Y))T(Y_1, \ldots, Y_k) =$$

$$= ((\nabla^2 T)(Y_1, \ldots, Y_k; X, Y) - (\nabla T \otimes \phi)(Y_1, \ldots, Y_k; X, Y))T(X_1, \ldots, X_k),$$

holds on $M$, where $\phi$ is a 1-form on $M$. From this it follows that at a point $x \in M$ if the tensor $T$ is non-zero, then there exists a unique $(0, 2)$-tensor $\psi$, defined on a neighborhood $U$ of $x$, such that

$$\nabla^2 T = \nabla T \otimes \phi + T \otimes \psi, \tag{2.4}$$

holds on $U$.

Let $f\colon (M, g) \to (\widetilde{M}, \widetilde{g})$ be an isometric immersion from an $n$-dimensional Riemannian manifold $(M, g)$ into $(n + d)$-dimensional Riemannian manifold $(\widetilde{M}, \widetilde{g})$, $n \geq 2$, $d \geq 1$. We denote by $\nabla$ and $\widetilde{\nabla}$ as Levi–Civita connection of $M^n$ and $\widetilde{M}^{n+d}$ respectively. Then the formulas of Gauss and Weingarten are given by

$$\widetilde{\nabla}_X Y = \nabla_X Y + \sigma(X, Y), \tag{2.5}$$

$$\widetilde{\nabla}_X N = -A_N X + \nabla^{\perp}_X N, \tag{2.6}$$

for any tangent vector fields $X$, $Y$ and the normal vector field $N$ on $M$, where $\sigma$, $A$ and $\nabla^{\perp}$ are the second fundamental form, the shape operator and the normal connection respectively. If the second fundamental form $\sigma$ is identically zero then the manifold is said to be $totallygeodesic$. The second fundamental form $\sigma$ and $A_N$ are related by

$$\widetilde{g}(\sigma(X,Y),N) = g(A_N X, Y),$$

for tangent vector fields $X$, $Y$. The first and second covariant derivatives of the second fundamental form $\sigma$ are given by

$$(\widetilde{\nabla}_X\sigma)(Y,Z) = \nabla^{\perp}_X(\sigma(Y,Z)) - \sigma(\nabla_X Y, Z) - \sigma(Y, \nabla_X Z), \tag{2.7}$$

$$(\widetilde{\nabla}^2\sigma)(Z,W,X,Y) = (\widetilde{\nabla}_X\widetilde{\nabla}_Y\sigma)(Z,W) =$$
$$= \nabla^{\perp}_X((\widetilde{\nabla}_Y\sigma)(Z,W)) - (\widetilde{\nabla}_Y\sigma)(\nabla_X Z, W) -$$
$$-(\widetilde{\nabla}_X\sigma)(Z,\nabla_Y W) - (\widetilde{\nabla}_{\nabla_X Y}\sigma)(Z,W) \tag{2.8}$$

respectively, where $\widetilde{\nabla}$ is called the van der Waerden – Bortolotti connection of $M$ [7]. If $\widetilde{\nabla}\sigma = 0$, then $M$ is said to have parallel second fundamental form [7]. We next define endomorphisms $R(X,Y)$ and $X \wedge_B Y$ of $\chi(M)$ by

$$R(X,Y)Z = \nabla_X\nabla_Y Z - \nabla_Y\nabla_X Z - \nabla_{[X,Y]}Z,$$
$$(X \wedge_B Y)Z = B(Y,Z)X - B(X,Z)Y \tag{2.9}$$

respectively, where $X, Y, Z \in \chi(M)$ and $B$ is a symmetric $(0,2)$-tensor.

Now, for a $(0,k)$-tensor field $T$, $k \geq 1$ and a $(0,2)$-tensor field $B$ on $(M,g)$, we define the tensor $Q(B,T)$ by

$$Q(B,T)(X_1,\ldots,X_k;X,Y) = -(T(X\wedge_B Y)X_1,\ldots,X_k) - \ldots$$
$$\ldots - T(X_1,\ldots,X_{k-1}(X\wedge_B Y)X_k). \tag{2.10}$$

Putting into the above formula $T = \sigma, \widetilde{\nabla}\sigma$ and $B = g$, $B = S$, we obtain the tensors $Q(g,\sigma)$, $Q(S,\sigma)$, $Q(g,\widetilde{\nabla}\sigma)$ and $Q(S,\widetilde{\nabla}\sigma)$.

**Definition 2.1.** *The immersion $f$ is said to be*

$$\textit{semiparallel } [9] \textit{ if } \quad \widetilde{R}\cdot\sigma = 0, \tag{2.11}$$

$$\textit{2-semiparallel } [14] \textit{ if } \quad \widetilde{R}\cdot\widetilde{\nabla}\sigma = 0, \tag{2.12}$$

$$\textit{pseudoparallel } [2] \textit{ if } \quad \widetilde{R}\cdot\sigma = L_1 Q(g,\sigma), \tag{2.13}$$

$$\textit{2-pseudoparallel } [14] \textit{ if } \quad \widetilde{R}\cdot\widetilde{\nabla}\sigma = L_1 Q(g,\widetilde{\nabla}\sigma) \tag{2.14}$$

*and*

$$\textit{Ricci-generalized pseudoparallel } [12] \textit{ if } \quad \widetilde{R}\cdot\sigma = L_2 Q(S,\sigma) \tag{2.15}$$

*respectively, where $\widetilde{R}$ denotes the curvature tensor with respect to connection $\widetilde{\nabla}$ and $\widetilde{R}(X,Y)\sigma(U,V) = (\widetilde{\nabla}_X\widetilde{\nabla}_Y - \widetilde{\nabla}_Y\widetilde{\nabla}_X - \widetilde{\nabla}_{[X,Y]})\sigma(U,V)$ and $(\widetilde{R}(X,Y)\widetilde{\nabla}\sigma)(U,V,W) = = \widetilde{R}(X,Y)(\widetilde{\nabla}_U\sigma)(V,W)$. Here $L_1$, $L_2$ are functions depending on $\sigma$ and $\widetilde{\nabla}\sigma$.*

Now we introduce the definition of 2-Ricci-generalized pseudoparallel.

**Definition 2.2.** *The immersion $f$ is said to be* 2*-Ricci-generalized pseudoparallel if*

$$\widetilde{R}\cdot\widetilde{\nabla}\sigma = L_2 Q(S,\widetilde{\nabla}\sigma), \tag{2.16}$$

*where $L_2$ is a function depending on $\widetilde{\nabla}\sigma$.*

From the Gauss and Weingarten formulas, we obtain

$$(\widetilde{R}(X,Y)Z)^T = R(X,Y)Z + A_{\sigma_{(X,Z)}}Y - A_{\sigma_{(Y,Z)}}X. \tag{2.17}$$

By (2.11), we have

$$(\widetilde{R}(X,Y)\cdot\sigma)(U,V) = R^{\perp}(X,Y)\sigma(U,V) - \sigma(R(X,Y)U,V) - \sigma(U,R(X,Y)V), \tag{2.18}$$

for all vector fields $X$, $Y$, $U$ and $V$ tangent to $M$, where

$$R^{\perp}(X,Y) = [\nabla^{\perp}_X,\nabla^{\perp}_Y] - \nabla^{\perp}_{[X,Y]}. \tag{2.19}$$

Similarly, we obtain

$$(\widetilde{R}(X,Y)\cdot\widetilde{\nabla}\sigma)(U,V,W) = R^{\perp}(X,Y)(\widetilde{\nabla}\sigma)(U,V,W) - (\widetilde{\nabla}\sigma)(R(X,Y)U,V,W) -$$

$$-(\widetilde{\nabla}\sigma)(U,R(X,Y)V,W) - (\widetilde{\nabla}\sigma)(U,V,R(X,Y)W), \tag{2.20}$$

for all vector fields $X$, $Y$, $U$, $V$, $W$ tangent to $M$, where $(\widetilde{\nabla}\sigma)(U,V,W) = (\widetilde{\nabla}_U\sigma)(V,W)$ [1].

**3. Preliminaries.** Let $M$ be a $n = (2m+1)$-dimensional almost contact metric manifold with an almost contact metric structure $(\phi,\xi,\eta,g)$, where $\phi$ is a $(1,1)$-tensor field, $\xi$ is a vector field, $\eta$ is a 1-form and g is the associated Riemannian metric such that [5],

$$\phi^2 = -I + \eta\otimes\xi, \qquad \eta(\xi) = 1, \qquad \eta\circ\phi = 0, \qquad \phi\xi = 0, \tag{3.1}$$

$$g(\phi X,\phi Y) = g(X,Y) - \eta(X)\eta(Y), \qquad g(X,\xi) = \eta(X), \qquad g(\phi X,Y) = -g(X,\phi Y), \tag{3.2}$$

for all vector fields $X$, $Y$ on $\widetilde{M}$.

An almost contact metric structure $(\phi,\xi,\eta,g)$ on $M$ is called a trans-Sasakian structure [13] if $(M\times R,J,G)$ belongs to the class $W_4$ [10], where $J$ is the almost complex structure on $M\times R$ defined by $J(X,\lambda d/dt) = (\phi X - \lambda\xi, \eta(X)d/dt)$ for all vector fields $X$ on $M$ and smooth function $\lambda$ on $M\times R$ and $G$ is the product metric on $M\times R$. This may be expressed by the condition [6]

$$(\nabla_X\phi)Y = \alpha(g(X,Y)\xi - \eta(Y)X) + \beta(g(\phi X,Y)\xi - \eta(Y)\phi X), \tag{3.3}$$

for some smooth functions $\alpha$ and $\beta$ on $M$ and we say that the trans-Sasakian structure is of type $(\alpha,\beta)$.

Let $M$ be a trans-Sasakian manifold. From (3.3), it is easy to see that

$$\nabla_X\xi = -\alpha\phi X + \beta(X - \eta(X)\xi). \tag{3.4}$$

If $\alpha = 1$, $\beta = 0$ it reduces to Sasakian manifold.

If $\alpha = 0$, $\beta = 1$ it reduces to Kenmotsu manifold.

In an $n$-dimensional trans-Sasakian manifold, we have

$$R(X,Y)\xi = (\alpha^2 - \beta^2)\{\eta(Y)X - \eta(X)Y\} + 2\alpha\beta\{\eta(Y)\phi(X) - \eta(X)\phi(Y)\} +$$
$$+ \left\{(Y\alpha)\phi X - (X\alpha)\phi Y + (Y\beta)\phi^2 X - (X\beta)\phi^2 Y\right\}, \tag{3.5}$$

$$R(\xi,X)Y = (\alpha^2 - \beta^2)\{g(X,Y)\xi - \eta(Y)X\} + (\xi\beta)\eta(Y)\{-X + \eta(X)\xi\}, \tag{3.6}$$

$$R(\xi,X)\xi = (\alpha^2 - \beta^2 - \xi\beta)\{\eta(X)\xi - X\}, \tag{3.7}$$

$$2\alpha\beta + \xi\alpha = 0, \tag{3.8}$$

$$S(X,\xi) = ((n-1)(\alpha^2 - \beta^2) - \xi\beta)\eta(X) - (n-2)X\beta - (\phi X)\alpha, \tag{3.9}$$

$$Q\xi = ((n-1)(\alpha^2 - \beta^2) - \xi\beta)\xi - (n-2)\operatorname{grad}\beta + \phi(\operatorname{grad}\alpha). \tag{3.10}$$

Further, in a trans-Sasakian manifold of type $(\alpha, \beta)$, we have

$$\phi(\operatorname{grad}\alpha) = (n-2)\operatorname{grad}\beta. \tag{3.11}$$

Using (3.11) the equations (3.5)–(3.7), (3.9) and (3.10) reduce to

$$R(X,Y)\xi = (\alpha^2 - \beta^2)\{\eta(Y)X - \eta(X)Y\}, \tag{3.12}$$

$$R(\xi,X)Y = (\alpha^2 - \beta^2)\{g(X,Y)\xi - \eta(Y)X\}, \tag{3.13}$$

$$R(\xi,X)\xi = (\alpha^2 - \beta^2)\{\eta(X)\xi - X\}, \tag{3.14}$$

$$S(X,\xi) = (n-1)(\alpha^2 - \beta^2)\eta(X), \tag{3.15}$$

$$Q\xi = (n-1)(\alpha^2 - \beta^2)\xi \tag{3.16}$$

respectively.

A submanifold $M$ of a trans-Sasakian manifold $\widetilde{M}$ is called an invariant submanifold of $\widetilde{M}$, if for each $x \in M$, $\phi(T_x M) \subset T_x M$. As a consequence, $\xi$ becomes tangent to $M$. In an invariant submanifold of a trans-Sasakian manifold

$$\sigma(X,\xi) = 0, \tag{3.17}$$

for any vector $X$ tangent to $M$.

**4. Recurrent invariant submanifolds of trans-Sasakian manifolds.** We consider invariant submanifold of a trans-Sasakian manifold satisfying the conditions $\sigma$ is recurrent, 2-recurrent, generalized 2-recurrent and $M$ has parallel third fundamental form. As a result of this we state the following theorem.

**Theorem 4.1.** *Let $M$ be an invariant submanifold of a trans-Sasakian manifold $\widetilde{M}$. Then $\sigma$ is recurrent if and only if it is totally geodesic.*

***Proof.*** Let $\sigma$ be recurrent, from (2.2) and we get

$$(\widetilde{\nabla}_X\sigma)(Y,Z)=\phi(X)\sigma(Y,Z),$$

where $\phi$ is a 1-form on $M$ and in view of (2.7) and taking $Z=\xi$ in the above equation, we have

$$\nabla^{\perp}_X\sigma(Y,\xi)-\sigma(\nabla_XY,\xi)-\sigma(Y,\nabla_X\xi)=\phi(X)\sigma(Y,\xi). \tag{4.1}$$

Using (3.4), (3.17) in (4.1), we obtain $(\alpha^2+\beta^2)\sigma(X,Y)=0$. Since $\alpha$ and $\beta$ are not simultaneously zero. Hence $(\alpha^2+\beta^2)\neq 0$ and $\sigma(X,Y)=0$. Thus $M$ is totally geodesic. The converse statement is trivial.

Theorem 4.1 is proved.

**Theorem 4.2.** *Let $M$ be an invariant submanifold of a trans-Sasakian manifold $\widetilde{M}$. Then $M$ has parallel third fundamental form if and only if it is totally geodesic.*

***Proof.*** Let $M$ has parallel third fundamental form. Then we obtain

$$(\widetilde{\nabla}_X\widetilde{\nabla}_Y\sigma)(Z,W)=0.$$

Taking $W=\xi$ and using (2.8) in the above equation, we have

$$\nabla^{\perp}_X((\widetilde{\nabla}_Y\sigma)(Z,\xi))-(\widetilde{\nabla}_Y\sigma)(\nabla_XZ,\xi)-(\widetilde{\nabla}_X\sigma)(Z,\nabla_Y\xi)-(\widetilde{\nabla}_{\nabla_XY}\sigma)(Z,\xi)=0. \tag{4.2}$$

By virtue of (2.7) in (4.2) and using (3.17), we get

$$2\nabla^{\perp}_X\alpha\sigma(Z,\phi Y)-2\nabla^{\perp}_X\beta\sigma(Z,Y)-2\alpha\sigma(\nabla_XZ,\phi Y)+2\beta\sigma(\nabla_XZ,Y)-\sigma(Z,\nabla_X\alpha\phi Y)+$$
$$+\sigma(Z,\nabla_X\beta Y)-\sigma(Z,\nabla_X\beta\eta(Y)\xi)-\alpha\sigma(Z,\phi\nabla_XY)+\beta\sigma(Z,\nabla_XY). \tag{4.3}$$

Putting $Y=\xi$ and using (3.4), (3.17) in (4.3), we get $(\alpha^2+\beta^2)^2\sigma(X,Z)=0$. Since $(\alpha^2+\beta^2)\neq 0$, then $\sigma(X,Z)=0$. Thus $M$ is totally geodesic. The converse statement is trivial.

Theorem 4.2 is proved.

***Corollary* 4.1.** *Let $M$ be an invariant submanifold of a trans-Sasakian manifold $\widetilde{M}$. Then $\sigma$ is 2-recurrent if and only if it is totally geodesic.*

***Proof.*** Let $\sigma$ be 2-recurrent, from (2.3), we have

$$(\widetilde{\nabla}_X\widetilde{\nabla}_Y\sigma)(Z,W)=\sigma(Z,W)\phi(X,Y). \tag{4.4}$$

Taking $W=\xi$ in (4.4) and using the proof of the Theorem 4.2, we get $(\alpha^2+\beta^2)^2\sigma(X,Z)=0$. Since $(\alpha^2+\beta^2)\neq 0$, then $\sigma(X,Z)=0$. Thus $M$ is totally geodesic. The converse statement is trivial.

Corollary 4.1 is proved.

**Theorem 4.3.** *Let $M$ be an invariant submanifold of a trans-Sasakian manifold $\widetilde{M}$. Then $\sigma$ is generalized 2-recurrent if and only if it is totally geodesic.*

***Proof.*** Let $\sigma$ be generalized 2-recurrent, from (2.4), we obtain

$$(\widetilde{\nabla}_X\widetilde{\nabla}_Y\sigma)(Z,W)=\psi(X,Y)\sigma(Z,W)+\phi(X)(\widetilde{\nabla}_Y\sigma)(Z,W), \tag{4.5}$$

where $\psi$ and $\phi$ are 2-recurrent and 1-form respectively. Taking $W=\xi$ in (4.5) and using (3.17), we get

$$(\widetilde{\nabla}_X \widetilde{\nabla}_Y \sigma)(Z, \xi) = \phi(X)(\widetilde{\nabla}_Y \sigma)(Z, \xi).$$

By virtue of (2.7) and (2.8) in above equation and in view of (3.17), we have

$$2\nabla_X^{\perp} \alpha\sigma(Z, \phi Y) - 2\nabla_X^{\perp} \beta\sigma(Z, Y) - 2\alpha\sigma(\nabla_X Z, \phi Y) + 2\beta\sigma(\nabla_X Z, Y) -$$

$$-\sigma(Z, \nabla_X \alpha\phi Y) + \sigma(Z, \nabla_X \beta Y) - \sigma(Z, \nabla_X \beta\eta(Y)\xi) - \alpha\sigma(Z, \phi\nabla_X Y) + \beta\sigma(Z, \nabla_X Y) =$$

$$= \{\alpha\sigma(Z, \phi Y) - \beta\sigma(Z, Y)\}.$$

Putting $Y = \xi$ and using (3.4), (3.17) in the above equation, we obtain $(\alpha^2 + \beta^2)^2\sigma(X, Z) = 0$. Since $(\alpha^2 + \beta^2) \neq 0$, then $\sigma(X, Z) = 0$. Thus $M$ is totally geodesic. The converse statement is trivial.

Theorem 4.3 is proved.

**5. 2-Semiparallel, pseudoparallel, 2-pseudoparallel, Ricci-generalized pseudoparallel and 2-Ricci-generalized pseudoparallel invariant submanifolds of trans-Sasakian manifolds.** We consider invariant submanifolds of trans-Sasakian manifolds satisfying the conditions $\widetilde{R} \cdot \widetilde{\nabla}\sigma = 0$, $\widetilde{R} \cdot \sigma = L_1 Q(g, \sigma)$, $\widetilde{R} \cdot \widetilde{\nabla}\sigma = L_1 Q(g, \widetilde{\nabla}\sigma)$ $\widetilde{R} \cdot \sigma = L_2 Q(S, \sigma)$ and $\widetilde{R} \cdot \widetilde{\nabla}\sigma = L_2 Q(S, \widetilde{\nabla}\sigma)$.

**Theorem 5.1.** *Let $M$ be an invariant submanifold of a trans-Sasakian manifold $\widetilde{M}$. Then the submanifold $M$ is 2-semiparallel if and only if it is totally geodesic.*

***Proof.*** Let $M$ be 2-semiparallel $\widetilde{R} \cdot \widetilde{\nabla}\sigma = 0$. Put $X = V = \xi$ in (2.20), we get

$$R^{\perp}(\xi, Y)(\widetilde{\nabla}\sigma)(U, \xi, W) - (\widetilde{\nabla}\sigma)(R(\xi, Y)U, \xi, W) -$$

$$-(\widetilde{\nabla}\sigma)(U, R(\xi, Y)\xi, W) - (\widetilde{\nabla}\sigma)(U, \xi, R(\xi, Y)W) = 0. \tag{5.1}$$

In view of (2.7), (3.4), (3.13), (3.14) and (3.17), we have the following equalities:

$$(\widetilde{\nabla}\sigma)(U, \xi, W) = (\widetilde{\nabla}_U \sigma)(\xi, W) =$$

$$= \nabla_U^{\perp}\sigma(\xi, W) - \sigma(\nabla_U \xi, W) - \sigma(\xi, \nabla_U W) =$$

$$= \alpha\sigma(\phi U, W) - \beta\sigma(U, W), \tag{5.2}$$

$$(\widetilde{\nabla}\sigma)(R(\xi, Y)U, \xi, W) = (\widetilde{\nabla}_{R(\xi,Y)U}\sigma)(\xi, W) =$$

$$= \nabla_{R(\xi,Y)U}^{\perp}\sigma(\xi, W) - \sigma(\nabla_{R(\xi,Y)U}\xi, W) - \sigma(\xi, \nabla_{R(\xi,Y)U}W) =$$

$$= -\alpha(\alpha^2 - \beta^2)\eta(U)\sigma(\phi Y, W) + \beta(\alpha^2 - \beta^2)\eta(U)\sigma(Y, W), \tag{5.3}$$

$$(\widetilde{\nabla}\sigma)(U, R(\xi, Y)\xi, W) = (\widetilde{\nabla}_U \sigma)(R(\xi, Y)\xi, W) =$$

$$= \nabla_U^{\perp}\sigma(R(\xi, Y)\xi, W) - \sigma(\nabla_U R(\xi, Y)\xi, W) - \sigma(R(\xi, Y)\xi, \nabla_U W) =$$

$$= \nabla_U^{\perp}\sigma\left((\alpha^2 - \beta^2)\{\eta(Y)\xi - Y\}, W\right) - \sigma\left(\nabla_U(\alpha^2 - \beta^2)\{\eta(Y)\xi - Y\}, W\right) +$$

$$+(\alpha^2 - \beta^2)\sigma(Y, \nabla_U W) \tag{5.4}$$

and

$$(\widetilde{\nabla}\sigma)(U,\xi,R(\xi,Y)W) = (\widetilde{\nabla}_U\sigma)(\xi,R(\xi,Y)W) =$$

$$= \nabla_U^{\perp}\sigma(\xi,R(\xi,Y)W) - \sigma(\nabla_U\xi,R(\xi,Y)W) - \sigma(\xi,\nabla_U R(\xi,Y)W) =$$

$$= -\alpha(\alpha^2-\beta^2)\eta(W)\sigma(\phi U,Y) + \beta(\alpha^2-\beta^2)\eta(W)\sigma(U,Y). \tag{5.5}$$

Substituting (5.2) – (5.5) into (5.1), we obtain

$$R^{\perp}(\xi,Y)\left\{\alpha\sigma(\phi U,W) - \beta\sigma(U,W)\right\} + \alpha(\alpha^2-\beta^2)\eta(U)\sigma(\phi Y,W) -$$

$$-\beta(\alpha^2-\beta^2)\eta(U)\sigma(Y,W) - \nabla_U^{\perp}\sigma\left((\alpha^2-\beta^2)\left\{\eta(Y)\xi - Y\right\},W\right) +$$

$$+\sigma\left(\nabla_U(\alpha^2-\beta^2)\left\{\eta(Y)\xi - Y\right\},W\right) - (\alpha^2-\beta^2)\sigma(Y,\nabla_U W) +$$

$$+\alpha(\alpha^2-\beta^2)\eta(W)\sigma(\phi U,Y) - \beta(\alpha^2-\beta^2)\eta(W)\sigma(U,Y) = 0. \tag{5.6}$$

Taking $W=\xi$ and using (3.4), (3.17) in (5.6), we get $(\alpha^2-\beta^2)(\alpha^2+\beta^2)\sigma(U,Y) = 0$. Since $(\alpha^2+\beta^2)\neq 0$, hence if $\alpha\neq\pm\beta$ and then $\sigma(U,Y)=0$, i.e., $M$ is totally geodesic. The converse statement is trivial.

Theorem 5.1 is proved.

**Theorem 5.2.** *Let $M$ be an invariant submanifold of a trans-Sasakian manifold $\widetilde{M}$. Then the submanifold $M$ is pseudoparallel if and only if it is totally geodesic.*

***Proof.*** Let $M$ be pseudoparallel $\widetilde{R}\cdot\sigma = L_1 Q(g,\sigma)$. Put $X=V=\xi$ in (2.10), (2.18) and adding, we get

$$R^{\perp}(\xi,Y)\sigma(U,\xi) - \sigma(R(\xi,Y)U,\xi) - \sigma(U,R(\xi,Y)\xi) =$$

$$= -L_1\Big\{g(\xi,\xi)\sigma(U,Y) - g(\xi,U)\sigma(\xi,Y) + g(\xi,Y)\sigma(\xi,U) - g(Y,U)\sigma(\xi,\xi)\Big\}. \tag{5.7}$$

Using (3.14) and (3.17) in (5.7), we get $[(\alpha^2-\beta^2)+L_1]\sigma(U,Y)=0$. If $L_1\neq -(\alpha^2-\beta^2)$ and $\alpha\neq\pm\beta$, then $\sigma(U,Y)=0$, i.e., $M$ is totally geodesic. The converse statement is trivial.

Theorem 5.2 is proved.

**Theorem 5.3.** *Let $M$ be an invariant submanifold of a trans-Sasakian manifold $\widetilde{M}$. Then the submanifold $M$ is 2-pseudoparallel if and only if it is totally geodesic.*

***Proof.*** Let $M$ be 2-pseudoparallel $\widetilde{R}\cdot\widetilde{\nabla}\sigma = L_1 Q(g,\widetilde{\nabla}\sigma)$. Put $X=V=\xi$ in (2.10), (2.20) and adding, in view of (3.1) and (3.17), we get

$$R^{\perp}(\xi,Y)(\widetilde{\nabla}\sigma)(U,\xi,W) - (\widetilde{\nabla}\sigma)(R(\xi,Y)U,\xi,W) -$$

$$-(\widetilde{\nabla}\sigma)(U,R(\xi,Y)\xi,W) - (\widetilde{\nabla}\sigma)(U,\xi,R(\xi,Y)W) =$$

$$= -L_1\Big[\eta(W)\Big\{\nabla_\xi^{\perp}\sigma(Y,U) - \sigma(\nabla_\xi Y,U) - \sigma(Y,\nabla_\xi U)\Big\} -$$

$$-\nabla_W^{\perp}\sigma(Y,U) + \sigma(\nabla_W Y,U) + \sigma(Y,\nabla_W U) - \eta(Y)\Big\{\nabla_\xi^{\perp}\sigma(W,U) - \sigma(\nabla_\xi W,U) -$$

$$-\sigma(W,\nabla_\xi U)\} - \eta(U)\left\{\nabla^\perp_\xi\sigma(Y,W) - \sigma(\nabla_\xi Y,W) - \sigma(Y,\nabla_\xi W)\right\}\Big]. \qquad (5.8)$$

Substituting (5.2) – (5.5) into (5.8), we obtain

$$R^\perp(\xi,Y)\{\alpha\sigma(\phi U,W) - \beta\sigma(U,W)\} + \alpha(\alpha^2-\beta^2)\eta(U)\sigma(\phi Y,W) -$$

$$-\beta(\alpha^2-\beta^2)\eta(U)\sigma(Y,W) - \nabla^\perp_U\sigma\left((\alpha^2-\beta^2)\{\eta(Y)\xi - Y\},W\right) +$$

$$+\sigma\left(\nabla_U(\alpha^2-\beta^2)\{\eta(Y)\xi - Y\},W\right) - (\alpha^2-\beta^2)\sigma(Y,\nabla_U W) +$$

$$+\alpha(\alpha^2-\beta^2)\eta(W)\sigma(\phi U,Y) - \beta(\alpha^2-\beta^2)\eta(W)\sigma(U,Y) =$$

$$= -L_1\Big[\eta(W)\Big\{\nabla^\perp_\xi\sigma(Y,U) - \sigma(\nabla_\xi Y,U) - \sigma(Y,\nabla_\xi U)\Big\} -$$

$$-\nabla^\perp_W\sigma(Y,U) + \sigma(\nabla_W Y,U) + \sigma(Y,\nabla_W U) -$$

$$-\eta(Y)\Big\{\nabla^\perp_\xi\sigma(W,U) - \sigma(\nabla_\xi W,U) - \sigma(W,\nabla_\xi U)\Big\} -$$

$$-\eta(U)\Big\{\nabla^\perp_\xi\sigma(Y,W) - \sigma(\nabla_\xi Y,W) - \sigma(Y,\nabla_\xi W)\Big\}\Big]. \qquad (5.9)$$

Taking $W = \xi$ and using (3.4), (3.17) in (5.9), we get $(\alpha^2-\beta^2)(\alpha^2+\beta^2)\sigma(U,Y) = 0$. Since $(\alpha^2+\beta^2) \neq 0$, hence if $\alpha \neq \pm\beta$ and then $\sigma(U,Y) = 0$, i.e., $M$ is totally geodesic. The converse statement is trivial.

Theorem 5.3 is proved.

**Theorem 5.4.** *Let $M$ be an invariant submanifold of a trans-Sasakian manifold $\widetilde{M}$. Then the submanifold $M$ is Ricci-generalized pseudoparallel if and only if it is totally geodesic.*

***Proof.*** Let $M$ be Ricci-generalized pseudoparallel $\widetilde{R}\cdot\widetilde{\nabla}\sigma = L_2 Q(S,\sigma)$. Put $X = V = \xi$ in (2.10), (2.18) and adding, we get

$$R^\perp(\xi,Y)\sigma(U,\xi) - \sigma(R(\xi,Y)U,\xi) - \sigma(U,R(\xi,Y)\xi) =$$

$$= -L_2\big\{S(\xi,\xi)\sigma(U,Y) - S(\xi,U)\sigma(\xi,Y) + S(\xi,Y)\sigma(\xi,U) - S(Y,U)\sigma(\xi,\xi)\big\}. \qquad (5.10)$$

Using (3.14), (3.15) and (3.17) in (5.10), we have $(\alpha^2-\beta^2)[1 + L_2(n-1)]\sigma(U,Y) = 0$. If $\alpha \neq \pm\beta$ and $L_2 \neq -\dfrac{1}{n-1}$, then $\sigma(U,Y) = 0$, i.e., $M$ is totally geodesic. The converse statement is trivial.

**Theorem 5.5.** *Let $M$ be an invariant submanifold of a trans-Sasakian manifold $\widetilde{M}$. Then the submanifold $M$ is 2-Ricci-generalized pseudoparallel, if and only if it is totally geodesic.*

***Proof.*** Let $M$ be 2-Ricci-generalized pseudoparallel $\widetilde{R}\cdot\widetilde{\nabla}\sigma = L_2 Q(S,\widetilde{\nabla}\sigma)$. Put $X = V = \xi$ in (2.10), (2.20) and adding, in view of (3.15) and (3.17) we obtain

$$R^\perp(\xi,Y)(\widetilde{\nabla}\sigma)(U,\xi,W) - (\widetilde{\nabla}\sigma)(R(\xi,Y)U,\xi,W) -$$

$$-(\widetilde{\nabla}\sigma)(U,R(\xi,Y)\xi,W) - (\widetilde{\nabla}\sigma)(U,\xi,R(\xi,Y)W) =$$

$$= -L_2\Big[(n-1)(\alpha^2-\beta^2)\eta(W)\Big\{\nabla^\perp_\xi\sigma(Y,U) - \sigma(\nabla_\xi Y,U) - \sigma(Y,\nabla_\xi U)\Big\} -$$

$$-(n-1)(\alpha^2-\beta^2)\left\{\nabla^{\perp}_{W}\sigma(Y,U)-\sigma(\nabla_W Y,U)-\sigma(Y,\nabla_W U)\right\}-$$

$$-(n-1)(\alpha^2-\beta^2)\eta(Y)\left\{\nabla^{\perp}_{\xi}\sigma(W,U)-\sigma(\nabla_\xi W,U)-\sigma(W,\nabla_\xi U)\right\}-$$

$$-(n-1)(\alpha^2-\beta^2)\eta(U)\Big\{\nabla^{\perp}_{\xi}\sigma(Y,W)-\sigma(\nabla_\xi Y,W)-\sigma(Y,\nabla_\xi W)\Big\}\Big]. \tag{5.11}$$

Substituting (5.2)–(5.5) into (5.11), we have

$$R^{\perp}(\xi,Y)\left\{\alpha\sigma(\phi U,W)-\beta\sigma(U,W)\right\}+\alpha(\alpha^2-\beta^2)\eta(U)\sigma(\phi Y,W)-$$

$$-\beta(\alpha^2-\beta^2)\eta(U)\sigma(Y,W)-\nabla^{\perp}_{U}\sigma\left((\alpha^2-\beta^2)\left\{\eta(Y)\xi-Y\right\},W\right)+$$

$$+\sigma\left(\nabla_U(\alpha^2-\beta^2)\left\{\eta(Y)\xi-Y\right\},W\right)-(\alpha^2-\beta^2)\sigma(Y,\nabla_U W)+$$

$$+\alpha(\alpha^2-\beta^2)\eta(W)\sigma(\phi U,Y)-\beta(\alpha^2-\beta^2)\eta(W)\sigma(U,Y)=$$

$$=-L_2\Big[(n-1)(\alpha^2-\beta^2)\eta(W)\Big\{\nabla^{\perp}_{\xi}\sigma(Y,U)-\sigma(\nabla_\xi Y,U)-\sigma(Y,\nabla_\xi U)\Big\}-$$

$$-(n-1)(\alpha^2-\beta^2)\Big\{\nabla^{\perp}_{W}\sigma(Y,U)-\sigma(\nabla_W Y,U)-\sigma(Y,\nabla_W U)\Big\}-$$

$$-(n-1)(\alpha^2-\beta^2)\eta(Y)\Big\{\nabla^{\perp}_{\xi}\sigma(W,U)-\sigma(\nabla_\xi W,U)-\sigma(W,\nabla_\xi U)\Big\}-$$

$$-(n-1)(\alpha^2-\beta^2)\eta(U)\Big\{\nabla^{\perp}_{\xi}\sigma(Y,W)-\sigma(\nabla_\xi Y,W)-\sigma(Y,\nabla_\xi W)\Big\}\Big]. \tag{5.12}$$

Taking $W=\xi$ and using (3.4), (3.15), (3.17) in (5.12), we get $(\alpha^2-\beta^2)(\alpha^2+\beta^2)\sigma(U,Y)=0$. Since $(\alpha^2+\beta^2)\neq 0$, hence if $\alpha\neq\pm\beta$ and then $\sigma(U,Y)=0$. i.e., $M$ is totally geodesic. The converse statement is trivial.

Theorem 5.5 is proved.

Using Theorems 4.1 to 4.3, 5.1 to 5.5, Corollary 4.1 and the result of [3], we have the following result.

***Corollary* 5.1.** *Let $M$ be an invariant submanifold of a trans-Sasakian manifold $\widetilde{M}$. Then the following statements are equivalent*:

(1) *$\sigma$ is parallel*;
(2) *$\sigma$ is 2-parallel*;
(3) *$\sigma$ is recurrent*;
(4) *$\sigma$ is 2-recurrent*;
(5) *$\sigma$ is generalized 2-recurrent*;
(6) *$M$ has parallel third fundamental form*;
(7) *$M$ is semiparallel*;
(8) *$M$ is 2-semiparallel, if $\alpha\neq\pm\beta$*;
(9) *$M$ is pseudoparallel, if $L_1\neq-(\alpha^2-\beta^2)$ and $\alpha\neq\pm\beta$*;
(10) *$M$ is 2-pseudoparallel, if $\alpha\neq\pm\beta$*;
(11) *$M$ is Ricci-generalized pseudoparallel, if $L_2\neq-\dfrac{1}{n-1}$ and $\alpha\neq\pm\beta$*;

(12) *$M$ is 2-Ricci-generalized pseudoparallel, if $\alpha \neq \pm\beta$;*

(13) *$M$ is totally geodesic.*

**Example of trans-Sasakian manifold.** We consider the 3-dimensional manifold $M = \{(x, y, z) \in R^3 : x \neq 0, y \neq 0\}$, where $(x, y, z)$ are the standard coordinates in $R^3$. Let $\{E_1, E_2, E_3\}$ be linearly independent global frame field on $M$ given by

$$E_1 = \frac{e^z}{x}\left(\frac{\partial}{\partial x} + y\frac{\partial}{\partial z}\right), \qquad E_2 = \frac{e^z}{y}\frac{\partial}{\partial y}, \qquad E_3 = \frac{\partial}{\partial z}.$$

Let $g$ be the Riemannian metric defined by

$$g(E_1, E_2) = g(E_2, E_3) = g(E_1, E_3) = 0,$$

$$g(E_1, E_1) = g(E_2, E_2) = g(E_3, E_3) = 1.$$

The $(\phi, \xi, \eta)$ is given by

$$\eta = dz - ydx, \qquad \xi = E_3 = \frac{\partial}{\partial z},$$

$$\phi E_1 = E_2, \qquad \phi E_2 = -E_1, \qquad \phi E_3 = 0.$$

The linearity property of $\phi$ and $g$ yields that

$$\eta(E_3) = 1, \qquad \phi^2 U = -U + \eta(U)E_3,$$

$$g(\phi U, \phi W) = g(U, W) - \eta(U)\eta(W),$$

for any vector fields $U, W$ on $M$. By definition of Lie bracket, we have

$$[E_1, E_2] = y\frac{e^z}{x}E_2 - \frac{e^{2z}}{xy}E_3, \qquad [E_1, E_3] = -E_1, \qquad [E_2, E_3] = -E_2.$$

Let $\nabla$ be the Levi – Civita connection with respect to above metric $g$ is given by Koszula formula

$$2g(\nabla_X Y, Z) = X(g(Y, Z)) + Y(g(Z, X)) - Z(g(X, Y)) -$$

$$-g(X, [Y, Z]) - g(Y, [X, Z]) + g(Z, [X, Y]).$$

Then we get

$$\nabla_{E_1}E_1 = E_3, \qquad \nabla_{E_1}E_2 = -\frac{e^{2z}}{2xy}E_3, \qquad \nabla_{E_1}E_3 = -E_1 + \frac{e^{2z}}{2xy}E_2,$$

$$\nabla_{E_2}E_1 = -y\frac{e^z}{x}E_2 + \frac{e^{2z}}{2xy}E_3, \quad \nabla_{E_2}E_2 = y\frac{e^z}{x}E_1 + E_3, \quad \nabla_{E_2}E_3 = -\frac{e^{2z}}{2xy}E_1 - E_2,$$

$$\nabla_{E_3}E_1 = \frac{e^{2z}}{2xy}E_2, \qquad \nabla_{E_3}E_2 = -\frac{e^{2z}}{2xy}E_1, \qquad \nabla_{E_3}E_3 = 0.$$

The tangent vectors $X$ and $Y$ to $M$ are expressed as linear combination of $E_1, E_2, E_3$, i.e., $X = a_1E_1 + a_2E_2 + a_3E_3$ and $Y = b_1E_1 + b_2E_2 + b_3E_3$, where $a_i$ and $b_j$ are scalars. Clearly $(\phi, \xi, \eta, g)$ and $X$, $Y$ satisfy equations (3.1), (3.2), (3.3) and (3.4) with $\alpha = -\dfrac{e^{2z}}{2xy}$ and $\beta = -1$. Thus $M$ is a trans-Sasakian manifold. In particular we consider the example of monkey saddle given by

$$M = \left\{(x, y, z) \in R^3 \colon z = x^3 - 3xy^2\right\}.$$

By the above $x \neq 0, y \neq 0 \Rightarrow z \neq 0$ and $M = R^3 - \{0\}$. We show that though $\alpha \neq -\beta$, $M$ is not totally geodesic. For if $X$ is a patch defined by $X(u, v) = (u, v, u^3 - 3uv^2)$ then any tangent vector $V$ to the monkey saddle is given by $V = C_1X_u + C_2X_v$, where $X_u = (1, 0, 3u - 3v^2)$ and $X_v = (0, 1, -6uv)$. $M$ will not be totally geodesic, if $\nabla_V V \neq 0$. On verification we can see that $\nabla_V V \neq 0$. Hence $M$ is not totally geodesic.

**Conclusion.** From the above discussion we conclude that $\alpha \neq \pm\beta$ is only a necessary condition but not a sufficient condition. Hence it needs further investigation.

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