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ON WEAKLY (μ, λ)-OPEN FUNCTIONS*

ПРО СЛАБКО (μ, λ)-ВІДКРИТІ ФУНКЦІЇ

In the paper, some characterizations and properties of almost (μ, λ)-open functions are investigated. Some conditions are given under which an almost (μ, λ)-open function is equivalent to a (μ, λ)-open function.

Вивчаються деякі характеристики та властивості майже (μ, λ)-відкритих функцій. Наведено деякі умови, за яких майже (μ, λ)-відкрита функція еквівалентна (μ, λ)-відкритій функції.

1. Introduction. During the last few years, different forms of open sets have been studied. A significant contribution to the theory of generalized open sets was extended by A. Császár [4, 5, 7, 8]. In [3], he introduced the concept of generalized neighbourhood systems and generalized topological spaces. He also introduced the concept of continuous functions and associated interior and closure operators on generalized topological spaces and had shown that the fundamental definitions and major part of many statements and constructions in set topology can be formulated by replacing topology with the help of generalized topology. The notion of (μ, λ)-open function was studied by Ekici [11]. Al-Omari and Noiri [1] introduced the definition of almost (μ, λ)-open functions. We investigate several characterizations of these functions. An endeavour has been made to obtain several conditions for an almost (μ, λ)-open function to be a (μ, λ)-open function.

We recall some notions defined in [3]. Let X be a nonempty set and $\exp X$ be the power set of X. We call a class $\mu \subsetneqq \exp X$ a generalized topology [3] (briefly, GT) if $\varnothing \in \mu$ and union of elements of μ belongs to μ. A set X, with a GT μ on it is said to be a generalized topological space (briefly, GTS) and is denoted by (X, μ). A GT μ on X is said to be strong [8] if $X \in \mu$.

For a GTS (X, μ), the elements of μ are called μ-open sets and the complement of μ-open sets are called μ-closed sets. For $A \subsetneqq X$, we denote by $c_\mu(A)$ the intersection of all μ-closed sets containing A, i.e., the smallest μ-closed set containing A; and by $i_\mu(A)$ the union of all μ-open sets contained in A, i.e., the largest μ-open set contained in A (see [3, 6]).

According to [3, 9], a GT is said to be a quasitopology (briefly, QT) iff $M, M' \in \mu$ implies $M \cap M' \in \mu$. A QT on X coincides with a topology on a subset $X_0 \subsetneqq X$.

It is easy to observe that i_μ and c_μ are idempotent and monotonic, where $\gamma\colon \exp X \to \exp X$ is said to be idempotent iff for each $A \subsetneqq X$, $\gamma(\gamma(A)) = \gamma(A)$, and monotonic iff $\gamma(A) \subsetneqq \gamma(B)$ whenever $A \subsetneqq B \subsetneqq X$. It is also well known from [6, 7] that if μ is a GT on X and $A \subsetneqq X$, $x \in X$, then $x \in c_\mu(A)$ iff ($x \in M \in \mu \Rightarrow M \cap A \neq \varnothing$) and that $c_\mu(X \setminus A) = X \setminus i_\mu(A)$.

2. Almost (μ, λ)-open and (μ, λ)-open functions.

Definition 2.1. *Let (X, μ) be a GTS. The $\mu(\theta)$-closure* [7] (*resp. $\mu(\theta)$-interior* [7]) *of a subset A in a GTS (X, μ) is denoted by $c_{\mu(\theta)}(A)$ (resp. $i_{\mu(\theta)}(A)$) and is defined to be the set $\{x \in X\colon c_\mu(U) \cap A \neq \varnothing$ for each $U \in \mu(x)\}$ (resp. $\{x \in X\colon$ there exists $U \in \mu(x)$ such that $c_\mu(U) \subsetneqq A\}$), where $\mu(x) = \{U \in \mu\colon x \in U\}$.*

Theorem 2.1 [15]. *In a GTS (X, μ), $X \setminus c_{\mu(\theta)}(A) = i_{\mu(\theta)}(X \setminus A)$ and $X \setminus i_{\mu(\theta)}(A) = c_{\mu(\theta)}(X \setminus A)$.*

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Definition 2.2. *A function* $f\colon (X,\mu)\to(Y,\lambda)$ *is said to be almost* (μ,λ)*-open* [1] *if for each* $U\in\mu$, $f(U)\subseteq i_\lambda(f(c_\mu(U)))$.

Theorem 2.2. *For a function* $f\colon (X,\mu)\to(Y,\lambda)$, *the following properties are equivalent*:

(a) f *is almost* (μ,λ)*-open*,

(b) $f(i_{\mu(\theta)}(A))\subseteq i_\lambda(f(A))$ *for any subset* A *of* X,

(c) $i_{\mu(\theta)}(f^{-1}(B))\subseteq f^{-1}(i_\lambda(B))$ *for every subset* B *of* Y,

(d) $f^{-1}(c_\lambda(B))\subseteq c_{\mu(\theta)}(f^{-1}(B))$ *for every subset* B *of* Y,

(e) *for each* $x\in X$ *and each* μ*-open set* U *containing* x, *there exists a* λ*-open set* V *containing* $f(x)$ *such that* $V\subseteq f(c_\mu(U))$.

Proof. (a) $\Rightarrow$ (b). Let A be any subset of X and $x\in i_{\mu(\theta)}(A)$. Then there exists a μ-open set U containing x such that $c_\mu(U)\subseteq A$. Thus $f(x)\in f(U)\subseteq f(c_\mu(U))\subseteq f(A)$. Since f is almost (μ,λ)-open, $f(U)\subseteq i_\lambda(f(c_\mu(U)))\subseteq i_\lambda(f(A))$ and so $x\in f^{-1}(i_\lambda(f(A)))$. Thus $i_{\mu(\theta)}(A)\subseteq f^{-1}(i_\lambda(f(A)))$ and hence $f(i_{\mu(\theta)}(A))\subseteq i_\lambda(f(A))$.

(b) $\Rightarrow$ (c). Let B be any subset of Y. Then by (b), $f(i_{\mu(\theta)}(f^{-1}(B)))\subseteq i_\lambda(B)$. Thus $i_{\mu(\theta)}(f^{-1}(B))\subseteq f^{-1}(i_\lambda(B))$.

(c) $\Rightarrow$ (d). Let B be any subset of Y. Then $X\setminus c_{\mu(\theta)}(f^{-1}(B))=i_{\mu(\theta)}(X\setminus f^{-1}(B))$ (by Theorem 2.1) $=i_{\mu(\theta)}(f^{-1}(Y\setminus B))\subseteq f^{-1}(i_\lambda(Y\setminus B))$ (by (c)) $=f^{-1}(Y\setminus c_\lambda(B))=X\setminus f^{-1}(c_\lambda(B))$. Thus $f^{-1}(c_\lambda(B))\subseteq c_{\mu(\theta)}(f^{-1}(B))$.

(d) $\Rightarrow$ (e). Let $x\in X$ and U be any μ-open set containing x. Let $B=Y\setminus f(c_\mu(U))$. Then by (d), $f^{-1}(c_\lambda(Y\setminus f(c_\mu(U))))\subseteq c_{\mu(\theta)}(f^{-1}(Y\setminus f(c_\mu(U))))$. Now, $f^{-1}(c_\lambda(Y\setminus f(c_\mu(U))))=$ $=X\setminus f^{-1}(i_\lambda(f(c_\mu(U))))$. Also, $c_{\mu(\theta)}(f^{-1}(Y\setminus f(c_\mu(U))))=c_{\mu(\theta)}(X\setminus f^{-1}(f(c_\mu(U))))\subseteq c_{\mu(\theta)}(X\setminus c_\mu(U))=X\setminus i_{\mu(\theta)}(c_\mu(U))$ (by Theorem 2.1) $\subseteq X\setminus U$. Therefore, $U\subseteq f^{-1}(i_\lambda(f(c_\mu(U))))$ and thus $f(U)\subseteq i_\lambda(f(c_\mu(U)))$. Since $f(x)\in f(U)$, there exists a $V\in\lambda$ containing $f(x)$ such that $V\subseteq f(c_\mu(U))$.

(e) $\Rightarrow$ (a). Let U be a μ-open set containing x. Then by (e), there exists a λ-open set V containing $f(x)$ such that $V\subseteq f(c_\mu(U))$. Thus $f(x)\in V\subseteq i_\lambda(f(c_\mu(U)))$ for each $x\in U$. Therefore we obtain, $f(U)\subseteq i_\lambda(f(c_\mu(U)))$. Hence f is almost (μ,λ)-open.

Example **2.1.** The concept of weak BR-openness as a natural dual to the weak BR-continuity due to Ekici [12] was introduced and studied in [2].

Theorem 2.3. *For a bijective function* $f\colon (X,\mu)\to(Y,\lambda)$ *the following properties are equivalent*:

(a) f *is almost* (μ,λ)*-open*,

(b) $c_\lambda(f(i_\mu(F)))\subseteq f(F)$ *for each* μ*-closed set* F *in* X,

(c) $c_\lambda(f(U))\subseteq f(c_\mu(U))$ *for each* $U\in\mu$.

Proof. (a) $\Rightarrow$ (b). Let F be a μ-closed subset of X. Then $X\setminus F$ is μ-open and $Y\setminus f(F)=$ $=f(X\setminus F)\subseteq i_\lambda(f(c_\mu(X\setminus F)))$ (by (a)) $=i_\lambda(f(X\setminus i_\mu(F)))=i_\lambda(Y\setminus f(i_\mu(F)))=Y\setminus c_\lambda(f(i_\mu(F)))$. Thus $c_\lambda(f(i_\mu(F)))\subseteq f(F)$.

(b) $\Rightarrow$ (c). Let U be any μ-open set in X. Then we have $c_\lambda(f(U))=c_\lambda(f(i_\mu(U)))\subseteq$ $\subseteq c_\lambda(f(i_\mu(c_\mu(U))))\subseteq f(c_\mu(U))$ (by (b)).

(c) $\Rightarrow$ (a). Let U be a μ-open set in X. Then we have, $Y\setminus i_\lambda(f(c_\mu(U)))=c_\lambda(Y\setminus f(c_\mu(U)))=$ $=c_\lambda(f(X\setminus c_\mu(U)))\subseteq f(c_\mu(X\setminus c_\mu(U))))$ (by (c)) $=f(X\setminus i_\mu(c_\mu(U)))\subseteq f(X\setminus U)=Y\setminus f(U)$. Therefore $f(U)\subseteq i_\lambda(f(c_\mu(U)))$. Hence f is almost (μ,λ)-open.

***Example* 2.2.** Let $X = \{a,b,c\}$, $\mu = \{\varnothing,\{a\},\{a,b\}\}$ and $\lambda = \{\varnothing,\{c\},\{a,b\},\{b,c\},X\}$. Then (X,μ) and (X,λ) are two GTS's. Consider the map $f\colon (X,\mu) \to (X,\lambda)$ defined by $f(a) = f(c) = c$ and $f(b) = b$. Then f is clearly not a bijection but f is almost (μ,λ)-open. We note that $c_\lambda(f(\{a,b\})) \nsubseteq f(c_\mu(\{a,b\}))$ (see Theorem 2.3(c)).

Definition 2.3. *A function $f\colon (X,\mu) \to (Y,\lambda)$ is said to be (μ,λ)-open* [11, 14] *if $f(U)$ is λ-open for each μ-open set U in X.*

***Remark* 2.1.** It follows from Definitions 2.2 and 2.3 that every (μ,λ)-open function is almost (μ,λ)-open but the converse is not true as follows from the next example.

***Example* 2.3.** Consider $X = \{a,b,c\}$, $\mu = \{\varnothing,\{a\},\{c\},\{a,c\}\}$ and $\lambda = \{\varnothing,\{a,b\},\{b,c\},X\}$. Then μ and λ are two GT's on X. Now the identity mapping $f\colon (X,\mu) \to (X,\lambda)$ is almost (μ,λ)-open but not (μ,λ)-open.

Definition 2.4. *A function $f\colon (X,\mu) \to (Y,\lambda)$ is said to be strongly (μ,λ)-continuous iff $f(c_\mu(A)) \subseteqq f(A)$ for each subset A of X.*

Theorem 2.4. *If a function $f\colon (X,\mu) \to (Y,\lambda)$ is strongly (μ,λ)-continuous and almost (μ,λ)-open then f is (μ,λ)-open.*

Proof. Let U be any μ-open set in X. We have to show that $f(U)$ is λ-open in Y. Since f is almost (μ,λ)-open and strongly (μ,λ)-continuous, $f(U) \subseteqq i_\lambda(f(c_\mu(U))) \subseteqq i_\lambda(f(U))$. Thus $f(U)$ is λ-open.

Definition 2.5. *A GTS (X,μ) is said to be μ-regular* [13, 16] *iff for each $x \in X$ and each $U \in \mu$ containing x there exists a $V \in \mu$ containing x such that $x \in V \subseteqq c_\mu(V) \subseteqq U$.*

Theorem 2.5. *Let (X,μ) be μ-regular. Then the function $f\colon (X,\mu) \to (Y,\lambda)$ is (μ,λ)-open iff f is almost (μ,λ)-open.*

Proof. One part of the theorem is trivial due to Remark 2.1. For the converse, let f be almost (μ,λ)-open and U be any μ-open set in X. Then for each $x \in U$, there exists a $V_x \in \mu$ containing x such that $x \in V_x \subseteqq c_\mu(V_x) \subseteqq U$. Hence $U = \cup\{V_x\colon x \in U\} = \cup\{c_\mu(V_x)\colon x \in U\}$ and hence $f(U) = \cup\{f(V_x)\colon x \in U\} \subseteqq \cup\{i_\lambda(f(c_\mu(V_x)))\colon x \in U\} \subseteqq i_\lambda(\cup\{f(c_\mu(V_x)))\colon x \in U\} \subseteqq i_\lambda(f(\cup\{c_\mu(V_x)\colon x \in U\})) = i_\lambda(f(U))$. Thus f is (μ,λ)-open.

Definition 2.6. *A function $f\colon (X,\mu) \to (Y,\lambda)$ is said to satisfy the almost (μ,λ)-open interiority condition if for each $U \in \mu$, $i_\lambda(f(c_\mu(U))) \subseteqq f(U)$.*

Theorem 2.6. *If a function $f\colon (X,\mu) \to (Y,\lambda)$ is almost (μ,λ)-open and satisfy the almost (μ,λ)-open interiority condition, then f is (μ,λ)-open.*

Proof. Let U be a μ-open set in X. Since f is almost (μ,λ)-open, $f(U) \subseteqq i_\lambda(f(c_\mu(U))) = i_\lambda(i_\lambda(f(c_\mu(U)))) \subseteqq i_\lambda(f(U))$. Hence $f(U) = i_\lambda(f(U))$, i.e., $f(U)$ is λ-open.

***Example* 2.4.** Let $X = \{a,b,c\}$, $\mu = \{\varnothing,\{a\}\}$ and $\lambda = \{\varnothing,\{a\},\{a,b\}\}$. Then the identity map $f\colon (X,\mu) \to (X,\lambda)$ is (μ,λ)-open but it does not satisfy almost (μ,λ)-interiority condition.

For any subset A of a GTS (X,μ), the μ-frontier [17] of A is denoted by $Fr_\mu(A)$ and defined by $Fr_\mu(A) = c_\mu(A) \cap c_\mu(X \setminus A)$.

Definition 2.7. *A function $f\colon (X,\mu) \to (Y,\lambda)$ is said to be complementary weakly (μ,λ)-open if $f(Fr_\mu(U))$ is λ-closed for each $U \in \mu$.*

That the notions of complementary weakly (μ,λ)-open and almost (μ,λ)-open functions are independent as shown in the next two examples.

***Example* 2.5.** (a) Let $X = \{a,b,c\}$, $\mu = \{\varnothing,\{c\}\}$ and $\lambda = \{\varnothing,\{a,b\},\{a,c\},X\}$. Then it is easy to see that the identity map $f\colon (X,\mu) \to (X,\lambda)$ is almost (μ,λ)-open but not complementary weakly (μ,λ)-open.

(b) Let $X = \{a, b, c\}$, $\mu = \{\varnothing, \{a\}, \{a, b\}, \{b, c\}, X\}$ and $\lambda = \{\varnothing, \{a\}, \{a, b\}, X\}$. Consider the function $f : (X, \mu) \to (X, \lambda)$ defined by $f(a) = b$ and $f(b) = f(c) = c$. Then it can be checked that f is complementary weakly (μ, λ)-open but not almost (μ, λ)-open.

Theorem 2.7. *If $f : (X, \mu) \to (Y, \lambda)$ is an almost (μ, λ)-open and complementary weakly (μ, λ)-open bijection and (Y, λ) is a QT then f is (μ, λ)-open.*

Proof. Let $x \in X$ and U be a μ-open set containing x. Since f is almost (μ, λ)-open, by Theorem 2.2 there exists $V \in \lambda$ containing $f(x)$ such that $V \subseteqq f(c_\mu(U))$. Now, $Fr_\mu(U) = c_\mu(U) \cap c_\mu(X \setminus U) = c_\mu(U) \cap (X \setminus U)$. Since $x \in U$, $x \notin Fr_\mu(U)$ and hence $f(x) \notin f(Fr_\mu(U))$. Put $W = V \cap (Y \setminus f(Fr_\mu(U)))$. Since f is complementary weakly (μ, λ)-open and (Y, λ) is a QT, W is a λ-open set containing $f(x)$. It is now sufficient to show that $W \subseteqq f(U)$. Let $y \in W$. Then $y \in V \subseteqq f(c_\mu(U))$ and $y \notin f(Fr_\mu(U)) = f(c_\mu(U) \cap (X \setminus U)) = f(c_\mu(U)) \cap (Y \setminus f(U))$. Thus we have $y \in (Y \setminus f(c_\mu(U))) \cup f(U)$. Thus $y \in f(U)$. Hence f is (μ, λ)-open.

***Example* 2.6.** Consider the Example 2.5(b). Then (Y, λ) is a QT and f is complementary weakly (μ, λ)-open which is not bijective. Also f is not (μ, λ)-open.

Definition 2.8. *A function $f : (X, \mu) \to (Y, \lambda)$ is said to be contra (μ, λ)-closed if $f(F)$ is λ-open for every μ-closed set F in (X, μ).*

Theorem 2.8. *If a function $f : (X, \mu) \to (Y, \lambda)$ is contra (μ, λ)-closed, then f is almost (μ, λ)-open.*

Proof. Let U be a μ-open set in (X, μ). Then $f(U) \subseteqq f(c_\mu(U)) = i_\lambda(f(c_\mu(U)))$. Thus f is almost (μ, λ)-open.

The converse of the above theorem need not be true as shown in the next example.

***Example* 2.7.** Let $X = \{a, b, c\}$, $\mu = \{\varnothing, \{a\}, \{a, b\}\}$ and $\lambda = \{\varnothing, \{a, b\}, \{a, c\}, X\}$. Then μ and λ are two GT's on X. The identity map $f : (X, \mu) \to (X, \lambda)$ is almost (μ, λ)-open but not contra (μ, λ)-closed.

3. Some properties of almost (μ, λ)-open functions.

Definition 3.1. *A GTS (X, μ) is said to be μ-hyperconnected* [11] *if $c_\mu(U) = X$ for each nonempty μ-open set U.*

Theorem 3.1. *Let (X, μ) be μ-hyperconnected. If for a function $f : (X, \mu) \to (Y, \lambda)$, $f(X)$ is λ-open in (Y, λ), then f is almost (μ, λ)-open. If μ is a strong GT then the converse is also true.*

Proof. Let $f(X)$ be λ-open in (Y, λ). Let $U \in \mu$. Then $f(U) \subseteqq f(X) = i_\lambda(f(X)) = i_\lambda(f(c_\mu(U)))$. Thus $f(U) \subseteqq i_\lambda(f(c_\mu(U)))$. Hence f is almost (μ, λ)-open.

Conversely, let f be an almost (μ, λ)-open function. Since μ is strong, $X \in \mu$ and thus $f(X) \subseteqq i_\lambda(f(c_\mu(X))) = i_\lambda(f(X))$. Thus $f(X)$ is λ-open.

***Example* 3.1.** Let $X = \{a, b, c\}$, $\mu = \{\varnothing, \{a\}, \{a, b\}\}$ and $\lambda = \{\varnothing, \{a\}, \{c\}, \{a, c\}\}$. Then μ and λ are two GT's on X such that $X \notin \mu$. Consider the function $f : (X, \mu) \to (X, \lambda)$ defined by $f(a) = a$; $f(b) = c$ and $f(c) = b$. It is easy to check that f is almost (μ, λ)-open and (X, μ) is μ-hyperconnected but $f(X)$ is not λ-open.

Definition 3.2. *A GTS (X, μ) is said to be μ-connected* [19] *if X can not be written as the union of two nonempty disjoint μ-open sets of X.*

Theorem 3.2. *If $f : (X, \mu) \to (Y, \lambda)$ is an almost (μ, λ)-open bijection and (Y, λ) is λ-connected, then (X, μ) is μ-connected.*

Proof. Suppose that (X, μ) is not μ-connected. Then there exist disjoint μ-open sets U_1 and U_2 such that $X = U_1 \cup U_2$. Hence we have $f(U_1) \cap f(U_2) = \varnothing$ and $Y = f(U_1) \cup f(U_2)$. Since f is almost (μ, λ)-open, $f(U_i) \subseteqq i_\lambda(f(c_\mu(U_i)))$ for $i = 1, 2$. Since each U_i is μ-closed, $U_i = c_\mu(U_i)$, and hence $f(U_i) = i_\lambda(f(U_i))$ for $i = 1, 2$. So, $f(U_i)$ are λ-open for $i = 1, 2$. Thus Y has been decomposed into two nonempty disjoint λ-open sets which contradicts that (Y,λ) is λ-connected. Thus (X,μ) is μ-connected.

***Remark* 3.1** [11]**.** If a GTS (X,μ) is μ-hyperconnected, then (X,μ) is μ-connected.

***Corollary* 3.1.** *If $f\colon (X,\mu)\to(Y,\lambda)$ is an almost (μ,λ)-open bijection and (Y,λ) is λ-hyperconnected, then (X,μ) is connected.*

***Example* 3.2.** Let $X=\{a,b,c\}$, $\mu=\{\varnothing,\{c\},\{a,b\},\{b,c\},X\}$ and $\lambda=\{\varnothing,\{a\},\{a,b\},\{a,c\},X\}$. Then (X,μ) and (X,λ) are two GTS's. Then the map $f\colon (X,\mu)\to(X,\lambda)$ defined by $f(a)=f(c)=a$ and $f(b)=b$ is clearly not a bijection but f is almost (μ,λ)-open and (X,λ) is λ-connected but (X,μ) is not μ-connected.

Definition 3.3. *A subset A of a GTS (X,μ) is said to be weakly μ-compact* (*briefly ω_μ-compact*) [18] *if every cover of A by μ-open subsets of X has a finite subfamily the union of whose μ-closures covers A.*

Definition 3.4. *A GTS (X,μ) is said to be extremally μ-disconnected* [10] *if the μ-closure of any μ-open set is μ-open.*

Lemma 3.1. *If a function $f\colon (X,\mu)\to(Y,\sigma)$ is (μ,λ)-open then for each $B\subseteqq Y$, $f^{-1}(c_\lambda(B))\subseteqq c_\mu(f^{-1}(B))$.*

Theorem 3.3. *Let (X,μ) be an extremally μ-disconnected space where μ is a QT. Let $f\colon (X,\mu)\to(Y,\lambda)$ be an one-to-one (μ,λ)-open, almost (μ,λ)-open mapping such that $f^{-1}(y)$ is ω_μ-compact for each $y\in Y$. Then for every ω_λ-compact subset G of Y, $f^{-1}(G)$ is ω_μ-compact.*

Proof. Let $\{V_\alpha : \alpha\in\Lambda\}$ be a μ-open cover of $f^{-1}(G)$. Then for each $y\in G$, $f^{-1}(y)\subseteqq \cup\{c_\mu(V_\alpha)\colon \alpha\in\Lambda_y\}=H_y$ for some finite subset Λ_y of Λ. Then H_y is μ-open as X is extremally μ-disconnected. So by Theorem 2.2 ((a) $\Leftrightarrow$ (e)), there exists a λ-open set U_y containing $y\in Y$ such that $f^{-1}(U_y)\subseteqq c_\mu(H_y)$. Then $\{U_y : y\in G\}$ is a cover of G by λ-open subsets of Y. Thus by ω_λ-compactness of G, there exists a finite subset K of G such that $G\subseteqq\cup\{c_\lambda(U_y)\colon y\in K\}$. Hence by Lemma 3.1, $f^{-1}(G)\subseteqq\cup\{c_\mu(f^{-1}(U_y))\colon y\in K\}\subseteqq\cup\{c_\mu(H_y)\colon y\in K\}$. Thus $f^{-1}(G)\subseteqq\cup\{c_\mu(V_\alpha)\colon \alpha\in\Lambda_y, y\in K\}$. Hence $f^{-1}(G)$ is ω_μ-compact.

Conclusion. The definitions of various types of weakly open functions may be introduced from the definition of weakly (μ,λ)-open functions by replacing the generalized topologies μ and λ (resp. on X and Y) suitably.

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