UDC 514.7

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CRITICAL POINT EQUATION ON ALMOST KENMOTSU MANIFOLDS

РІВНЯННЯ КРИТИЧНОЇ ТОЧКИ НА МНОГОВИДАХ, ЩО Є МАЙЖЕ МНОГОВИДАМИ КЕНМОЦУ

We study the critical point equation (CPE) conjecture on almost Kenmotsu manifolds. First, we prove that if a three-dimensional $(k,\mu)'$-almost Kenmotsu manifold satisfies the CPE, then the manifold is either locally isometric to the product space $\mathbb{H}^2(-4)\times\mathbb{R}$ or the manifold is Kenmotsu manifold. Further, we prove that if the metric of an almost Kenmotsu manifold with conformal Reeb foliation satisfies the CPE conjecture, then the manifold is Einstein.

Вивчається гіпотеза про рівняння критичної точки (РКТ) на многовидах, що є майже многовидами Кенмоцу. Насамперед доведено, що у випадку, коли тривимірний $(k,\mu)'$-майже многовид Кенмоцу задовольняє РКТ, цей многовид є або локально ізометричним до добутку просторів $\mathbb{H}^2(-4)\times\mathbb{R}$, або многовидом Кенмоцу. Крім того, доведено, що у випадку, коли метрика многовиду, що є майже многовидом Кенмоцу з конформним розшаруванням Ріба, задовольняє РКТ гіпотезу, цей многовид є многовидом Ейнштейна.

1. Introduction. By an almost contact metric manifold of odd dimensional we mean that a smooth manifold together with an almost contact structure (ϕ,ξ,η,g) given by a $(1,1)$ tensor field ϕ, a characteristic vector field ξ, a 1-form η and a compatible metric g satisfying the conditions [3, 4]

$$\phi^2=-I+\eta\otimes\xi,\quad \phi(\xi)=0,\quad \eta(\xi)=1,\quad \eta\circ\phi=0,$$

$$g(\phi X,\phi Y)=g(X,Y)-\eta(X)\eta(Y),$$

for any vector fields X and Y of T_pM, where T_pM denotes the tangent vector space of M at any point $p\in M$. In 1972, Kenmotsu [13] introduced a new type of almost contact metric manifolds named Kenmotsu manifolds nowadays. Later such manifolds were generalized to almost Kenmosu manifolds by Janssens and Vanhecke [12]. Recently, Dileo and Pastore [9] introduced the notion of $(k,\mu)'$-nullity distribution on an almost Kenmotsu manifold $(M^{2n+1},\phi,\xi,\eta,g)$, which is defined for any $p\in M^{2n+1}$ and $k,\mu\in\mathbb{R}$ as follows:

$$N_p(k,\mu)'=\big\{Z\in T_pM^{2n+1}: R(X,Y)Z=$$

$$=k[g(Y,Z)X-g(X,Z)Y]+\mu[g(Y,Z)h'X-g(X,Z)h'Y]\big\},$$

where two symmetric $(1,1)$-type tensor fields defined by $h'=h\circ\phi$ and $2h=\pounds_\xi\phi$. Since then several authors such as Dileo and Pastore [8], De and Mandal [5–7], Wang and Liu [16–19] studied almost Kenmotsu manifolds satisfying some nullity distributions.

A Riemannian manifold (M,g) of dimension $(2n+1)\geq 3$ with constant scalar curvature and unit volume together with a non-constant smooth potential function λ satisfying

$$\left(\frac{r}{2n}g-S\right)\lambda-\operatorname{Hess}\lambda=S-\frac{r}{2n+1}g, \tag{1.1}$$

where S, r and $\operatorname{Hess}\lambda$ are, respectively, Ricci tensor, scalar curvature and the Hess ian of the smooth function λ on M is called a critical point equation (CPE). Note that if $\lambda=0$, then (1.1) becomes

Einstein metric. Therefore, we consider only the non-trivial potential function λ. In [2], Besse conjectured that the solution of the CPE is Einstein. Barros and Ribeiro [1] proved that the CPE conjecture is true for half conformally flat. Recently, Hwang [11] proved that the CPE conjecture is also true under certain condition on the bounds of the potential function λ. A necessary and sufficient condition on the norm of the gradient of the potential function for a CPE metric is to be Einstein obtained by Neto [14]. Ghosh and Patra [10] consider the CPE conjecture in the frame-work of K-contact manifolds and (k,μ)-contact manifolds.

Motivated by the above studies in this paper we study the CPE conjecture on almost Kenmotsu manifolds. In Section 3, we prove that a three-dimensional $(k,\mu)'$-almost Kenmotsu manifold satisfying the CPE conjecture is either locally isometric to the product space $\mathbb{H}^2(-4)\times\mathbb{R}$, or Kenmotsu. In the final section, we prove that if the metric of an almost Knemotsu manifold with conformal Reeb foliation satisfies the CPE conjecture, then the manifold is Einstein.

2. Almost Kenmotsu manifolds. Let us consider (M^{2n+1},ϕ,ξ,η) be an almost contact manifold. The fundamental 2-form Φ on an almost contact metric manifold is defined by $\Phi(X,Y) = g(X,\phi Y)$ for any vector fields X, Y of T_pM^{2n+1}. An almost Kenmotsu manifold is defined as an almost contact metric manifold such that $d\eta=0$ and $d\Phi = 2\eta\wedge\Phi$. An almost contact metric manifold is said to be normal if the $(1,2)$-type torsion tensor N_ϕ vanishes, where $N_\phi = [\phi,\phi] + 2d\eta\otimes\xi$, where $[\phi,\phi]$ is the Nijenhuis torsion of ϕ [3]. A normal almost Kenmotsu manifold is a Kenmotsu manifold. Also Kenmotsu manifolds can be characterized by $(\nabla_X\phi)Y = g(\phi X,Y)\xi - \eta(Y)\phi X$, for any vector fields X, Y. It is well known [13] that a Kenmotsu manifold M^{2n+1} is locally a warped product $I\times_f N^{2n}$, where N^{2n} is a Kähler manifold, I is an open interval with coordinate t and the warping function f, defined by $f=ce^t$ for some positive constant c. Let $\mathcal{D}$ be the distribution orthogonal to ξ and defined by $\mathcal{D} = \operatorname{Ker}(\eta) = \operatorname{Im}(\phi)$. In an almost Kenmotsu manifold $\mathcal{D}$ is an integrable distribution as η is closed. Let in an almost Kenmotsu manifold the two tensor fields h and l are defined by $h = \dfrac{1}{2}\mathcal{L}_\xi\phi$ and $l = R(\cdot,\xi)\xi$. The tensor fields l and h are symmetric and satisfy the following relations [8]:

$$h\xi = 0,\quad l\xi = 0,\quad tr(h) = 0,\quad tr(h\phi) = 0,\quad h\phi + \phi h = 0, \tag{2.1}$$

$$\nabla_X\xi = -\phi^2X - \phi hX (\Rightarrow \nabla_\xi\xi = 0), \tag{2.2}$$

$$\phi l\phi - l = 2(h^2 - \phi^2), \tag{2.3}$$

$$R(X,Y)\xi = \eta(X)(Y - \phi hY) - \eta(Y)(X - \phi hX) + (\nabla_Y\phi h)X - (\nabla_X\phi h)Y, \tag{2.4}$$

for any vector fields X, Y.

An almost Kenmotsu manifold $(M^{2n+1},\phi,\xi,\eta,g)$ with its characteristic vector field ξ belonging to the $(k,\mu)'$-nullity distribution is known as $(k,\mu)'$-almost Kenmotsu manifolds and the curvature tensor satisfies

$$R(X,Y)\xi = k[\eta(Y)X - \eta(X)Y] + \mu[\eta(Y)h'X - \eta(X)h'Y]. \tag{2.5}$$

Now we provide some related results on almost Kenmotsu manifolds such that ξ belongs to some nullity distributions. The $(1,1)$-type symmetric tensor field $h' = h\circ\phi$ is anticommuting with ϕ and $h'\xi = 0$. Also it is clear that

$$h = 0 \Leftrightarrow h' = 0, \qquad h'^2 = (k+1)\phi^2 (\Leftrightarrow h^2 = (k+1)\phi^2). \tag{2.6}$$

Let $X \in \mathcal{D}$ be the eigen vector of h' corresponding to the eigen value λ. It follows from (2.6) that $\lambda^2 = -(k+1)$ is an constant. Therefore, $k \leq -1$ and $\lambda = \pm\sqrt{-k-1}$. We denote by $[\lambda]'$ and $[-\lambda]'$ the corresponding eigenspaces associated with h' corresponding to the non-zero eigen value λ and $-\lambda$, respectively. We have the following lemmas.

Lemma 2.1 (Propositions 4.1 and 4.3 of [9])**.** *Let $(M^{2n+1}, \phi, \xi, \eta, g)$ be an almost Kenmotsu manifold such that ξ belongs to the $(k,\mu)'$-nullity distribution and $h' \neq 0$. Then $k < -1$, $\mu = -2$ and* $\operatorname{Spec}(h') = \{0, \lambda, -\lambda\}$, *with 0 as simple eigen value and $\lambda = \sqrt{-k-1}$. The distributions $[\xi] \oplus [\lambda]'$ and $[\xi] \oplus [-\lambda]'$ are integrable with totally geodesic leaves. The distributions $[\lambda]'$ and $[-\lambda]'$ are integrable with totally umbilical leaves. Furthermore, the sectional curvature are given as following:*

(a) *$K(X,\xi) = k - 2\lambda$ if $X \in [\lambda]'$ and $K(X,\xi) = k + 2\lambda$ if $X \in [-\lambda]'$;*

(b) *$K(X,Y) = k - 2\lambda$ if $X,\ Y \in [\lambda]'$; $K(X,Y) = k + 2\lambda$ if $X,\ Y \in [-\lambda]'$ and $K(X,Y) = = -(k+2)$ if $X \in [\lambda]'$, $Y \in [-\lambda]'$;*

(c) *M^{2n+1} has constant negative scalar curvature $r = 2n(k-2n)$.*

Lemma 2.2 (Lemma 3 of [19])**.** *Let $(M^{2n+1}, \phi, \xi, \eta, g)$ be an almost Kenmotsu manifold with ξ belonging to the $(k,\mu)'$-nullity distribution. If $h' \neq 0$, then the Ricci operator Q of M^{2n+1} is given by*

$$Q = -2nid + 2n(k+1)\eta \otimes \xi - 2nh'. \tag{2.7}$$

Moreover, the scalar curvature of M^{2n+1} is $2n(k-2n)$.

Lemma 2.3 (Lemma 4.1 of [9])**.** *Let $(M^{2n+1}, \phi, \xi, \eta, g)$ be an almost Kenmotsu manifold with $h' \neq 0$ and ξ belonging to the $(k,-2)'$-nullity distribution. Then, for any $X,\ Y \in T_pM$,*

$$(\nabla_X h')Y = -g(h'X + h'^2X, Y)\xi - \eta(Y)(h'X + h'^2X). \tag{2.8}$$

Lemma 2.4 (Proposition 3 of [8])**.** *An almost Kenmotsu manifold M^3 such that $\nabla\xi = -\phi^2$ is a Kenmotsu manifold.*

3. $(k,\mu)'$-Almost Kenmotsu manifolds satisfying the CPE conjecture. In this section, we consider $(k,\mu)'$-almost Kenmotsu manifolds satisfying the critical point equation in dimension three. Before proving our main result we recall the following result.

Lemma 3.1 [10]**.** *Let (g,λ) be a non-trivial solution of the CPE given by* (1.1) *on a* $(2n+1)$-*dimensional Riemannian manifold M. Then the curvature tensor R can be expressed as*

$$R(X,Y)D\lambda = (X\lambda)QY - (Y\lambda)QX + (\lambda+1)(\nabla_X Q)Y -$$

$$-(\lambda+1)(\nabla_Y Q)X + (Xf)Y - (Yf)X, \tag{3.1}$$

where $f = -r\left(\dfrac{\lambda}{2n} + \dfrac{1}{2n+1}\right)$.

Now we prove the following theorem.

Theorem 3.1. *If the metric of a three dimensional $(k,\mu)'$-almost Kenmotsu manifold satisfies the critical point equation, then the manifold is either Kenmotsu manifold, or locally isometric to the product space $\mathbb{H}^2(-4) \times \mathbb{R}$.*

Proof. Replacing X by ξ in (3.1) and making use of (2.7), we have

$$R(\xi, Y)D\lambda = (\xi\lambda)QY - 2k(Y\lambda)\xi + (\lambda+1)(\nabla_\xi Q)Y -$$
$$-(\lambda+1)(\nabla_Y Q)\xi + (\xi f)Y - (Yf)\xi. \tag{3.2}$$

Taking covariant differentiation of (1.1) along arbitrary vector field X and using (2.2), we obtain

$$(\nabla_X Q)Y = 2(k+1)\eta(Y)(X + h'X) - 2(\nabla_X h')Y -$$
$$-2(k+1)\{g(X,Y) - 2\eta(X)\eta(Y) + g(h'X, Y)\}\xi.$$

By using the above equation, we get

$$(\nabla_X Q)Y - (\nabla_Y Q)X =$$
$$= -2(k+1)\{\eta(X)(Y + h'Y) - \eta(Y)(X + h'X)\} -$$
$$-2\{(\nabla_X h')Y - (\nabla_Y h')X\} \tag{3.3}$$

for any vector fields X, Y. Putting $X = \xi$ in (3.3), we have

$$(\nabla_\xi Q)Y - (\nabla_Y Q)\xi = -2(k+1)\{Y + h'Y - \eta(Y)\xi\} -$$
$$-2\{(\nabla_\xi h')Y - (\nabla_Y h')\xi\}. \tag{3.4}$$

From (2.8), we obtain

$$(\nabla_\xi h')Y - (\nabla_Y h')\xi = h'Y + h'^2 Y. \tag{3.5}$$

Taking inner product of (3.4) and using (3.5), we get

$$g((\nabla_\xi Q)Y - (\nabla_Y Q)\xi, \xi) = 0. \tag{3.6}$$

It follows from (3.2) and (3.6) that

$$g(R(\xi, Y)D\lambda, \xi) = 2nk\xi(\lambda)\eta(Y) - 2nkY(\lambda) + \xi(f)\eta(Y) - Y(f). \tag{3.7}$$

On the other hand, from (2.5) and Lemma 2.1 we have

$$g(R(\xi, Y)D\lambda, \xi) = -g(R(\xi, Y)\xi, D\lambda) =$$
$$= k[g(D\lambda, Y) - \xi(\lambda)\eta(Y)] - 2g(h'D\lambda, Y). \tag{3.8}$$

Making use of (3.7) and (3.8), we get

$$3kD\lambda - k\xi(\lambda)\xi - 2h'D\lambda = 2k\xi(\lambda)\xi + \xi(f)\xi - Df. \tag{3.9}$$

Now, from Lemma 3.1, we have $f = -r\left(\dfrac{\lambda}{2} + \dfrac{1}{3}\right)$. Differentiating this equation, we get

$$\xi(f) = -(k-2)\xi(\lambda) \text{ and } Df = -(k-2)D\lambda, \tag{3.10}$$

where we have used Lemma 2.1. Relations (3.9) and (3.10) both gives

$$h'D\lambda = (k+1)(D\lambda - \xi(\lambda)\xi). \tag{3.11}$$

Applying h' on both sides of (3.11) and using (2.6), we have

$$-(k+1)(D\lambda - \xi(\lambda)\xi) = (k+1)h'D\lambda. \tag{3.12}$$

Substituting the value of $h'D\lambda$ from (3.11) in (3.12) yields

$$(k+1)(k+2)(D\lambda - \xi(\lambda)\xi) = 0.$$

We consider three cases:

Case 1. Let $k = -1$, then from (2.6) we have $h' = 0$. Using this in (2.2) gives $\nabla\xi = -\phi^2$. Hence from Lemma 2.4 the manifold becomes Kenmotsu manifold.

Case 2. Let $k = -2$. Then, from Remark 5.1 of [9], we can state that the manifold is locally isometric to the Riemannian product $\mathbb{H}^2(-4) \times \mathbb{R}$.

Case 3. Let $D\lambda = \xi(\lambda)\xi$. Taking trace of the equation (1.1), we have $\triangle_g\lambda = -\dfrac{r\lambda}{2n}$. Using this and Lemma 2.2 in (1.1) gives

$$\nabla_X D\lambda = (\lambda+1)QX + fX. \tag{3.13}$$

Putting $D\lambda = \xi(\lambda)\xi$ in (3.13) yields

$$(\lambda+1)QX = \{X\xi(\lambda) - \xi(\lambda)\eta(X)\}\xi + \xi(\lambda)h'X +$$
$$+\left\{\xi(\lambda) + 2(k-2)\left(\frac{\lambda}{2} + \frac{1}{3}\right)\right\}X.$$

Comparing this relation with (2.7), we have

$$X\xi(\lambda) - \xi(\lambda)\eta(X) = 2(k+1)(\lambda+1)\eta(X), \tag{3.14}$$

$$\xi\lambda + 2(k-2)\left(\frac{\lambda}{2} + \frac{1}{3}\right) = -2(\lambda+1), \tag{3.15}$$

$$\xi(\lambda) = -2(\lambda+1), \tag{3.16}$$

for any vector field X. By using (3.16) in (3.15), we have λ is a constant, which is a contradiction.

Theorem 3.1 is proved.

4. Almost Kenmotsu manifolds with conformal Reeb foliation satisfying the CPE conjecture. This section is devoted to study almost Kenmotsu manifolds with conformal Reeb foliation satisfying the CPE conjecture of dimension ≥ 5. An almost contact Riemannian manifold M is said to be an η-Einstein manifold if the Ricci tensor S satisfies the condition

$$S(X,Y) = \gamma g(X,Y) + \delta\eta(X)\eta(Y),$$

where γ, δ are smooth functions and X, Y are vector fields on the manifold. In particular, if $\delta = 0$, then M is an Einstein manifold. Pastore and Saltarelli [15] prove that on an almost Kenmotsu manifold the Reeb foliation is conformal if and only if $h = 0$. We present the following result.

Lemma 4.1 [15]. *Let $(M^{2n+1}, \phi, \xi, \eta, g)$, $n > 1$, be an η-Einstein almost Kenmotsu manifold with conformal Reeb foliation, then either the manifold is Einstein, or δ is not constant, $X(\delta) = 0$ for any vector field $X \perp \xi$, $\xi(\delta) = -2\delta$ and in this case the Ricci operator is given by $QX = -(2n+\delta)X + \delta\eta(X)\xi$, where δ is locally given by $\delta = ce^{-2t}$ for some non-zero constant c.*

Now we prove the following theorem.

Theorem 4.1. *Let $(M^{2n+1}, \phi, \xi, \eta, g)$, $n > 1$, be an almost Kenmotsu manifold with conformal Reeb foliation. If M satisfies the critical point equation, then the manifold is an Einstein manifold provided the scalar curvature $r \neq -2n(2n+1)$.*

Proof. Since $h = 0$, we have, from (2.4),

$$R(X,Y)\xi = \eta(X)Y - \eta(Y)X \tag{4.1}$$

for any vector fields X, Y. From this we obtain

$$Q\xi = -2n\xi. \tag{4.2}$$

Since

$$f = -r\left(\frac{\lambda}{2n} + \frac{1}{2n+1}\right),$$

then we get

$$(\xi f) = -\frac{r}{2n}(\xi\lambda) \quad \text{and} \quad (Yf) = -\frac{r}{2n}(Y\lambda). \tag{4.3}$$

Replacing ξ instead of X in (3.1) and using (4.2) yields

$$R(\xi,Y)D\lambda = (\xi\lambda)QY + 2n(Y\lambda)\xi + (\lambda+1)(\nabla_\xi Q)Y - \\ -(\lambda+1)(\nabla_Y Q)\xi + (\xi f)Y - (Yf)\xi. \tag{4.4}$$

Taking inner product of (4.4) with ξ and making use of (4.3) gives

$$g(R(\xi,Y)D\lambda,\xi) = -\left(2n + \frac{r}{2n}\right)(\xi\lambda)\eta(Y) + \left(2n + \frac{r}{2n}\right)(Y\lambda).$$

Also, from (4.1), we obtain

$$g(R(\xi,Y)D\lambda,\xi) = -g(R(\xi,Y)\xi,D\lambda) = (\xi\lambda)\eta(Y) - (Y\lambda).$$

Comparing the above two equations, we have

$$\left(2n + 1 + \frac{r}{2n}\right)\{(Y\lambda) - (\xi\lambda)\eta(Y)\} = 0,$$

from which it follows that

$$\left(2n + 1 + \frac{r}{2n}\right)\{D\lambda - (\xi\lambda)\xi\} = 0. \tag{4.5}$$

Let us assume that the scalar curvature $r \neq -2n(2n+1)$, then we have from (4.5) that $D\lambda = (\xi\lambda)\xi$. By using this in (3.13), we get

$$(\lambda+1)QX = \left((\xi\lambda) + \frac{r\lambda}{2n} + \frac{r}{2n+1}\right)X + (X(\xi\lambda) - (\xi\lambda)\eta(X))\xi. \tag{4.6}$$

This shows the manifold is an η-Einstein manifold. Without loss of any generality we may assume that the manifold is not Einstein. From Lemma 4.1 we see that the second case is true, that is,

$$QX = -(2n + \delta)X + \delta\eta(X)\xi, \tag{4.7}$$

where δ is locally given by $\delta = ce^{-2t}$ for some non-zero constant c. Now comparing the relations (4.6) and (4.7), we have

$$(\xi\lambda) + \frac{r\lambda}{2n} + \frac{r}{2n+1} = -(\lambda + 1)(2n + \delta), \tag{4.8}$$

$$X(\xi\lambda) - (\xi\lambda)\eta(X) = \delta(\lambda + 1)\eta(X) \tag{4.9}$$

for any vector field X. With the help of (4.8) and (4.9) we get

$$\xi(\xi\lambda) = -\frac{r\lambda}{2n} - \frac{r}{2n+1} - (\lambda + 1)2n. \tag{4.10}$$

Tracing (4.6) gives

$$\xi(\xi\lambda) = -2n(\xi\lambda) - \frac{r\lambda}{2n}. \tag{4.11}$$

In view of (4.10) and (4.11), we obtain

$$\xi\lambda = \frac{r}{2n(2n+1)} + \lambda + 1. \tag{4.12}$$

Making use of (4.12) in (4.10) yields

$$(\lambda + 1)\left(\frac{r}{2n} + 1 + 2n\right) = 0.$$

Since λ being a non-constant smooth function, then from the above equation we have $r = -2n(2n + 1)$, which is a contradiction. Hence, using Lemma 4.1, we complete the proof of theorem.

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Received 03.12.16