UDC 517.98

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STRONGLY ALTERNATIVE DUNFORD – PETTIS SUBSPACES OF OPERATOR IDEALS

СИЛЬНО АЛЬТЕРНАТИВНІ ПРОСТОРИ ДАНФОРДА – ПЕТТІСА ОПЕРАТОРНИХ ІДЕАЛІВ

Introducing the concept of strong alternative Dunford – Pettis property (strong DP1) for the subspace $\mathcal{M}$ of operator ideals $\mathcal{U}(X, Y)$ between Banach spaces X and Y, we show that $\mathcal{M}$ is a strong DP1 subspace if and only if all evaluation operators $\phi_x\colon \mathcal{M} \to Y$ and $\psi_{y^*}: \mathcal{M} \to X^*$ are DP1 operators, where $\phi_x(T) = Tx$ and $\psi_{y^*}(T) = T^*y^*$ for $x \in X$, $y^* \in Y^*$, and $T \in \mathcal{M}$. Some consequences related to the concept of alternative Dunford – Pettis property in subspaces of some operator ideals are obtained.

Введено поняття сильної альтернативної властивості Данфорда – Петтіса (сильна DP1) для підпростору $\mathcal{M}$ операторних ідеалів $\mathcal{U}(X, Y)$ між банаховими просторами X та Y, за допомогою якого показано, що $\mathcal{M}$ є сильним DP1 підпростором тоді і тільки тоді, коли всі оператори оцінки $\phi_x\colon \mathcal{M} \to Y$ та $\psi_{y^*}: \mathcal{M} \to X^*$ є DP1 операторами, де $\phi_x(T) = Tx$ та $\psi_{y^*}(T) = T^*y^*$ при $x \in X$, $y^* \in Y^*$ та $T \in \mathcal{M}$. Отримано деякі наслідки щодо поняття альтернативної властивості Данфорда – Петтіса в підпросторах деяких операторних ідеалів.

1. Introduction. A Banach space X has the Dunford – Pettis property (DP) if for each weakly null sequences $(x_n) \subseteq X$ and $(x_n^*) \subseteq X^*$, one has $x_n^*(x_n) \to 0$ as $n \to \infty$. Also the Banach space X has the alternative Dunford – Pettis property (DP1) if for each weakly convergent sequence $x_n \to x$ in X with $\|x_n\| = \|x\| = 1$, for all integer n, and each weakly null sequence (x_n^*) in X^*, we have $x_n^*(x_n) \to 0$.

It is clear that the Banach space X has the DP1 if and only if for each weakly null sequences (x_n) in X and (x_n^*) in X^* and each $x \in X$ with $\|x_n + x\| = \|x\| = 1$, we have $x_n^*(x_n) \to 0$.

Evidently, DP implies the DP1, but the converse in general, is false. For example, every (infinite dimensional) Hilbert space has DP1, but does not have the DP and the space of trace class operators on an infinite dimensional Hilbert space provides another Banach space with the DP1 and without the DP [6]. Also there are Banach spaces such as C^*-algebras and von Neumann algebras, that the DP1 and DP on them coincide [1, 6].

A bounded linear operator $T\colon X \to Y$ between Banach spaces X and Y is said to be completely continuous (or Dunford – Pettis) operator, if for each weakly convergent sequence $x_n \to x$ in X, we have $\|Tx_n - Tx\| \to 0$, that is T carries weakly convergent sequences to norm convergent sequences. But if under the additional condition $\|x_n\| = \|x\| = 1$, the conclusion $\|Tx_n - Tx\| \to 0$ is obtained, we say that T is a DP1 operator.

The concept of DP1 for Banach spaces and for operators introduced by Freedman in [6], and he obtained some properties of them. In particular, the Banach space X has the DP1 if and only if every weakly compact operator $T\colon X \to Y$ into arbitrary Banach space Y is a DP1 operator [6] (Theorem 1.4). This is similar to Theorem 1 of [4] which stay that a Banach space X has the DP property if and only if every weakly compact operator $T\colon X \to Y$ is completely continuous. We refer the reader for additional properties of these concepts to [1, 2, 4, 6, 8].

In [10], the author in a joint work with J. Zafarani, has introduced the concept of strongly completely continuous for subspaces of operator ideals which is the main motivation of this article.

In fact, if $\mathcal{U}$ is any operator ideal with ideal norm $A(\cdot)$, and by meaning of [3] or [12], for any Banach spaces X and Y; $\mathcal{U}(X,Y)$ denotes the component of $\mathcal{U}$ consisting of all bounded linear operators $T\colon X \to Y$ that belongs to $\mathcal{U}$ and $K(X,Y)$ denotes the Banach space of all compact operators from X to Y; then a linear subspace $\mathcal{M} \subseteq \mathcal{U}(X,Y)$ is called strongly completely continuous in $\mathcal{U}(X,Y)$ (resp. in $K(X,Y)$), if for all Banach spaces W and Z and all compact operators $R : Y \to W$ and $S\colon Z \to X$, the left and right multiplication operators L_R and R_S as operators from $\mathcal{M}$ into $\mathcal{U}(X,W)$ and $\mathcal{U}(Z,Y)$ (resp. into $K(X,W)$ and $K(Z,Y)$) respectively, are compact, where $L_R(T) = RT$ and $R_S(T) = TS$, for $T \in \mathcal{M}$. Here, the linear subspace $\mathcal{M} \subseteq \mathcal{U}(X,Y)$ is called strongly DP1 in $\mathcal{U}(X,Y)$ (resp. in $K(X,Y)$), if under the same conditions, the operators L_R and R_S are DP1.

Evidently, everywhere one talks about $\mathcal{U}(X,Y)$ or linear subspace $\mathcal{M}$ of it, the related norm is ideal norm $A(\cdot)$, while the operator norm $\|\cdot\|$ is applied when the space is a linear subspace of $L(X,Y)$ of all bounded linear operators from X into Y. Thus, if $\mathcal{M}$ is a linear subspace of $\mathcal{U}(X,Y)$, we endowed always $\mathcal{M}$ by ideal norm $A(\cdot)$ and the weak topology of $\mathcal{M}$ is refered to this norm. Also, DP1 ness of every operator on $\mathcal{M}$, such as left and right multiplication operators, depends on the norm of the image space.

In [9] and [10], the authors proved that for several operator ideal $\mathcal{U}$, a closed subspace $\mathcal{M} \subseteq \subseteq \mathcal{U}(X,Y)$ is strongly completely continuous in $\mathcal{U}(X,Y)$ (or in $K(X,Y)$) if and only if all evaluation operators $\phi_x\colon \mathcal{M} \to Y$ and $\psi_{y^*}\colon \mathcal{M} \to X^*$ are compact operators, where $x \in X$ and $y^* \in Y^*$ are arbitrary and for each $T \in \mathcal{M}$, $\phi_x(T) = Tx$, $\psi_{y^*}(T) = T^*y^*$. Here, we will prove that similar results hold for strongly DP1 property.

Throughout this article X, Y, Z, V and W denote arbitrary Banach spaces. The closed unit ball of a Banach space X is denoted by X_1, X^* is the dual of X and T^* refers to the adjoint of the operator T. $\mathcal{U}$ is an arbitrary (Banach) operator ideal and $\mathcal{U}(X,Y)$ is applied for component of $\mathcal{U}$. We use the notations $\|T\|$ and $A(T)$ for operator norm and ideal norm of any operator $T \in \mathcal{U}$ respectively and note that in general, $\|T\| \leq A(T)$, for all $T \in \mathcal{U}$. Also for arbitrary Banach spaces X and Y, $L(X,Y)$ and $K(X,Y)$ are used for the Banach spaces of all bounded linear and compact operators between X and Y, respectively, and $K_{w^*}(X^*,Y)$ is the space of all compact weak*-weak continuous operators from X^* to Y. The abbreviation $K(X)$ is used for $K(X,X)$. Our notations are standard and we refer the reader to [3, 5, 7] for undefined notations and terminologies.

2. Main results. In this section we will show that in many operator ideals the strong DP1 ness of its subspaces is necessary or sufficient for the DP1 ness of all evaluation operators on that subspace.

Theorem 2.1. *Let $\mathcal{M} \subseteq \mathcal{U}(X,Y)$ be a closed subspace such that all of the evaluation operators ϕ_x and ψ_{y^*} are* DP1. *Then $\mathcal{M}$ is strongly* DP1 *in $K(X,Y)$.*

Proof. Let $T_n \to T$ weakly in $\mathcal{M}$ and $A(T_n) = A(T) = 1$. Then by assumption, for each $x \in X$, $T_n x \to Tx$ in norm. The boundedness of the sequence $(\|T_n\|)$ then implies that the sequence T_n converges uniformly to T on compact subsets of X. This shows that $T_n S \to TS$ in norm of $K(Z,Y)$, for every compact operator $S\colon Z \to X$. So R_S is DP1.

Similarly, $T_n^* \to T^*$ uniformly on compact subsets of Y^* and so $T_n^* R^* \to T^* R^*$ in norm of $K(W^*, X^*)$, for every compact operator $R\colon Y \to W$. Thus $RT_n \to RT$ and the proof is completed.

As a consequence of the theorem, we have the following refined corollary for closed operator ideals. Recall that an operator ideal $\mathcal{U}$ is closed if its components $\mathcal{U}(X,Y)$ are closed in $L(X,Y)$.

***Corollary* 2.1.** *Let $\mathcal{U}$ be a closed operator ideal and $\mathcal{M}$ be a linear subspace of $\mathcal{U}(X,Y)$ such that all of the evaluation operators ϕ_x and ψ_{y^*} are* DP1. *Then $\mathcal{M}$ is strongly* DP1 *in $\mathcal{U}(X,Y)$.*

Proof. We first note that by definition of operator ideal, L_R and R_S are operators into $\mathcal{U}(X,W)$ and $\mathcal{U}(Z,Y)$, respectively. Now suppose that $T_n \to T$ weakly in $\mathcal{M}$ and $A(T_n) = A(T) = 1$. Then by Theorem 2.1, $\|T_nS - TS\| \to 0$ and $\|RT_n - RT\| \to 0$ as $n \to \infty$. Since $\mathcal{U}$ is a closed operator ideal, by open mapping theorem, there exists a $\delta > 0$ such that $A(K) \leq \delta\|K\|$ for all operator K in $\mathcal{U}$. So $A(T_nS - TS)$ and $A(RT_n - RT)$ tend to 0. This shows that $\mathcal{M}$ is strongly DP1 in $\mathcal{U}(X,Y)$ and the proof is completed.

Although, by Theorem 2.1, the strong DP1 ness of $\mathcal{M} \subseteq \mathcal{U}(X,Y)$ in $K(X,Y)$ follows from the DP1 ness of all point evaluations on $\mathcal{M}$, but we do not know that in general, the same question about strong DP1 ness of $\mathcal{M} \subseteq \mathcal{U}(X,Y)$ in $\mathcal{U}(X,Y)$ is true or false. In the following two theorems, we partially give an affirmative answer to this question.

Theorem 2.2. *Let X and Y^* have the approximation property and $\mathcal{M} \subseteq \mathcal{U}(X,Y)$ be a linear subspace. If all of the evaluation operators ϕ_x and ψ_{y^*} are* DP1, *then $\mathcal{M}$ is strongly* DP1 *in $\mathcal{U}(X,Y)$.*

Proof. Let $R\colon Y \to W$ be a compact operator. Since Y^* has the approximation property, there exists a sequence $R_n : Y \to W$ of finite rank operators such that $\|R_n - R\| \to 0$ as $n \to \infty$.

We claim that each multiplication operator $L_{R_n}\colon \mathcal{M} \to \mathcal{U}(X,W)$ is a DP1 operator. Since any finite sum of DP1 operators is also DP1, it is enough to consider the particular case $R_n = y^* \otimes w$, where $y^* \in Y^*$ and $w \in W$ are arbitrary.

Now suppose that (T_k) is a weakly null sequence in $\mathcal{M}$, $T \in \mathcal{M}$ and $A(T + T_k) = A(T) = 1$. Then by assumption $\|T_k^* y^*\| \to 0$ and so

$$A(L_{R_n}T_k) = A(R_nT_k) = A(T_k^* y^* \otimes w) = \|T_k^* y^*\| \cdot \|w\| \to 0,$$

as $k \to \infty$. Thus L_{R_n} is DP1, for each n. Also,

$$A(L_RT_k) \leq A((L_R - L_{R_n})T_k) + A(L_{R_n}T_k) \leq \|R_n - R\|A(T_k) + A(L_{R_n}T_k) \to 0,$$

as $k \to \infty$ for suitable n. Hence L_R is a DP1 operator.

Similarly, if $S\colon Z \to X$ is a compact operator, the approximation property of X yields the existence of a sequence $S_n\colon Z \to X$ of finite rank operators such that $\|S_n - S\| \to 0$ as $n \to \infty$.

Since each S_n is a finite sum of operators $z^* \otimes x$ with $z^* \in Z^*$ and $x \in X$, a similar method finishes the proof of the argument.

Now we will show that for each two arbitrary Banach spaces X and Y (without any approximate assumption), if $\mathcal{U}$ is an injective operator ideal, then a similar result holds for closed subspaces of $\mathcal{U}(X,Y)$. Recall that an operator ideal $\mathcal{U}$ is injective if for each Banach spaces X, V and W and each isometric embedding $J\colon V \to W$, the operator $L_J\colon \mathcal{U}(X,V) \to \mathcal{U}(X,W)$ is also an (isometric) embedding, and furthermore, an operator $T \in L(X,V)$ belongs to $\mathcal{U}$ if $JT \in \mathcal{U}$. Many usual operator ideals are injective. For instance, the (weakly) compact operators, the (weakly) Banach – Saks operators, the unconditionally converging operators and the p-summing operators between Banach spaces, with $1 \leq p < \infty$, are standard examples. For additional examples see [3, 12]. So we have the following theorem:

Theorem 2.3. *If $\mathcal{U}$ is an injective operator ideal and $\mathcal{M} \subseteq \mathcal{U}(X,Y)$ is a linear subspace such that all of the evaluation operators are* DP1, *then $\mathcal{M}$ is strongly* DP1 *in $\mathcal{U}(X,Y)$.*

Proof. Let $R\colon Y \to V$ be a compact operator and $W = l^\infty(V_1^*)$. Then W has the approximation property and there exists a natural isometric embedding $J\colon V \to W$. Since $JR\colon Y \to W$ is compact, the proof of Theorem 2.2 shows that the left multiplication operator $L_{JR}\colon \mathcal{M} \to \mathcal{U}(X,W)$ is a DP1 operator. But $L_{JR} = L_J \circ L_R$ and $L_J : \mathcal{U}(X,V) \to \mathcal{U}(X,W)$ is an isometric embedding. So L_R is a DP1 operator.

Similarly, for each compact operator $S\colon Z \to X$, if $J\colon Z^* \to l^\infty((Z^{**})_1)$ is an isometric embedding and (S_n) is an approximated sequence of finite rank operators for JS^*, then the DP1 ness of all ϕ_x with the fact that each S_n is a finite sum of one rank operators $x \otimes z$ with $x \in X$ and $z \in l^\infty((Z^{**})_1)$, shows that each L_{S_n} as operator from $\widetilde{\mathcal{M}} = \{T^* : T \in \mathcal{M}\}$ is DP1. Hence L_{JS^*} and so L_{S^*} as operator from $\widetilde{\mathcal{M}}$ is DP1.

Theorem 2.3 is proved.

The following theorem shows that the converse of the above theorems is also valid in every operator ideal $\mathcal{U}$.

Theorem 2.4. *Let $\mathcal{M} \subseteq \mathcal{U}(X,Y)$ be a linear subspace such that for some Banach spaces W and Z and each finite rank operators $R\colon Y \to W$ and $S\colon Z \to X$, the left and right multiplication operators L_R and R_S, as operators from $\mathcal{M}$ into $\mathcal{U}(X,W)$ and $\mathcal{U}(Z,Y)$ respectively, be* DP1. *Then all of the evaluation operators ϕ_x and ψ_{y^*} are* DP1.

Proof. Suppose that $x \in X$ is a fixed element. We will show that the evaluation operator ϕ_x is DP1. Consider an element $z^* \in Z^*$ such that $\|z^*\| = 1$ and $J : Y \to \mathcal{U}(Z,Y)$ via $Jy = z^* \otimes y$ as an isometric embedding. If one define the operator S on Z by $S = z^* \otimes x$, then $R_S = J\phi_x$ and then by assumption is DP1 operator. Since J is an isometric embedding, ϕ_x is also DP1. The same argument proves that all evaluation operators ψ_{y^*} is DP1.

***Corollary* 2.2.** *Let X and Y be two Banach spaces and $\mathcal{U}$ be an operator ideal that satisfy one of the following assertions*:

(1) *X and Y^* have the approximation property,*

(2) *$\mathcal{U}$ is a closed operator ideal or,*

(3) *$\mathcal{U}$ is an injective operator ideal.*

If $\mathcal{M} \subseteq \mathcal{U}(X,Y)$ is a normed linear subspace, then the following are equivalent:

(a) *all evaluation operators ϕ_x and ψ_{y^*} are* DP1,

(b) *$\mathcal{M}$ is strongly* DP1 *in $\mathcal{U}(X,Y)$*,

(c) *$\mathcal{M}$ is strongly* DP1 *in $K(X,Y)$*,

(d) *for some Banach spaces W and Z and all finite rank operators $R\colon Y \to W$ and $S\colon Z \to X$, left and right multiplication operators L_R and R_S, as operators from $\mathcal{M}$ into $\mathcal{U}(X,W)$ and $\mathcal{U}(Z,Y)$ (or into $K(X,W)$ and $K(Z,Y)$) respectively, are* DP1 *operators.*

***Corollary* 2.3.** *Let X and Y be two reflexive Banach spaces and $\mathcal{M} \subseteq \mathcal{U}(X,Y)$ be a closed subspace. If $\mathcal{M}$ has the* DP1, *then $\mathcal{M}$ is strongly* DP1 *in $K(X,Y)$.*

Proof. Since X and Y are reflexive Banach spaces, Theorem 2.2 of [8] shows that all evaluation operators are DP1. So by Theorem 2.1, $\mathcal{M}$ is strongly DP1 in $K(X,Y)$.

Evidently, if one of the additional conditions of Corollary 2.2 satisfies, then $\mathcal{M}$ is strongly DP1 in $\mathcal{U}(X,Y)$.

The following corollary is similar to Theorem 6 of [13] which stay that if $\mathcal{A}$ is a closed subalgebra of the space $K(X)$ of all compact operators on a reflexive Banach space X, which has the DP, then

$\mathcal{A}$ is completely continuous; that is, for each $S \in \mathcal{A}$, the left and right multiplication operators $L_S\colon \mathcal{A} \to \mathcal{A}$ and $R_S\colon \mathcal{A} \to \mathcal{A}$ are compact.

***Corollary* 2.4.** *Let X be a reflexive Banach space and $\mathcal{A} \subseteq K(X)$ be a closed subalgebra which has the* DP1 *property. Then for each $S \in \mathcal{A}$, the left and right multiplication operators $L_S\colon \mathcal{A} \to \mathcal{A}$ and $R_S\colon \mathcal{A} \to \mathcal{A}$ are* DP1.

Proof. By Corollary 2.3, $\mathcal{A}$ is strongly DP1 in $K(X)$. Since the concept of strongly DP1 in $K(X)$ is in general stronger than the DP1 of the left and right multiplication operators on closed subalgebras of $K(X)$, the prove of this corollary is completed.

The next Corollary 2.5 proves that for some Banach spaces X and Y, having the Schauder decompositions [7], and some operator ideal between them, the strong DP1 is also a sufficient condition for the DP1 property. The concept of Schauder decomposition, as a generalization of Schauder basis of Banach spaces, provides a good location for introducing the concept of $\mathcal{P}$-property for subspaces of operator spaces, which has an essential role in the results of [8, 11].

If $\sum_{n=1}^{\infty} \oplus X_n$ and $\sum_{n=1}^{\infty} \oplus Y_n$ are Schauder decompositions of X and Y respectively, by meaning of [11], we say that a closed subspace $\mathcal{M} \subseteq \mathcal{U}(X,Y)$ has the $\mathcal{P}$-property if for all integers m_0 and n_0 and every operators $T, S \in \mathcal{M}$,

$$\|P_W T P_V + P_{W'} S P_{V'}\| \le \max\{\|P_W T P_V\|, \|P_{W'} S P_{V'}\|\},$$

where $V = X_1 \oplus \ldots \oplus X_{m_0}$ and $W = Y_1 \oplus \ldots \oplus Y_{n_0}$, V' and W' are complementary subspaces of V and W in X and Y respectively. Also for each complemented subspace V of X, the symbols P_V and $P_{V'}$ refer to the the natural projections of X onto V and complementary subspace V' of V respectively.

Finally, we need to remember the following theorem of [8]. For undefined terminologies about Schauder decomposition, we refer the reader to [7].

Theorem 2.5 (Theorems 2.4 and 2.6 of [8])**.** *Let X and Y have monotone finite dimensional Schauder decompositions such that the decomposition of X is shrinking. Let $\mathcal{M}$ be a closed subspace of $K_{w^*}(X^*, Y)$ or $K(X,Y)$ which has the $\mathcal{P}$-property. If all of the related evaluation operators are* DP1 *operators, then $\mathcal{M}$ has the* DP1 *property.*

There is a similar result for a closed subspace $\mathcal{M} \subseteq K(H_1, H_2)$, where H_1 and H_2 are two arbitrary Hilbert spaces and one can find it in Theorem 2.10 of [8].

***Corollary* 2.5.** *Let X and Y satisfy the hypothesis of Theorem 2.5. If $\mathcal{M}$ is a closed subspace of $K_{w^*}(X^*, Y)$ or $K(X,Y)$ which has the $\mathcal{P}$-property and is strongly* DP1, *then $\mathcal{M}$ has the* DP1 *property.*

Proof. By Theorem 2.4, all related evaluation operators on $\mathcal{M}$ are DP1. Now an appeal to Theorem 2.5 finishes the proof.

There are Banach spaces X and Y having the Schauder decompositions such that some classes of operators between them have the $\mathcal{P}$-property and one can fined them in [8] or [11]. These leads to the following two corollaries:

***Corollary* 2.6.** *Let X be an l_p-direct sum and Y be an l_q-direct sum of finite dimensional Banach spaces with $1 < p \le q < \infty$. If $\mathcal{M}$ is a closed subspace of $K(X,Y)$, then the following are equivalent*:

(a) *$\mathcal{M}$ has the* DP1 *property,*

(b) *$\mathcal{M}$ is a strongly* DP1 *subspace of* $K(X,Y)$,

(c) *all evaluation operators ϕ_x and ψ_{y^*} are* DP1 *operators.*

Proof. The statement $(a) \Rightarrow (b)$ follows from Corollary 2.3, and $(b) \Rightarrow (c)$ follows from Theorem 2.4. Also an appeal to Corollary 2.7 of [8] proves $(c) \Rightarrow (a)$.

***Corollary* 2.7.** *Let H_1 and H_2 be two Hilbert spaces and $\mathcal{M}$ be a closed subspace of $K(H_1,H_2)$. Then the following are equivalent*:

(a) *$\mathcal{M}$ has the* DP1 *property,*

(b) *$\mathcal{M}$ is a strongly* DP1 *subspace of* $K(H_1,H_2)$,

(c) *all evaluation operators ϕ_x and ψ_y are* DP1 *operators.*

Proof. Apply Theorems 2.1, 2.4 and also Theorem 2.10 of [8].

Finally, we give an example of a closed linear subspace of an operator ideal such that all evaluation operators are DP1 but all are not completely continuous. So the results of this article can be informative.

***Example* 2.1.** Let $\mathcal{U}$ be an arbitrary operator ideal and consider the closed linear subspace $\mathcal{M} := l_2$ of $\mathcal{U}(l_2,l_2)$ via the isometrically embedding

$$x \mapsto T_x, \quad T_x(y) = \langle y, e_1\rangle x,$$

where $x, y \in l_2$ and e_1 is the first element of the standard orthonormal basis (e_n) of l_2.

Since the Hilbert space l_2 has the Kadec – Klee property (that is, weak and norm convergence of sequences in the unit sphere of l_2 coincide); for each weakly convergent sequence $x_n \to x$ in l_2 with $\|x_n\| = \|x\| = 1$, we have $\|x_n - x\| \to 0$ as $n \to \infty$. Thus all of the bounded evaluation operators on $\mathcal{M} = l_2$ are DP1. On the other hand, for each n, $e_n = T_{e_n}(e_1) \in \phi_{e_1}(\mathcal{M}_1)$, so that the evaluation operator ϕ_{e_1} is not compact and so is not completely continuous.

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Received 28.01.11