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GLOBAL NONEXISTENCE OF SOLUTIONS OF A SYSTEM OF NONLINEAR VISCOELASTIC WAVE EQUATIONS WITH DEGENERATE DAMPING AND SOURCE TERMS

ГЛОБАЛЬНЕ НЕІСНУВАННЯ РОЗВ'ЯЗКІВ СИСТЕМИ НЕЛІНІЙНИХ В'ЯЗКОЕЛАСТИЧНИХ ХВИЛЬОВИХ РІВНЯНЬ ІЗ ВИРОДЖЕНИМ ЗАТУХАННЯМ ТА ДЖЕРЕЛАМИ

The global existence and nonexistence of solutions of a system of nonlinear wave equations with degenerate damping and source terms supplemented with initial and Dirichlet boundary conditions was shown by Rammaha and Sakuntasathien in a bounded domain $\Omega \subset \mathbb{R}^n$, $n = 1, 2, 3$, in the case where the initial energy is negative. A global nonexistence result on a solution with positive initial energy for a system of viscoelastic wave equations with nonlinear damping and source terms was obtained by Messaoudi and Said-Houari. Our result extends these previous results. We prove that the solutions of a system of wave equations with viscoelastic term, degenerate damping, and strong nonlinear sources acting in both equations at the same time are globally nonexistent, provided that the initial data are large enough in a bounded domain Ω of $\mathbb{R}^n$, $n \geq 1$, the initial energy is positive, and the strong nonlinear functions f_1 and f_2 satisfy appropriate conditions. The main tool of the proof is based on methods used by Vitillaro and developed by Said-Houari.

Глобальне існування та неіснування розв'язків системи нелінійних хвильових рівнянь із виродженим затуханням та джерелами, доповненої початковими умовами та граничними умовами Діріхле, було встановлено Rammaha та Sakuntasathien у обмеженій області $\Omega \subset \mathbb{R}^n$, $n = 1, 2, 3$, при від'ємній початковій енергії. Результат про глобальне неіснування розв'язку системи нелінійних в'язкоеластичних хвильових рівнянь із нелінійним затуханням та джерелами при додатній початковій енергії було отримано у роботі Messaoudi та Said-Houari. Наш результат узагальнює ці попередні результати. Доведено, що розв'язки системи хвильових рівнянь із в'язкоеластичним членом, виродженим затуханням та сильно нелінійними джерелами, що діють одночасно в обох рівняннях, глобально не існують, якщо початкові дані є достатньо великими в обмеженій області Ω в $\mathbb{R}^n$, $n \geq 1$, початкова енергія є додатною, а сильно нелінійні функції f_1 та f_2 задовольняють відповідні умови. Доведення базується на методах, що були використані у роботі Vitillaro та розвинуті у роботі Said-Houari.

1. Introduction. In this work we consider the following system of viscoelastic wave equations with degenerate damping and strong nonlinear source terms:

$$\begin{gathered} u_{tt} - \Delta u + \int\limits_0^t g(t-s)\,\Delta u(x,s)\,ds + \left(a\,|u|^k + b\,|v|^l\right)|u_t|^{m-1}u_t = f_1(u,v), \\ v_{tt} - \Delta v + \int\limits_0^t h(t-s)\,\Delta v(x,s)\,ds + \left(c\,|v|^\theta + d\,|u|^\varrho\right)|v_t|^{r-1}v_t = f_2(u,v), \end{gathered} \tag{1.1}$$

where $m, r > 0$, $k, l, \theta, \varrho \geq 1$ and the two functions $f_1(u,v)$ and $f_2(u,v)$ given by

$$\begin{gathered} f_1(u,v) = a_1|u+v|^{2(\rho+1)}(u+v) + b_1|u|^\rho u|v|^{(\rho+2)}, \\ f_2(u,v) = a_1|u+v|^{2(\rho+1)}(u+v) + b_1|u|^{(\rho+2)}|v|^\rho v, \qquad a_1, b_1 > 0, \quad \rho > -1. \end{gathered} \tag{1.2}$$

In (1.1), $u = u(t,x)$, $v = v(t,x)$, where $x \in \Omega$ is a bounded domain of $\mathbb{R}^n$, $n \geq 1$, with a smooth boundary $\partial\Omega$ and $t > 0$, $a, b, c, d > 0$.

System (1.1) is supplemented with the following initial conditions:

$$(u(0), v(0)) = (u_0, v_0), (u_t(0), v_t(0)) = (u_1, v_1), \quad x \in \Omega, \tag{1.3}$$

and boundary conditions

$$u(x) = v(x) = 0, \quad x \in \partial\Omega. \tag{1.4}$$

Viscoelastic materials have properties between two types (elastic materials and viscous fluids). This two types of materials are usually considered in basic texts on continuum mechanics. At each material point of an elastic material the stress at the present time depends only on the present value of the strain. On the other hand, for an incompressible viscous fluid the stress at a given point is a function of the present value of the velocity gradient at that point. Such materials have memory: the stress depends not only on the present values of the strain and /or velocity gradient, but also on the entire temporal history of motion.

This type of problems arise usually in viscoelasticity, it has been considered first by Dafermos [9], where the general decay was discussed. A related problems to (1.1) have attracted a great deal of attention in the last decades, and many results have been appeared on the existence and long time behavior of solutions. See in this directions [3 – 5, 8, 11, 15, 19 – 21] and references therein.

In the absence of viscoelastic term, some special cases of the single wave equations with nonlinear damping and nonlinear source terms in the form

$$u_{tt} - \Delta u + a|u_t|^{m-1}u_t = b|u|^{p-1}u, \tag{1.5}$$

arise in quantum field theory which describe the motion of charged mesons in an electromagnetic field. Equation (1.5) together with initial and boundary conditions of Dirichlet type, has been extensively studied and results concerning existence, blow up and asymptotic behavior of smooth, as well as weak solutions have been established by several authors over the past decades.

The study of single wave equation with the presence of different mechanisms of dissipation, damping and nonlinear sources has been extensively studied and results concerning existence, nonexistence and asymptotic behavior of solutions have been established by several authors and many results appeared in the literature over the past decades. See [2, 10, 12, 13, 16, 18, 23] and references therein.

Concerning the system of equations, in [1] Agre and Rammaha studied the following system:

$$u_{tt} - \Delta u + |u_t|^{m-1}u_t = f_1(u, v),$$

$$v_{tt} - \Delta v + |v_t|^{r-1}v_t = f_2(u, v),$$

in $\Omega \times (0, T)$ with initial and boundary conditions and the nonlinear functions f_1 and f_2 satisfying appropriate conditions. They proved under some restrictions on the parameters and the initial data many results on the existence of a weak solution. They also showed that any weak solution with negative initial energy blows up in finite time using the same techniques as in [10].

In [23], author considered the same problem treated in [1], and he improved the blow up result obtained in [1], for a large class of initial data in which the initial energy can take positive values.

In the work [18], authors considered the nonlinear viscoelastic system:

$$\begin{aligned} &u_{tt} - \Delta u + \int\limits_0^t g(t-s)\Delta u(x,s)ds + |u_t|^{m-1} u_t = f_1(u,v), \\ & \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad x \in \Omega, \quad t > 0, \\ &v_{tt} - \Delta v + \int\limits_0^t h(t-s)\Delta v(x,s)ds + |v_t|^{r-1} v_t = f_2(u,v), \end{aligned} \tag{1.6}$$

where

$$f_1(u,v) = a|u+v|^{2(\rho+1)}(u+v) + b|u|^{\rho}u|v|^{(\rho+2)},$$

$$f_2(u,v) = a|u+v|^{2(\rho+1)}(u+v) + b|u|^{(\rho+2)}|v|^{\rho}v,$$

and they prove a global nonexistence theorem for certain solutions with positive initial energy, the main tool of the proof is a method used in [23].

Concerning the study of decay of solutions of systems of evolution equations, let us mention the work of B. Said-Houari, S. A. Messaoudi and A. Guesmia in [25], where they treated the nonlinear viscoelastic system in (1.6) and under some restrictions on the nonlinearity of the damping and the source terms, they prove that, for certain class of relaxation functions and for some restrictions on the initial data, the rate of decay of the total energy depends on those of the relaxation functions.

Recently, in [22] M. A. Rammaha and Sawanya Sakuntasathien focus on the global well-posedness of the system of nonlinear wave equations

$$u_{tt} - \Delta u + \left(d\,|u|^k + e\,|v|^l\right)|u_t|^{m-1}u_t = f_1(u,v),$$

$$v_{tt} - \Delta v + \left(d'\,|v|^\theta + e'\,|u|^\rho\right)|v_t|^{r-1}v_t = f_2(u,v),$$

in a bounded domain $\Omega \subset \mathbb{R}^n$, $n = 1,2,3$, with Dirichlet boundary conditions. The nonlinearities $f_1(u,v)$ and $f_2(u,v)$ act as a strong source in the system. Under some restriction on the parameters in the system, they obtain several results on the existence and uniqueness of solutions. In addition, they prove that weak solutions blow up in finite time whenever the initial energy is negative and the exponent of the source term is more dominant than the exponents of both damping terms. This type of problems are not only important from the theoretical point of view, but also arise in many physical applications and describe a great deal of models in applied science, many questions in physics and engineering give rise to problems that deal with system of nonlinear wave equations.

The presente paper is organized as follows:

In Section 2 we introduce and present some notation and prepare some material needed for our proof. In Section 3 we state and prove our main result, where we prove a global nonexistence (blow up for all time) of solution of system (1.1) – (1.4) with positive initial energy for some conditions on the functions f_1 and f_2.

2. Assumptions, notations and preliminaries. In this section, we introduce and present some notations and some technical lemmas to be used throughout this paper. The constants c_i, $i = 0, 1, 2, \ldots$, used throughout this paper are positive generic constants, which may be different in various occurrences.

We assume that the relaxation functions g, $h\colon \mathbb{R}^+ \longrightarrow \mathbb{R}^+$ are of class C^1 and nonnicreasing differentiable satisfying:

$$\begin{aligned} &1 - \int_0^\infty g(s)ds = l > 0, \quad g(t) \geq 0, \quad g'(t) \leq 0, \\ & \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad t \geq 0. \\ &1 - \int_0^\infty h(s)ds = k > 0, \quad h(t) \geq 0, \quad h'(t) \leq 0, \end{aligned} \tag{2.1}$$

Lemma 2.1. *There exists a function $F(u,v)$ such that*

$$F(u,v) = \frac{1}{2(\rho+2)}\left[uf_1(u,v) + vf_2(u,v)\right] = \frac{1}{2(\rho+2)}\left[a_1|u+v|^{2(\rho+2)} + 2b_1|uv|^{\rho+2}\right] \geq 0,$$

where

$$\frac{\partial F}{\partial u} = f_1(u,v), \quad \frac{\partial F}{\partial v} = f_2(u,v).$$

We introduce the "*modified*" energy functional $E(t)$ associated to our system

$$2E(t) = \|u_t\|_2^2 + \|v_t\|_2^2 + J(u,v) - 2\int_\Omega F(u,v)\,dx, \tag{2.2}$$

where

$$J(u,v) = \left(1 - \int_0^t g(s)\,ds\right)\|\nabla u\|_2^2 + \left(1 - \int_0^t h(s)\,ds\right)\|\nabla v\|_2^2 + (g \circ \nabla u) + (h \circ \nabla v)$$

and

$$(g \circ u)(t) = \int_0^t g(t-\tau)\|u(t) - u(\tau)\|_2^2\,d\tau,$$

$$(h \circ v)(t) = \int_0^t h(t-\tau)\|v(t) - v(\tau)\|_2^2\,d\tau.$$

Let us point out that the integral $\int_\Omega F(u,v)\,dx$ in (2.2) makes sense because $H_0^1(\Omega) \subset L^{2(\rho+2)}(\Omega)$, for

$$\begin{aligned} &-1 < \rho && \text{if} \quad n = 1, 2, \\ &-1 < \rho \leq \frac{4-n}{n-2} && \text{if} \quad n \geq 3. \end{aligned} \tag{2.3}$$

Lemma 2.2 [23]. *There exist two positive constants c_0 and c_1 such that*

$$\frac{c_0}{2(\rho+2)}\left(|u|^{2(\rho+2)}+|v|^{2(\rho+2)}\right)\leq F(u,v)\leq\frac{c_1}{2(\rho+2)}\left(|u|^{2(\rho+2)}+|v|^{2(\rho+2)}\right).$$

The following technical lemma will play an important role in the sequel.

Lemma 2.3. *Suppose that* (2.3) *holds. Then there exists $\eta>0$ such that for any $(u,v)\in$ $\in H_0^1(\Omega)\times H_0^1(\Omega)$ the inequality*

$$2(\rho+2)\int\limits_{\Omega}F(u,v)\,dx\leq\eta\left(\|\nabla u\|_2^2+\|\nabla v\|_2^2\right)^{\rho+2} \tag{2.4}$$

holds.

Direct computations, using Minkowski, Hôlder's and Young's inequalities and the embedding $H_0^1(\Omega)\hookrightarrow L^{2(\rho+2)}(\Omega)$ yields the proof of this previous Lemma 2.3.

Lemma 2.4. *Let $\nu>0$ be a real positive number and $L(t)$ be a solution of the ordinary differential inequality*

$$\frac{dL(t)}{dt}\geq\xi L^{1+\nu}(t) \tag{2.5}$$

defined in $[0,\infty)$.

If $L(0)>0$, then the solution ceases to exist for $t\geq L(0)^{-\nu}\xi^{-\nu}\nu^{-1}$.

Proof. Direct integration of (2.5) gives

$$L^{-\nu}(0)-L^{-\nu}(t)\geq\xi\nu t.$$

Thus we obtain the following estimate:

$$L^{\nu}(t)\geq\left[L^{-\nu}(0)-\xi\nu t\right]^{-1}. \tag{2.6}$$

It is clear that the right-hand side of (2.6) is unbounded when

$$\xi\nu t=L^{-\nu}(0).$$

Lemma 2.4 is proved.

3. Blow up results.

Lemma 3.1. *Suppose that* (2.3) *holds. Let (u,v) be the solution of the system* (1.1)–(1.4) *then the energy functional is a nonincreasing function, that is for all $t\geq 0$,*

$$E'(t)=-\int\limits_{\Omega}\left(|u(t)|^k+|v(t)|^l\right)|u_t(t)|^{m+1}\,dx-\int\limits_{\Omega}\left(|v(t)|^{\theta}+|u(t)|^{\varrho}\right)|v_t(t)|^{r+1}\,dx+$$

$$+\frac{1}{2}\left(g'\circ\nabla u\right)+\frac{1}{2}\left(h'\circ\nabla v\right)-\frac{1}{2}g(s)\|\nabla u\|_2^2-\frac{1}{2}h(s)\|\nabla v\|_2^2. \tag{3.1}$$

Our main result reads as follows:

Theorem 3.1. *Suppose that* (2.3) *holds. Assume further that*

$$\rho > \max\left(\frac{k+m-3}{2}, \frac{l+m-3}{2}, \frac{\theta+r-3}{2}, \frac{\varrho+r-3}{2}\right), \tag{3.2}$$

and that there exists p, *such that* $2 < p < 2(\rho+2)$, *for which*

$$\max\left(\int\limits_0^\infty g(s)\,ds, \int\limits_0^\infty h(s)\,ds\right) < \frac{(p/2)-1}{(p/2)-1+(1/2p)}, \tag{3.3}$$

holds. Then any solution of problem (1.1)–(1.4), *with initial data satisfying*

$$\|\nabla u_0\|_2^2 + \|\nabla v_0\|_2^2 > \alpha_1^2 \qquad \text{and} \qquad E(0) < E_2$$

blows up for all time, where the constants α_1 *and* E_2 *are defined in* (3.4).

We take $a_1 = b_1 = 1$ for convenience. We introduce the following:

$$B = \eta^{\frac{1}{2(\rho+2)}}, \qquad \alpha_1 = B^{-\frac{(\rho+2)}{(\rho+1)}}, \qquad E_1 = \left(\frac{1}{2} - \frac{1}{2(\rho+2)}\right)\alpha_1^2, \tag{3.4}$$

$$E_2 = \left(\frac{1}{p} - \frac{1}{2(\rho+2)}\right)\alpha_1^2,$$

where η is the optimal constant in (2.4).

Lemma 3.2 [23]. *Suppose that* (2.3), (3.2) *and* (3.3) *hold. Let* (u,v) *be a solution of* (1.1)–(1.4). *Assume further that* $E(0) < E_2$ *and*

$$\|\nabla u_0\|_2^2 + \|\nabla v_0\|_2^2 > \alpha_1^2.$$

Then there exists a constant $\alpha_2 > \alpha_1$ *such that*

$$J(t) > \alpha_2^2, \tag{3.5}$$

and

$$2(\rho+2)\int\limits_\Omega F(u,v)\,dx \geq (B\alpha_2)^{2(\rho+2)} \quad \forall t \geq 0.$$

***Proof of Theorem* 3.1.** We suppose that the solution exists for all time and we reach to a contradiction. For this purpose, we set

$$H(t) = E_2 - E(t). \tag{3.6}$$

By using the definition of $H(t)$, we get

$$H'(t) = -E'(t) =$$

$$= \int_{\Omega} \left(|u(t)|^{k} + |v(t)|^{l} \right) |u_t(t)|^{m+1}\, dx + \int_{\Omega} \left(|v(t)|^{\theta} + |u(t)|^{\varrho} \right) |v_t(t)|^{r+1}\, dx -$$

$$-\frac{1}{2}\left(g' \circ \nabla u\right) - \frac{1}{2}\left(h' \circ \nabla v\right) + \frac{1}{2} g(s) \|\nabla u\|_2^2 + \frac{1}{2} h(s) \|\nabla v\|_2^2 \geq 0 \quad \forall t \geq 0.$$

Consequently, since $E'(t)$ is absolutely continuous

$$H(0) = E_2 - E(0) > 0.$$

Then

$$0 < H(0) \leq H(t) = E_2 - \frac{1}{2}\left(\|u_t\|_2^2 + \|v_t\|_2^2\right) - \frac{J(t)}{2} +$$

$$+ \frac{1}{2(\rho+2)} \left[\|u+v\|_{2(\rho+2)}^{2(\rho+2)} + 2\|uv\|_{\rho+2}^{\rho+2} \right].$$

From (2.1) and (3.5), we obtain

$$E_2 - \frac{1}{2}\left(\|u_t\|_2^2 + \|v_t\|_2^2\right) - \frac{J(t)}{2} < E_2 - \frac{1}{2}\alpha_2^2 < E_2 - \frac{1}{2}\alpha_1^2 <$$

$$< E_1 - \frac{1}{2}\alpha_1^2 = -\frac{1}{2(\rho+2)}\alpha_1^2 < 0 \quad \forall t \geq 0.$$

Hence, by the above inequality and Lemma 2.2, we have for all $t \geq 0$

$$0 < H(0) \leq H(t) \leq \frac{1}{2(\rho+2)} \left[\|u+v\|_{2(\rho+2)}^{2(\rho+2)} + 2\|uv\|_{\rho+2}^{\rho+2} \right] \leq$$

$$\leq \frac{c_1}{2(\rho+2)} \left(\|u\|_{2(\rho+2)}^{2(\rho+2)} + \|v\|_{2(\rho+2)}^{2(\rho+2)} \right).$$

Then we define the functional

$$M(t) = \frac{1}{2} \int_{\Omega} \left(u^2 + v^2\right)(x,t)\, dx.$$

We introduce

$$L(t) = H^{1-\sigma}(t) + \varepsilon M'(t), \tag{3.7}$$

for ε small to be chosen later and

$$0 < \sigma \leq \min \left\{ \frac{1}{2},\ \frac{2\rho+3-(k+m)}{2(m+1)(\rho+2)},\ \frac{2\rho+3-(l+m)}{2(m+1)(\rho+2)}, \right.$$

$$\left. \frac{2\rho+3-(\varrho+r)}{2(r+1)(\rho+2)}, \frac{2\rho+3-(\theta+r)}{2(r+1)(\rho+2)}, \frac{2\rho+2}{4(\rho+2)} \right\}. \tag{3.8}$$

We will show that $L(t)$ satisfies a differential inequality in Lemma 2.4. By taking a derivative of (3.7) and using (1.1), we obtain

$$L'(t) = (1-\sigma) H^{-\sigma}(t) H'(t) + \varepsilon\left(\|u_t\|_2^2 + \|v_t\|_2^2\right) - \varepsilon\left(\|\nabla u\|_2^2 + \|\nabla v\|_2^2\right) -$$

$$-\varepsilon \int_\Omega u\left(|u(t)|^k + |v(t)|^l\right)|u_t|^{m-1} u_t dx - \varepsilon \int_\Omega v\left(|v(t)|^\theta + |u(t)|^\varrho\right)|v_t|^{r-1} v_t dx +$$

$$+\varepsilon \int_\Omega \left(u f_1(u,\ v) + v f_2(u,\ v)\right) dx + \varepsilon \int_\Omega \nabla u(t) \int_0^t g(t-s) \nabla u(\tau)\, dxds +$$

$$+\varepsilon \int_\Omega \nabla v(t) \int_0^t h(t-s) \nabla v(\tau)\, dxds.$$

Then

$$L'(t) = (1-\sigma) H^{-\sigma}(t) H'(t) + \varepsilon\left(\|u_t\|_2^2 + \|v_t\|_2^2\right) - \varepsilon\left(\|\nabla u\|_2^2 + \|\nabla v\|_2^2\right) -$$

$$-\varepsilon \int_\Omega u\left(|u(t)|^k + |v(t)|^l\right)|u_t|^{m-1} u_t dx - \varepsilon \int_\Omega v\left(|v(t)|^\theta + |u(t)|^\varrho\right)|v_t|^{r-1} v_t dx +$$

$$+\varepsilon\left(\|u+v\|_{2(\rho+2)}^{2(\rho+2)} + 2\|uv\|_{\rho+2}^{\rho+2}\right) + \varepsilon\left(\int_0^t g(s)\, ds\right)\|\nabla u\|_2^2 + \left(\int_0^t h(s)\, ds\right)\|\nabla v\|_2^2 +$$

$$+\varepsilon \int_0^t g(t-s) \int_\Omega \nabla u(t) . \left[\nabla u(\tau) - \nabla u(t)\right] dxds +$$

$$+\varepsilon \int_0^t h(t-s) \int_\Omega \nabla v(t) . \left[\nabla v(\tau) - \nabla v(t)\right] dxds.$$

By Cauchy – Schwarz and Young's inequalities, we estimate

$$\int_0^t g(t-s) \int_\Omega \nabla u(t) . \left[\nabla u(\tau) - \nabla u(t)\right] dxds \leq$$

$$\leq \int_0^t g(t-s) \|\nabla u\|_2 \left\|\nabla u(\tau) - \nabla u(t)\right\|_2 d\tau \leq$$

$$\leq \lambda (g \circ \nabla u) + \frac{1}{4\lambda}\left(\int_0^t g(s)\, ds\right)\|\nabla u\|_2^2, \quad \lambda > 0,$$

and

$$\int\limits_0^t h(t-s)\int\limits_\Omega \nabla v(t).\left[\nabla v(\tau)-\nabla v(t)\right]dxds\leq$$

$$\leq \lambda(h\circ\nabla v)+\frac{1}{4\lambda}\left(\int\limits_0^t h(s)\,ds\right)\|\nabla v\|_2^2,\quad \lambda>0.$$

Adding and substituting $pE(t)$, using the definition of $H(t)$, E_2 lead to

$$L'(t)\geq(1-\sigma)H^{-\sigma}(t)H'(t)+\varepsilon\left(1+\frac{p}{2}\right)\left(\|u_t\|_2^2+\|v_t\|_2^2\right)+$$

$$+\varepsilon\left(\frac{p}{2}-\lambda\right)\left[(g\circ\nabla u)+(h\circ\nabla v)\right]+p\varepsilon H(t)-p\varepsilon E_2-$$

$$-\varepsilon\int\limits_\Omega u\left(|u(t)|^k+|v(t)|^l\right)|u_t|^{m-1}u_tdx-\varepsilon\int\limits_\Omega v\left(|v(t)|^\theta+|u(t)|^\varrho\right)|v_t|^{r-1}v_tdx+$$

$$+\varepsilon\left(1-\frac{p}{2(\rho+2)}\right)\left(\|u+v\|_{2(\rho+2)}^{2(\rho+2)}+2\|uv\|_{\rho+2}^{\rho+2}\right)+$$

$$+\varepsilon\left[\left(\frac{p}{2}-1\right)-\left(\frac{p}{2}-1+\frac{1}{4\lambda}\right)\int\limits_0^\infty g(s)\,ds\right]\|\nabla u\|_2^2+$$

$$+\varepsilon\left[\left(\frac{p}{2}-1\right)-\left(\frac{p}{2}-1+\frac{1}{4\lambda}\right)\int\limits_0^\infty h(s)\,ds\right]\|\nabla v\|_2^2, \tag{3.9}$$

for some λ such that

$$a_1=\frac{p}{2}-\lambda>0,\quad a_2=\left[\left(\frac{p}{2}-1\right)-\left(\frac{p}{2}-1+\frac{1}{4\lambda}\right)\max\left(\int\limits_0^\infty g(s)\,ds,\int\limits_0^\infty h(s)\,ds\right)\right]>0.$$

Then, estimate (3.9) becomes

$$L'(t)\geq(1-\sigma)H^{-\sigma}(t)H'(t)+\varepsilon\left(1+\frac{p}{2}\right)\left(\|u_t\|_2^2+\|v_t\|_2^2\right)+$$

$$+\varepsilon a_1\left[(g\circ\nabla u)+(h\circ\nabla v)\right]+p\varepsilon H(t)-p\varepsilon E_2-$$

$$-\varepsilon\int\limits_\Omega u\left(|u(t)|^k+|v(t)|^l\right)|u_t|^{m-1}u_tdx-$$

$$-\varepsilon\int\limits_\Omega v\left(|v(t)|^\theta+|u(t)|^\varrho\right)|v_t|^{r-1}v_tdx+$$

$$+\varepsilon\left(1-\frac{p}{2\left(\rho+2\right)}\right)\left(\|u+v\|_{2(\rho+2)}^{2(\rho+2)}+2\|uv\|_{\rho+2}^{\rho+2}\right)+\varepsilon a_2\left(\|\nabla u\|_2^2+\|\nabla v\|_2^2\right). \qquad (3.10)$$

By taking $c_3 = 1-\dfrac{p}{\rho+2}-2E_2\left(B\alpha_2\right)^{-2(\rho+2)}>0$, since $\alpha_2 > B^{-\frac{2(\rho+2)}{\rho+1}}$. Therefore, (3.10) takes the form

$$L'\left(t\right)\geq\left(1-\sigma\right)H^{-\sigma}\left(t\right)H'\left(t\right)+\varepsilon\left(1+\frac{p}{2}\right)\left(\|u_t\|_2^2+\|v_t\|_2^2\right)+$$

$$+\varepsilon a_1\left[\left(g\circ\nabla u\right)+\left(h\circ\nabla v\right)\right]+\varepsilon a_2\left(\|\nabla u\|_2^2+\|\nabla v\|_2^2\right)+p\varepsilon H\left(t\right)+$$

$$+\varepsilon c_3\left(\|u+v\|_{2(\rho+2)}^{2(\rho+2)}+2\|uv\|_{\rho+2}^{\rho+2}\right)-\varepsilon\int\limits_{\Omega}u\left(\left|u\left(t\right)\right|^k+\left|v\left(t\right)\right|^l\right)\left|u_t\right|^{m-1}u_t dx-$$

$$-\varepsilon\int\limits_{\Omega}v\left(\left|v\left(t\right)\right|^{\theta}+\left|u\left(t\right)\right|^{\varrho}\right)\left|v_t\right|^{r-1}v_t dx.$$

We use the Young's inequality as follows:

$$XY\leq\frac{\delta^{\alpha}X^{\alpha}}{\alpha}+\frac{\delta^{-\beta}Y^{\beta}}{\beta},$$

where $X,Y\geq 0$, $\delta>0$, and $\alpha,\beta\in\mathbb{R}^+$ such that $\dfrac{1}{\alpha}+\dfrac{1}{\beta}=1$, we get for all $\delta_1>0$

$$\left|u\left|u_t\right|^{m-1}u_t\right|\leq\frac{\delta_1^{m+1}}{m+1}\left|u\right|^{m+1}+\frac{m}{m+1}\delta_1^{-(m+1)/m}\left|u_t\right|^{m+1}$$

and

$$\int\limits_{\Omega}\left(\left|u\left(t\right)\right|^k+\left|v\left(t\right)\right|^l\right)\left|u\left|u_t\right|^{m-1}u_t\right|dx\leq\frac{\delta_1^{m+1}}{m+1}\int\limits_{\Omega}\left(\left|u\left(t\right)\right|^k+\left|v\left(t\right)\right|^l\right)\left|u\right|^{m+1}dx+$$

$$+\frac{m}{m+1}\delta_1^{-(m+1)/m}\int\limits_{\Omega}\left(\left|u\left(t\right)\right|^k+\left|v\left(t\right)\right|^l\right)\left|u_t\right|^{m+1}dx.$$

Similarly, for all $\delta_2>0$

$$\left|v\left|v_t\right|^{r-1}v_t\right|\leq\frac{\delta_2^{r+1}}{r+1}\left|v\right|^{r+1}+\frac{r}{r+1}\delta_2^{-(r+1)/r}\left|v_t\right|^{r+1},$$

which gives

$$\int\limits_{\Omega}\left(\left|v\left(t\right)\right|^{\theta}+\left|u\left(t\right)\right|^{\varrho}\right)\left|v\left|v_t\right|^{r-1}v_t\right|dx\leq\frac{\delta_2^{r+1}}{r+1}\int\limits_{\Omega}\left(\left|v\left(t\right)\right|^{\theta}+\left|u\left(t\right)\right|^{\varrho}\right)\left|v\right|^{r+1}dx+$$

$$+\frac{r}{r+1}\delta_2^{-(r+1)/r}\int\limits_{\Omega}\left(\left|v\left(t\right)\right|^{\theta}+\left|u\left(t\right)\right|^{\varrho}\right)\left|v_t\right|^{r+1}dx.$$

Then

$$L'(t) \geq (1-\sigma) H^{-\sigma}(t) H'(t) + \varepsilon \left(1+\frac{p}{2}\right)\left(\|u_t\|_2^2 + \|v_t\|_2^2\right) +$$

$$+\varepsilon a_1 \left[(g \circ \nabla u) + (h \circ \nabla v)\right] + \varepsilon a_2 \left(\|\nabla u\|_2^2 + \|\nabla v\|_2^2\right) + p\varepsilon H(t) +$$

$$+\varepsilon c_3 \left(\|u+v\|_{2(\rho+2)}^{2(\rho+2)} + 2\|uv\|_{\rho+2}^{\rho+2}\right) - \varepsilon \frac{\delta_1^{m+1}}{m+1} \int\limits_\Omega \left(|u(t)|^k + |v(t)|^l\right) |u|^{m+1} dx -$$

$$-\varepsilon \frac{m}{m+1} \delta_1^{-(m+1)/m} \int\limits_\Omega \left(|u(t)|^k + |v(t)|^l\right) |u_t|^{m+1} dx -$$

$$-\varepsilon \frac{\delta_2^{r+1}}{r+1} \int\limits_\Omega \left(|v(t)|^\theta + |u(t)|^\varrho\right) |v|^{r+1} dx -$$

$$-\varepsilon \frac{r}{r+1} \delta_2^{-(r+1)/r} \int\limits_\Omega \left(|v(t)|^\theta + |u(t)|^\varrho\right) |v_t|^{r+1} dx.$$

Let us choose δ_1 and δ_2 such that

$$\delta_1^{-(m+1)/m} = M_1 H(t)^{-\sigma}, \quad \delta_2^{-(r+1)/r} = M_2 H(t)^{-\sigma}, \tag{3.11}$$

for M_1 and M_2 large constants to be fixed later. Thus, by using (3.11), we get

$$L'(t) \geq ((1-\sigma) - M\varepsilon) H^{-\sigma}(t) H'(t) + \varepsilon \left(1+\frac{p}{2}\right)\left(\|u_t\|_2^2 + \|v_t\|_2^2\right) +$$

$$+\varepsilon a_1 \left[(g \circ \nabla u) + (h \circ \nabla v)\right] + \varepsilon a_2 \left(\|\nabla u\|_2^2 + \|\nabla v\|_2^2\right) + p\varepsilon H(t) +$$

$$+\varepsilon c_3 \left(\|u+v\|_{2(\rho+2)}^{2(\rho+2)} + 2\|uv\|_{\rho+2}^{\rho+2}\right) -$$

$$-\varepsilon M_1^{-m} H^{\sigma m}(t) \int\limits_\Omega \left(|u(t)|^k + |v(t)|^l\right) |u|^{m+1} dx -$$

$$-\varepsilon \frac{m}{m+1} \delta_1^{-(m+1)/m} \int\limits_\Omega \left(|u(t)|^k + |v(t)|^l\right) |u_t|^{m+1} dx -$$

$$-\varepsilon M_2^{-r} H^{\sigma r}(t) \int\limits_\Omega \left(|v(t)|^\theta + |u(t)|^\varrho\right) |v|^{r+1} dx -$$

$$-\varepsilon \frac{r}{r+1} \delta_2^{-(r+1)/r} \int\limits_\Omega \left(|v(t)|^\theta + |u(t)|^\varrho\right) |v_t|^{r+1} dx, \tag{3.12}$$

where $M = m/(m+1)M_1 + r/(r+1)M_2$. Consequently we have

$$\int_{\Omega}\left(|u(t)|^{k}+|v(t)|^{l}\right)|u|^{m+1}\,dx=\|u\|_{k+m+1}^{k+m+1}+\int_{\Omega}|v|^{l}\,|u|^{m+1}\,dx$$

and

$$\int_{\Omega}\left(|v(t)|^{\theta}+|u(t)|^{\varrho}\right)|v|^{r+1}\,dx=\|v\|_{\theta+r+1}^{\theta+r+1}+\int_{\Omega}|u|^{\varrho}\,|v|^{r+1}\,dx.$$

Also by using Young's inequality, we obtain

$$\int_{\Omega}|v|^{l}\,|u|^{m+1}\le\frac{l}{l+m+1}\delta_{1}^{(l+m+1)/l}\,\|v\|_{l+m+1}^{l+m+1}+\frac{m+1}{l+m+1}\delta_{1}^{-(l+m+1)/(m+1)}\,\|u\|_{l+m+1}^{l+m+1},$$

$$\int_{\Omega}|u|^{\varrho}\,|v|^{r+1}\le\frac{\varrho}{\varrho+r+1}\delta_{2}^{(\varrho+r+1)/\varrho}\,\|u\|_{\varrho+r+1}^{\varrho+r+1}+\frac{r+1}{\varrho+r+1}\delta_{2}^{-(\varrho+r+1)/(r+1)}\,\|v\|_{\varrho+r+1}^{\varrho+r+1}.$$

Consequently

$$H^{\sigma m}(t)\int_{\Omega}\left(|u(t)|^{k}+|v(t)|^{l}\right)|u|^{m+1}\,dx=$$

$$=H^{\sigma m}(t)\,\|u\|_{k+m+1}^{k+m+1}+\frac{l}{l+m+1}\delta_{1}^{(l+m+1)/l}H^{\sigma m}(t)\,\|v\|_{l+m+1}^{l+m+1}+$$

$$+\frac{m+1}{l+m+1}\delta_{1}^{-(l+m+1)/(m+1)}H^{\sigma m}(t)\,\|u\|_{l+m+1}^{l+m+1} \tag{3.13}$$

and

$$H^{\sigma r}(t)\int_{\Omega}\left(|v(t)|^{\theta}+|u(t)|^{\varrho}\right)|v|^{r+1}\,dx=$$

$$=H^{\sigma r}(t)\,\|v\|_{\theta+r+1}^{\theta+r+1}+\frac{\varrho}{\varrho+r+1}\delta_{2}^{\frac{\varrho+r+1}{\varrho}}H^{\sigma r}(t)\,\|u\|_{\varrho+r+1}^{\varrho+r+1}+$$

$$+\frac{r+1}{\varrho+r+1}\delta_{2}^{-\frac{(\varrho+r+1)}{r+1}}H^{\sigma r}(t)\,\|v\|_{\varrho+r+1}^{\varrho+r+1}. \tag{3.14}$$

Since (3.2) holds, we obtain by using (3.8)

$$\begin{gathered}H^{\sigma m}(t)\,\|u\|_{k+m+1}^{k+m+1}\le c_{5}\left(\|u\|_{2(\rho+2)}^{2\sigma m(\rho+2)+k+m+1}+\|v\|_{2(\rho+2)}^{2\sigma m(\rho+2)}\,\|u\|_{k+m+1}^{k+m+1}\right),\\ H^{\sigma r}(t)\,\|v\|_{\theta+r+1}^{\theta+r+1}\le c_{6}\left(\|v\|_{2(\rho+2)}^{2\sigma r(\rho+2)+\theta+r+1}+\|u\|_{2(\rho+2)}^{2\sigma r(\rho+2)}\,\|v\|_{\theta+r+1}^{\theta+r+1}\right).\end{gathered} \tag{3.15}$$

This implies

$$\begin{gathered}\frac{l}{l+m+1}\delta_1^{\frac{l+m+1}{l}} H^{\sigma m}(t)\,\|v\|_{l+m+1}^{l+m+1} \leq \\ \leq c_7\frac{l}{l+m+1}\delta_1^{\frac{l+m+1}{l}}\left(\|v\|_{2(\rho+2)}^{2\sigma m(\rho+2)+l+m+1} + \|u\|_{2(\rho+2)}^{2\sigma m(\rho+2)}\|v\|_{l+m+1}^{l+m+1}\right), \\ \frac{\varrho}{\varrho+r+1}\delta_2^{\frac{\varrho+r+1}{\varrho}} H^{\sigma r}(t)\,\|u\|_{\varrho+r+1}^{\varrho+r+1} \leq \\ \leq c_8\frac{\varrho}{\varrho+r+1}\delta_2^{\frac{\varrho+r+1}{\varrho}}\left(\|u\|_{2(\rho+2)}^{2\sigma r(\rho+2)+\varrho+r+1} + \|v\|_{2(\rho+2)}^{2\sigma r(\rho+2)}\|u\|_{\varrho+r+1}^{\varrho+r+1}\right).\end{gathered} \tag{3.16}$$

By using (3.8) and the algebraic inequality

$$z^\nu \leq (z+1) \leq \left(1+\frac{1}{a}\right)(z+a) \qquad \forall z \geq 0, \quad 0<\nu\leq 1, \quad a\geq 0, \tag{3.17}$$

we have, for all $t \geq 0$,

$$\begin{gathered}\|u\|_{2(\rho+2)}^{2\sigma m(\rho+2)+k+m+1} \leq d\left(\|u\|_{2(\rho+2)}^{2(\rho+2)} + H(0)\right) \leq d\left(\|u\|_{2(\rho+2)}^{2(\rho+2)} + H(t)\right), \\ \|v\|_{2(\rho+2)}^{2\sigma r(\rho+2)+\theta+r+1} \leq d\left(\|v\|_{2(\rho+2)}^{2(\rho+2)} + H(t)\right) \quad \forall t\geq 0,\end{gathered} \tag{3.18}$$

where $d = 1 + 1/H(0)$. Similarly

$$\begin{gathered}\|v\|_{2(\rho+2)}^{2\sigma m(\rho+2)+l(m+1)} \leq d\left(\|v\|_{2(\rho+2)}^{2(\rho+2)} + H(0)\right) \leq d\left(\|v\|_{2(\rho+2)}^{2(\rho+2)} + H(t)\right), \\ \|u\|_{2(\rho+2)}^{2\sigma r(\rho+2)+\varrho(r+1)} \leq d\left(\|u\|_{2(\rho+2)}^{2(\rho+2)} + H(t)\right) \quad \forall t\geq 0.\end{gathered} \tag{3.19}$$

Also, since

$$(X+Y)^s \leq C\left(X^s+Y^s\right), \qquad X,Y\geq 0, \quad s>0, \tag{3.20}$$

by using (3.8) and (3.17) we conclude

$$\|v\|_{2(\rho+2)}^{2\sigma m(\rho+2)}\|u\|_{k+m+1}^{k+m+1} \leq c_9\left(\|v\|_{2(\rho+2)}^{2(\rho+2)} + \|u\|_{k+m+1}^{2(\rho+2)}\right) \leq c_{10}\left(\|v\|_{2(\rho+2)}^{2(\rho+2)} + \|u\|_{2(\rho+2)}^{2(\rho+2)}\right), \tag{3.21}$$

similarly

$$\|u\|_{2(\rho+2)}^{2\sigma r(\rho+2)}\|v\|_{\theta+r+1}^{\theta+r+1} \leq c_{11}\left(\|u\|_{2(\rho+2)}^{2(\rho+2)} + \|v\|_{2(\rho+2)}^{2(\rho+2)}\right), \tag{3.22}$$

$$\|u\|_{2(\rho+2)}^{2\sigma m(\rho+2)}\|v\|_{l+m+1}^{l+m+1} \leq c_{12}\left(\|u\|_{2(\rho+2)}^{2(\rho+2)} + \|v\|_{2(\rho+2)}^{2(\rho+2)}\right), \tag{3.23}$$

and

$$\|v\|_{2(\rho+2)}^{2\sigma r(\rho+2)}\|u\|_{\varrho+r+1}^{\varrho+r+1} \leq c_{13}\left(\|v\|_{2(\rho+2)}^{2(\rho+2)} + \|u\|_{2(\rho+2)}^{2(\rho+2)}\right). \tag{3.24}$$

Taking into account (3.13)–(3.24), then (3.12) takes the form

$$L'(t) \geq ((1-\sigma) - M\varepsilon) H^{-\sigma}(t) H'(t) + 2\varepsilon \left(\|u_t\|_2^2 + \|v_t\|_2^2\right) +$$

$$+\varepsilon \left[2 - CM_1^{-m}\left(1 + \frac{l}{l+m+1}\delta_1^{\frac{l+m+1}{l}} + \frac{m+1}{l+m+1}\delta_1^{-\frac{(l+m+1)}{m+1}}\right) - \right.$$

$$\left. - CM_2^{-r}\left(1 + \frac{\varrho}{\varrho+r+1}\delta_2^{\frac{\varrho+r+1}{\varrho}} + \frac{r+1}{\varrho+r+1}\delta_2^{-\frac{(\varrho+r+1)}{r+1}}\right)\right] H(t) +$$

$$+\varepsilon \left[c_4 - CM_1^{-m}\left(1 + \frac{l}{l+m+1}\delta_1^{\frac{l+m+1}{l}} + \frac{m+1}{l+m+1}\delta_1^{-\frac{(l+m+1)}{m+1}}\right) - \right.$$

$$\left. - CM_2^{-r}\left(1 + \frac{\varrho}{\varrho+r+1}\delta_2^{\frac{\varrho+r+1}{\varrho}} + \frac{r+1}{\varrho+r+1}\delta_2^{-\frac{(\varrho+r+1)}{r+1}}\right)\right] \left(\|u\|_{2(\rho+2)}^{2(\rho+2)} + \|v\|_{2(\rho+2)}^{2(\rho+2)}\right). \quad (3.25)$$

At this point, and for large values of M_1 and M_2, we can find positive constants Λ_1 and Λ_2 such that (3.25) becomes

$$L'(t) \geq ((1-\sigma) - M\varepsilon) H^{-\sigma}(t) H'(t) + 2\varepsilon \left(\|u_t\|_2^2 + \|v_t\|_2^2\right) +$$

$$+\varepsilon\Lambda_1 \left(\|u(t)\|_{2(\rho+2)}^{2(\rho+2)} + \|v(t)\|_{2(\rho+2)}^{2(\rho+2)}\right) + \varepsilon\Lambda_2 H(t). \quad (3.26)$$

Once M_1 and M_2 are fixed (hence, Λ_1 and Λ_2), we pick ε small enough so that $((1-\sigma) - M\varepsilon) \geq 0$ and

$$L(0) = H^{1-\sigma}(0) + \int\limits_\Omega [u_0.u_t + v_0.v_t]\, dx > 0.$$

Consequently, there exists $\Gamma > 0$ such that (3.26) becomes

$$L'(t) \geq \varepsilon\Gamma \left(H(t) + \|u_t\|_2^2 + \|v_t\|_2^2 + \|u\|_{2(\rho+2)}^{2(\rho+2)} + \|v\|_{2(\rho+2)}^{2(\rho+2)}\right). \quad (3.27)$$

Thus, we have $L(t) \geq L(0) > 0$, for all $t \geq 0$.

Next, by Holder's and Young's inequalities, we estimate

$$\left(\int\limits_\Omega u.u_t(x,t)\, dx + \int\limits_\Omega v.v_t(x,t)\, dx\right)^{1/(1-\sigma)} \leq$$

$$\leq C\left(\|u\|_{2(\rho+2)}^{\tau/(1-\sigma)} + \|u_t\|_2^{s/(1-\sigma)} + \|v\|_{2(\rho+2)}^{\tau/(1-\sigma)} + \|v_t\|_2^{s/(1-\sigma)}\right), \quad (3.28)$$

for $\dfrac{1}{\tau} + \dfrac{1}{s} = 1$. We takes $s = 2(1-\sigma)$, to get $\dfrac{\tau}{1-\sigma} = \dfrac{2}{1-2\sigma}$. By using (3.6) and (3.17) we get

$$\|u\|_{2(\rho+2)}^{2/(1-2\sigma)} \leq d\left(\|u\|_{2(\rho+2)}^{2(\rho+2)} + H(t)\right)$$

and

$$\|v\|_{2(\rho+2)}^{2/(1-2\sigma)} \leq d\left(\|v\|_{2(\rho+2)}^{2(\rho+2)} + H(t)\right) \quad \forall t \geq 0.$$

Therefore, (3.28) becomes

$$\left(\int_{\Omega} uu_t(x,t)\,dx + \int_{\Omega} vv_t(x,t)\,dx\right)^{1/(1-\sigma)} \leq$$

$$\leq c_{14}\left(\|u\|_{2(\rho+2)}^{2(\rho+2)} + \|v\|_{2(\rho+2)}^{2(\rho+2)} + \|u_t\|_2^2 + \|v_t\|_2^2 + H(t)\right) \quad \forall t \geq 0.$$

Also, by noting that

$$L^{1/(1-\sigma)}(t) = \left(H^{1-\sigma}(t) + \varepsilon\int_{\Omega}(u.u_t + v.v_t)(x,t)\,dx\right)^{1/(1-\sigma)} \leq$$

$$\leq c_{15}\left(H(t) + \left|\int_{\Omega}(u.u_t(x,t) + v.v_t(x,t))\,dx\right|^{1/(1-\sigma)}\right) \leq$$

$$\leq c_{16}\left[H(t) + \|u\|_{2(\rho+2)}^{2(\rho+2)} + \|v\|_{2(\rho+2)}^{2(\rho+2)} + \|u_t\|_2^2 + \|v_t\|_2^2\right] \quad \forall t \geq 0, \tag{3.29}$$

and combining with (3.29) and (3.27), we arrive at

$$L'(t) \geq a_0 L^{1/(1-\sigma)}(t) \quad \forall t \geq 0. \tag{3.30}$$

Finally, a simple integration of (3.30) gives the desired result.

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