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ALMOST MGP-INJECTIVE RINGS

МАЙЖЕ MGP-ІН'ЄКТИВНІ КІЛЬЦЯ

A ring R is called right almost MGP-injective (or AMGP-injective for short) if, for any $0 \neq a \in R$, there exists an element $b \in R$ such that $ab = ba \neq 0$ and any right R-monomorphism from abR to R extends to an endomorphism of R. In this paper, several properties of these rings are given, some interesting results are obtained. Using the concept of right AMGP-injective rings, we present some new characterizations of QF-rings, semisimple Artinian rings and simple Artinian rings.

Кільце R називається правим майже MGP-ін'єктивним кільцем (або правим AMGP-ін'єктивним кільцем), якщо для всіх $0 \neq a \in R$ існує елемент $b \in R$ такий, що $ab = ba \neq 0$ і будь-який правий R-мономорфізм з abR в R продовжується до ендоморфізму в R. В роботі наведено деякі властивості таких кілець та отримано деякі цікаві результати. З використанням поняття AMGP-ін'єктивних кілець наведено деякі нові характеристики QF-кілець, напівпростих артінових кілець та простих артінових кілець.

1. Introduction. Throughout this paper, R is an associative ring with identity, and all modules are unitary. As usual, $J = J(R)$, Z_l (Z_r) and S_l (S_r) denote respectively the Jacobson radical, the left (right) singular ideal and the left (right) socle of R. The left (respectively, right) annihilators of a subset X of R is denoted by $\mathbf{l}(X)$ $\big($respectively, $\mathbf{r}(X)\big)$.

Recall that a ring R is *right P-injective* [1] if every R-homomorphism from a principal right ideal of R to R extends to an endomorphism of R. A ring R is *right generalized principally injective* (briefly *right GP-injective*) [2] if, for any $0 \neq a \in R$, there exists a positive integer n such that $a^n \neq 0$ and any right R-homomorphism from a^nR to R extends to an endomorphism of R. GP-injective rings are studied in papers [2–6]. In [6], GP-injective rings are called *YJ-injective* rings. It is easy to see that right P-injective rings are right GP-injective, but right GP-injective rings need not be right P-injective by [5] (Example 1).

In [7], the concepts of right P-injective rings and right GP-injective rings are generalized to *right MP-injective* rings and *right MGP-injective* rings, respectively. Following [7], a ring R is called right *MP-injective* if, for every R-monomorphism from a principal right ideal of R to R extends to an endomorphism of R; a ring R is called right MGP-injective if, for any $0 \neq a \in R$, there exists a positive integer n such that $a^n \neq 0$ and any R-monomorphism from a^nR to R extends to a endomorphism of R. In this paper, we shall generalize the concept of right MGP-injective rings to *right AMGP-injective rings*, some properties of these rings will be given, conditions under which right AMGP-injective rings are QF-rings, semisimple Artinian rings and simple Artinian rings will be given, respectively. And right AMGP-injective left Noertherian rings will be investigated.

2. AMGP-injective rings.

Definition 2.1. *A ring R is called right almost MGP-injective* (*or AMGP-injective for short*) *if, for any $0 \neq a \in R$, there exists an element $b \in R$ such that $ab = ba \neq 0$ and any right R-monomorphism from abR to R extends to an endomorphism of R.*

Theorem 2.1. *For a ring R, the following conditions are equivalent*:

(1) *R is right AMGP-injective*;

(2) *for any $0 \neq a \in R$, there exists $b \in R$ such that $ab = ba \neq 0$ and $c \in Rab$ for every $c \in R$ with $\mathbf{r}(ab) = \mathbf{r}(c)$.*

Proof. (1) $\Rightarrow$ (2). Let $0 \neq a \in R$. Since R is right AMGP-injective, there exists an element $b \in R$ such that $ab = ba \neq 0$ and every monomorphism from abR to R extends to R. Suppose that $\mathbf{r}(ab) = \mathbf{r}(c)$. Then $f\colon abR \to R; abr \mapsto cr$, is a monomorphism, which extends to an endomorphism g of R. So $c = f(ab) = g(ab) = g(1)ab \in Rab$.

(2) $\Rightarrow$ (1). Let $0 \neq a \in R$. By (2), there exists $b \in R$ such that $ab = ba \neq 0$ and $c \in Rab$ for every $c \in R$ with $\mathbf{r}(ab) = \mathbf{r}(c)$. Let $f\colon abR \to R$ be monic. Then $\mathbf{r}(ab) = \mathbf{r}\big(f(ab)\big)$, and so $f(ab) = cab$ for some $c \in R$. It follows that $f = c\cdot$, as required.

Theorem 2.1 is proved.

It is obvious that right MGP-injective rings are AMGP-injective. Our next example shows that a right AMGP-injective rings need not be right MGP-injective.

***Example* 2.1.** Let $M = \oplus_{i=1}^{\infty}\mathbb{Z}_{p_i}$, where p_i is the ith prime number, and let

$$R = \left\{ \begin{bmatrix} n & x \\ 0 & n \end{bmatrix} \middle| \, n \in \mathbb{Z},\ x \in M \right\}.$$

Then, by [7] (Example 3.3), R is not right MGP-injective. For any $0 \neq a = \begin{bmatrix} n & x \\ 0 & n \end{bmatrix} \in R$. If $n \neq 0$, then there exists $y \in M$ such that $ny \neq 0$. Now let $b = \begin{bmatrix} 0 & y \\ 0 & 0 \end{bmatrix}$, then $0 \neq ab = ba = \begin{bmatrix} 0 & ny \\ 0 & 0 \end{bmatrix} \in J(R)$. If $n = 0$, then $a \in J(R)$. Thus, by the proof of [8] (Example 3.1), for any $0 \neq a \in R$, there is $a, b \in R$, such that $ba = ab \neq 0$ and $\mathbf{lr}(ba) = R(ba)$, and so R is right AMGP-injective by Theorem 2.1.

Recall that a ring R is called *right mininjective* [9] if every R-homomorphism from a minimal right ideal of R into R extends to R.

Theorem 2.2. *Let R be right AMGP-injective. Then*:

(1) *R is right mininjective*;

(2) $J(R) \subseteq Z_r$.

Proof. (1). It is obvious.

(2). Let $a \in J(R)$, then we will show that $a \in Z_r$. If not, then there exists $0 \neq b \in R$ such that $\mathbf{r}(a) \cap bR = 0$. Clearly $ab \neq 0$. Since R is right AMGP-injective, there exists $c \in R$ such that $abc \neq 0$ and $u \in Rabc$ for every $u \in R$ with $\mathbf{r}(abc) = \mathbf{r}(u)$. Since $\mathbf{r}(abc) = \mathbf{r}(bc)$, so $bc = dabc$ for some $d \in R$. Thus $(1 - da)bc = 0$. Since $a \in J(R)$, $1 - da$ is invertible, and so $bc = 0$. Hence $abc = 0$, a contradiction.

Theorem 2.2 is proved.

We note that the ring $\mathbb{Z}$ of integers is right mininjective but not right AMGP-injective; so right mininjective rings need not be right AMGP-injective.

***Corollary* 2.1.** *Let R be a right AMGP-injective ring. Suppose that, for any sequence $\{a_1, a_2, \ldots\} \subseteq R$, the chain $\mathbf{r}(a_1) \subseteq \mathbf{r}(a_2a_1) \subseteq \ldots$ terminates. Then $J(R) = Z_r$.*

Proof. Since R is right AMGP-injective, by Theorem 2.2, $J(R) \subseteq Z_r$. Since the chain $\mathbf{r}(a_1) \subseteq \mathbf{r}(a_2a_1) \subseteq \ldots$ terminates for any sequence $\{a_1, a_2, \ldots\} \subseteq R$, by [7] (Lemma 3.10), Z_r is right T-nilpotent, and so Z_r is nil. It follows that $Z_r \subseteq J(R)$, and hence $J(R) = Z_r$.

Lemma 2.1. *Let R be right AMGP-injective. If $a \notin Z_r$, then the inclusion $\mathbf{r}(a) \subset \mathbf{r}(a - aca)$ is strict for some $c \in R$.*

Proof. Since $\mathbf{r}(a)$ is not an essential right ideal, there exists a nonzero right ideal I of R such that $\mathbf{r}(a) \oplus I$ is essential in R_R. Take $0 \neq b \in I$, then $ab \neq 0$. By the right AMGP-injectivity, there is an element c_1 in R such that $abc_1 \neq 0$ and any right R-monomorphism from abc_1R to R extends to an endomorphism of R. Observing that $bR \cap \mathbf{r}(a) = 0$, we have a right R-monomorphism $g\colon abc_1R \to R$ given by $g(abc_1r) = bc_1r$. Thus $bc_1 = cabc_1$ for some $c \in R$, and so $bc_1 \in \mathbf{r}(1 - ca)$, whence $bc_1 \in \mathbf{r}(a - aca)$. Note that $bc_1 \notin \mathbf{r}(a)$, thus we have that the inclusion $\mathbf{r}(a) \subset \mathbf{r}(a - aca)$ is strict.

Lemma 2.1 is proved.

Theorem 2.3. *If R is right AMGP-injective, then the following statements are equivalent:*

(1) *R is right perfect;*

(2) *the ascending chain $\mathbf{r}(a_1) \subseteq \mathbf{r}(a_2a_1) \subseteq \mathbf{r}(a_3a_2a_1) \subseteq \ldots$ terminates for every infinite sequence $a_1, a_2, a_3, \ldots$ of R.*

Proof. By Corollary 2.1, Lemma 2.1 and [7] (Lemma 2.10), we can complete the proof in a similar way to that of [7] (Theorem 2.11).

Recall that a ring R is called right $Kasch$ [10] if every simple right R-module embeds in R, equivalently if $\mathbf{l}(T) \neq 0$ for every maximal right ideal T of R. Left Kasch rings can be defined similarly; a ring R is called *right minfull* [9] if it is semiperfect, right mininjective, and $\mathrm{Soc}(eR) \neq 0$ for each local idempotent $e \in R$.

***Corollary* 2.2.** *If R is a right AMGP-injective ring with ACC on right annihilators, then:*

(1) *R is semiprimary;*

(2) *R is left and right Kasch.*

Proof. (1) It is well known that Z_r is nilpotent for any ring R with ACC on right annihilators. By Theorem 2.3 and Theorem 2.2(2), R is semiprimary.

(2). By (1), R is semiprimary, so R is semiperfect with essential right socle. Noting that R is right mininjective by Theorem 2.2(1), hence it is right minfull, and thus (2) follows from [10] (Theorem 3.12(1)).

***Corollary* 2.3.** *Let R be a right AMGP-injective ring. Then R is right Noetherian if and only if R is right Artinian.*

Proof. Let R be a right Noetherian right AMGP-injective ring. Then by Corollary 2.2, R is a right Noetherian semiprimary ring, and so R is right Artinian.

***Corollary* 2.4.** *Let R be a right AMGP-injective ring with ACC on right annihilators and $S_l \subseteq \subseteq S_r$. Then R is left Artinian if and only if S_l is a finitely generated left ideal.*

Proof. By Corollary 2.2, R is semiprimary. By Theorem 2.2 and [9] (Theorem 1.14(4)), $S_r \subseteq S_l$, and so $S_l = S_r$ by the hypothesis. Now the result follows from [11] (Lemma 6).

Recall that a ring R is called a *left minannihilator ring* [9], if every minimal left ideal K is a left annihilator, equivalently, if $\mathbf{lr}(K) = K$.

***Corollary* 2.5.** *Let R be a right AMGP-injective ring with ACC on right annihilators. If R is a left minannihilator ring, then:*

(1) *R is left Artinian;*

(2) *R is right Artinian if and only if S_r is finitely generated as a right ideal of R.*

Proof. (1). By Corollary 2.2, R is semiprimary. By [9] (Corollary 3.15), R is left finite dimensional with $S_l = S_r$. Now, by [11] (Lemma 6), R is left Artinian.

(2). The assertion follows from (1) and [11] (Lemma 6).

Definition 2.2. *A ring R is called right weakly P-injective (or right WP-injective for short) if, for any $0 \neq a \in R$, there exists $b \in R$, such that $ab = ba \neq 0$ and any right R-homomorphism from abR to R extends to an endomorphism of R.*

Theorem 2.4. *For a ring R, the following conditions are equivalent*:

(1) *R is right WP-injective*;

(2) *for any $0 \neq a \in R$, there exists $b \in R$ such that $ab = ba \neq 0$ and $\mathbf{lr}(ab) = Rab$.*

Proof. (1) $\Rightarrow$ (2). For any $0 \neq a \in R$, since R is right WP-injective, there exists an element $b \in R$, such that $ab = ba \neq 0$ and any R-homomorphism from abR to R extends to R. Now let $x \in \mathbf{lr}(ab)$, we define $f\colon abR \to R$ by $abr \mapsto xr$, then f is a well defined right R-homomorphism and hence f extends to an endomorphism g of R. Take $c = g(1)$, then $x = cab \in Rab$. This shows that $\mathbf{lr}(ab) = Rab$.

(2) $\Rightarrow$ (1). For any $0 \neq a \in R$, by (2), there exists $b \in R$ such that $ab = ba \neq 0$ and $\mathbf{lr}(ab) = Rab$. Suppose $f \in \mathrm{Hom}_R(abR, R)$, then $f(ab) \in \mathbf{lr}(ab)$, and so there exists $c \in R$ such that $f(ab) = cab$. Let $g\colon R \to R; x \mapsto cx$, then g extends f.

Theorem 2.4 is proved.

Clearly, right GP-injective rings are both right WP-injective and right MGP-injective, and right WP-injective rings are right AMGP-injective. It is easy to see that the ring in Example 2.1 is right WP-injective by Theorem 2.4, but it is not right MGP-injective by [7] (Example 3.3). Hence a right WP-injective rings need not be right GP-injective. By Theorem 2.4, we see that if R is a right WP-injective ring, then it is a left minannihilator ring, so by Corollary 2.5, we have the following corollary.

***Corollary* 2.6.** *Let R be a right WP-injective ring with ACC on right annihilators. Then*:

(1) *R is left Artinian*;

(2) *R is right Artinian if and only if S_r is finitely generated as a right ideal of R.*

Recall that a ring R is QF if it is right or left self-injective and right or left Artinian; a ring R is *semiregular* if $R/J(R)$ is von Neumann regular and idempotents can be lifted modulo $J(R)$; a ring R is right *CF* if every cyclic right R-module embeds in a free module; a ring R is called *right* (*left*) $\min -CS$ if every minimal right (left) ideal of R is essential in a direct summand of R_R (${}_RR$); a ring R is called *right* $\min -PF$ *ring* if R is a semiperfect, right mininjective ring in which $S_r \subseteq^{ess} R_R$ and $\mathbf{lr}(K) = K$ for every simple left ideal $K \subseteq Re$, where $e^2 = e$ is local. These concepts can be found in [10]. It is well known that right CF-rings are left P-injective [10] (Lemma 7.2 (1)); and a ring R is QF if and only if R is right Artinian and right and left mininjective [9] (Corollary 4.8). According to [12], a ring R is *right 2-simple injective* if every R-homomorphism from a 2-generated right ideal of R to R with simple image extends to an endomorphism of R.

Theorem 2.5. *Let R be a right AMGP-injective ring. Then the following are equivalent*:

(1) *R is a QF-ring*;

(2) *R is a left mininjective ring with ACC on right annihilators*;

(3) *R is right* $\min -CS$, *left minannihilator ring with ACC on right annihilators*;

(4) *R is a two-sided* $\min -CS$ *ring with ACC on right annihilators*;

(5) *R is a right* 2*-simple injective ring with ACC on right annihilators*;

(6) *R is right CF-ring and the ascending chain $\mathbf{r}(a_1) \subseteq \mathbf{r}(a_2a_1) \subseteq \mathbf{r}(a_3a_2a_1) \subseteq \ldots$ terminates for every sequence $\{a_1, a_2, \ldots\} \subseteq R$*;

(7) *R is a semiregular right CF-ring.*

Proof. It is obvious that (1) implies (2) through (5).

(2) $\Rightarrow$ (1). By Corollary 2.2(1), R is semiprimary, so it is a semilocal, left and right mininjective ring with ACC on right annihilators in which $S_r \subseteq^{ess} R_R$. By [10] (Theorem 3.31), R is a QF-ring.

(3) $\Rightarrow$ (1). Since R is a semiprimary left minannihilator ring, it is a right min-PF ring with $S_r = S_l$ by [10] (Corollary 3.25). Then R is a right minannihilator ring by [10] (Lemma 4.4) because

it is right min-CS. Hence R is left min-PF, again by [10] (Corollary 3.25). Now [10] (Theorem 3.38) shows that R is QF.

(4) $\Rightarrow$ (1). By Corollary 2.2(2), R is left and right Kasch, and hence $S_r = S_l$ by [10] (Lemma 4.5(2)) because R is left and right min-CS. Thus R is a left and right min-PF ring by [10] (Corollary 4.6), so R is QF, again by [10] (Theorem 3.38).

(5) $\Rightarrow$ (1). Suppose (5) holds. Then since R is a right AMGP-injective ring with ACC on right annihilators, by Corollary 2.2(1), R is semiprimary. Noting that R is right 2-simple injective, by [12] (Theorem 17(17)), R is a QF-ring.

(1) $\Rightarrow$ (6). Assume (1). Then since every injective module over a QF-ring is projective, so every right R-module embeds in a free module, and hence R is a right CF-ring. Note that a QF-ring is right Noetherian, the last assertion of (6) is clear.

(6) $\Rightarrow$ (7). By Theorem 2.3, R is right perfect, so that it is semiregular.

(7) $\Rightarrow$ (1). Note that the right AMGP-injectivity implies that $J(R) \subseteq Z_r$ by Theorem 2.2(2). Thus, R is right Artinian by [13] (Corollary 2.9). Since R is right and left mininjective, by [9] (Corollary 4.8), R is QF.

***Corollary* 2.7.** *Let R be a right WP-injective ring. Then R is a QF-ring if and only if R is a right* $\min -CS$ *ring with ACC on right annihilators.*

Theorem 2.6. *Let R be a left Noetherian right AMGP-injective ring. Then*:

(1) $\mathbf{r}(J) \subseteq^{ess} R_R$;

(2) *J is nilpotent*;

(3) $\mathbf{r}(J) \subseteq^{ess} {}_RR$.

Proof. Let $0 \neq x \in R$. Since R is left Noetherian, the nonempty set $\mathcal{F} = \{\mathbf{l}(xa) \mid a \in R$ such that $xa \neq 0\}$ has a maximal element, say $\mathbf{l}(xy)$.

We claim that $Jxy = 0$. If not, then there exists $t \in J$ such that $txy \neq 0$. Since R is right AMGP-injective, there exists a $z \in R$ such that $ztxy \neq 0$ and $b \in R(ztxy)$ for every $b \in R$ with $\mathbf{r}(ztxy) = \mathbf{r}(b)$. Write $ztxy = sxy$, where $s = zt \in J$. We proceed with the following two cases.

Case 1. $\mathbf{r}(xy) = \mathbf{r}(sxy)$. Then $xy = csxy$, i. e., $(1-cs)xy = 0$. Since $s \in J$, $1-cs$ is invertible. So we have $xy = 0$. This is a contradiction.

Case 2. $\mathbf{r}(xy) \neq \mathbf{r}(sxy)$. Then there exists $u \in \mathbf{r}(sxy)$ but $u \notin \mathbf{r}(xy)$. Thus, $sxyu = 0$ and $xyu \neq 0$. This shows that $s \in \mathbf{l}(xyu)$ and $\mathbf{l}(xyu) \in \mathcal{F}$. Noting that $s \notin \mathbf{l}(xy)$, so the inclusion $\mathbf{l}(xy) \subset \mathbf{l}(xyu)$ is strict. This contracts the maximality of $\mathbf{l}(xy)$ in $\mathcal{F}$.

Thus, $Jxy = 0$, and so $0 \neq xy \in xR \cap \mathbf{r}(J)$, proving (1).

(2). By (1) and [14] (Lemma 2.1).

(3). If $0 \neq c \in R$, we must show that $Rc \cap \mathbf{r}(J) \neq 0$. This is clear if $Jc = 0$. Otherwise, since J is nilpotent by (2), there exists $m \geq 1$ such that $J^m c \neq 0$ but $J^{m+1}c = 0$. Then $0 \neq J^m c \subseteq Rc \cap \mathbf{r}(J)$, as required.

Theorem 2.6 is proved.

Theorem 2.7. *Let R be a left Noetherian right AMGP-injective ring. Then the following statements are equivalent*:

(1) *R is right Kasch*;

(2) *R is left C_2*;

(3) *R is left GC_2*;

(4) *R is semilocal*;

(5) *R is left Artinian*;

(6) *the ascending chain* $\mathbf{r}(a_1) \subseteq \mathbf{r}(a_2a_1) \subseteq \mathbf{r}(a_3a_2a_1) \subseteq \ldots$ *terminates for every sequence* $\{a_1, a_2, \ldots\} \subseteq R$.

Proof. (1) $\Rightarrow$ (2). By [10] (Proposition 1.46).

(2) $\Rightarrow$ (3); and (5)$\Rightarrow$ (6) are obvious.

(3) $\Rightarrow$ (4). Since left Noetherian ring is left finite dimensional, and left finite dimensional left GC_2 ring is semilocal [15] (Lemma 1.1), so (4) follows from (3).

(4) $\Rightarrow$ (5). Since R is left noetherian right MGP-injective, by Theorem 2.6(2), J is nilpotent. And so R is left Noetherian and semiprimary by hypothesis, as required.

(5) $\Rightarrow$ (1). Assume (5). Then R is semiperfect right mininjective ring and $S_r \subseteq^{ess} R_R$. So that R is a right minfull ring. By [10] (Theorem 3.12), R is right Kasch.

(6) $\Rightarrow$ (4). By Theorem 2.3.

Theorem 2.7 is proved.

Theorem 2.8. *Let* R *be a right* $AMGP$*-injective ring. Then following conditions are equivalent*:

(1) R *is a semisimple Artinian ring*;

(2) R *is a semiprime ring, and the ascending chain* $\mathbf{r}(a_1) \subseteq \mathbf{r}(a_2a_1) \subseteq \mathbf{r}(a_3a_2a_1) \subseteq \ldots$ *terminates for every sequence* $\{a_1, a_2, \ldots\} \subseteq R$.

Proof. (1) $\Rightarrow$ (2) is trivial.

(2) $\Rightarrow$ (1). By Theorem 2.3, R is right perfect, i.e., $R/J(R)$ is semisimple Artinian and $J(R)$ is right $T-$ nilpotent. If $J(R) \neq 0$, then, by [7] (Lemma 3.16), $J(R)$ is not nil, a contradiction. So $J(R) = 0$, and whence R is semisimple Artinian.

Theorem 2.9. *Let* R *be a right AMGP-injective ring. Then following conditions are equivalent*:

(1) R *is a simple Artinian ring*;

(2) R *is a prime ring, and the ascending chain* $\mathbf{r}(a_1) \subseteq \mathbf{r}(a_2a_1) \subseteq \mathbf{r}(a_3a_2a_1) \subseteq \ldots$ *terminates for every sequence* $\{a_1, a_2, \ldots\} \subseteq R$.

Proof. (1) $\Rightarrow$ (2) is obvious.

(2) $\Rightarrow$ (1). By Theorem 2.8 and [14] (Lemma 2.3 (2)).

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