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EXISTENCE AND EXPONENTIAL STABILITY OF PERIODIC SOLUTION FOR FUZZY BAM NEURAL NETWORKS WITH PERIODIC COEFFICIENT *

ІСНУВАННЯ ТА ЕКСПОНЕНЦІАЛЬНА СТІЙКІСТЬ ПЕРІОДИЧНОГО РОЗВ'ЯЗКУ ДЛЯ НЕЧІТКИХ НЕЙРОННИХ МЕРЕЖ КОСКО З ПЕРІОДИЧНИМИ КОЕФІЦІЄНТАМИ

A class of fuzzy bidirectional associated memory (BAM) networks with periodic coefficients is studied. Some sufficient conditions are established for the existence and global exponential stability of a periodic solution of such fuzzy BAM neural networks by using a continuation theorem based on the coincidence degree and the Lyapunov-function method. The sufficient conditions are easy to verify in pattern recognition and automatic control. Finally, an example is given to show the feasibility and efficiency of our results.

Вивчено клас нечітких нейронних мереж Коско з періодичним коефіцієнтом. За допомогою теореми про продовження, що базується на ступені збігу та методі функцій Ляпунова, встановлено достатні умови для існування та глобальної експоненціальної стійкості періодичного розв'язку таких нечітких нейронних мереж Коско. Ці достатні умови легко перевіряються при розпізнаванні образів та автоматичному керуванні. Наведено приклад, що демонструє застосовність та ефективність отриманих результатів.

1. Introduction. Recently, a class of two-layer hetero-associative networks called bidirectional associated memory (BAM) neural networks [1, 2] with or without transmission delays have been proposed by Kosko and used in many fields such as pattern recognition and automatic control. Many authors studied the stability of BAM neural networks with delays or without delays (see, for example, [1, 2, 3 – 12, 17]).

It is well known that fuzzy cellular neural networks (FCNNs) first introduced by T. Yang and L. B. Yang [13, 14] is another type cellular neural networks model, which combined fuzzy operations (fuzzy AND and fuzzy OR) with cellular neural networks. Recently researchers have found that FCNNs are useful in image processing, and some results have been reported on stability and periodicity of FCNNs [15, 16, 18, 19]. However, the papers above only consider the FCNNs with constant coefficients. At present, the investigation of BAM neural networks with periodic coefficients and delays has attracted more and more attention of the researcher [17, 22], to the best of our knowledge, few author consider the stability of fuzzy BAM neural networks with periodic coefficients. In this paper, we would like to investigate the fuzzy BAM neural networks with periodic coefficients by the following system:

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$$x_i'(t) = -a_i(t)x_i(t) + \bigwedge_{j=1}^{m} \alpha_{ij}(t)f_j(y_j(t)) + \bigwedge_{j=1}^{m} T_{ij}(t)u_j(t) + I_i(t)+$$

$$+ \bigvee_{j=1}^{m} \beta_{ij}(t)f_j(y_j(t)) + \bigvee_{j=1}^{m} H_{ij}(t)u_j(t),$$

$$y_j'(t) = -b_j(t)y_j(t) + \bigwedge_{i=1}^{n} p_{ji}(t)g_i(x_i(t)) + \bigwedge_{i=1}^{n} K_{ji}(t)u_i(t) + J_j(t)+ \tag{1.1}$$

$$+ \bigvee_{i=1}^{n} q_{ji}(t)g_i(x_i(t)) + \bigvee_{i=1}^{n} N_{ji}(t)u_i(t),$$

where $a_i(t) \geq 0$, $b_j(t) \geq 0$, $i = 1, 2, \ldots, n$, $j = 1, 2, \ldots, m$. $x_i(t)$ and $y_j(t)$ are the activations of the ith neuron in X-layer and the jth neuron in Y-layer at the time t, respectively. $\bigwedge$ and $\bigvee$ denote fuzzy AND and fuzzy OR operations, respectively. f_j, $j = 1, 2, \ldots, m$, g_i, $i = 1, 2, \ldots, n$, are signal transmission functions. $\alpha_{ij}(t)$ and $\beta_{ij}(t)$ are respectively the elements of fuzzy feedback MIN and fuzzy feedback MAX in X-layer at the time t. $T_{ij}(t)$ and $H_{ij}(t)$ are respectively the elements of fuzzy feed-forward MIN and fuzzy feed-forward MAX in X-layer at the time t. $p_{ji}(t)$ and $q_{ji}(t)$ are respectively the elements of fuzzy feedback MIN and fuzzy feedback MAX in Y-layer at the time t. $K_{ji}(t)$ and $N_{ji}(t)$ are respectively the elements of fuzzy feed-forward MIN and fuzzy feed-forward MAX in Y-layer at the time t. $u_j(t)$ and $u_i(t)$ denote the external inputs at the time t. $I_i(t)$ and $J_j(t)$ denote bias of the ith neurons in X-layer and bias of the jth neurons in Y-layer at the time t, respectively.

Throughout this paper, we always assume that $a_i(t)$, $b_j(t)$, $\alpha_{ij}(t)$, $\beta_{ij}(t)$, $T_{ij}(t)$, $H_{ij}(t)$, $p_{ji}(t)$, $q_{ji}(t)$, $K_{ji}(t)$, $N_{ji}(t)$, $u_i(t)$, $u_j(t)$, $I_i(t)$, $J_j(t)$ are continuous ω-periodic functions.

For the sake of convenience, we introduce the following notations: Let $r(t)$ be a ω-periodic solution defined on R

$$r^- = \min_{0 \leq t \leq \omega} |r(t)|, \quad r^+ = \max_{0 \leq t \leq \omega} |r(t)|,$$

$$\overline{r} = \frac{1}{\omega} \int_0^{\omega} r(t)dt, \quad \|r\|_2 = \left(\int_0^{\omega} |r(t)|^2 dt \right)^{1/2}.$$

Throughout this paper, we give the following assumptions:

(A$_1$) $f_j(\cdot)$ and $g_i(\cdot)$ are Lipschitz continuous on R with Lipschitcz constants L_j^f, $j = 1, 2, \ldots, m$, L_i^g, $i = 1, 2, \ldots, n$, and $f_j(0) = g_i(0) = 0$. That is, for all $x, y \in R$

$$|f_j(x) - f_j(y)| \leq L_j^f |x - y|, \quad |g_i(x) - g_i(y)| \leq L_i^g |x - y|.$$

(A$_2$) There exist constant $M_j > 0, R_j > 0$ such that $|f_j(y)| \leq M_j, |g_j(x)| \leq R_j$ for $j = 1, 2, \ldots, n$, $x, y \in R$.

For any solution

$$z(t) = (x(t)^T, y(t)^T)^T = (x_1(t), \ldots, x_n(t), y_1(t), \ldots, y_m(t))^T$$

and periodic solution

$$z^*(t) = (x^*(t)^T, y^*(t)^T)^T = (x_1^*(t), \ldots, x_n^*(t), y_1^*(t), \ldots, y_m^*(t))^T$$

of system (1.1), define $\|(\phi^T, \varphi^T)^T - (x^{*T}, y^{*T})^T\|$ as

$$\|(\phi^T, \varphi^T)^T - (x^{*T}, y^{*T})^T\| = \sum_{i=1}^{n} \max_{t \in [0,\omega]} |\phi_i(t) - x_i^*(t)| + \sum_{j=1}^{m} \max_{t \in [0,\omega]} |\varphi_j(t) - y_j^*(t)|.$$

Definition 1.1. *The periodic solution* $(x^{*T}(t), y^{*T}(t))^T$ *of system* (1.1) *is said to be globally exponentially stable, if there exist constants* $\gamma > 0$ *and* $M \geq 1$ *such that*

$$|x_i(t) - x_i^*(t)| \leq M\|(\phi^T, \varphi^T)^T - (x^{*T}, y^{*T})^T\| e^{-\gamma t} \quad \forall t > 0, \quad i = 1, 2, \ldots, n,$$

$$|y_j(t) - y_j^*(t)| \leq M\|(\phi^T, \varphi^T)^T - (x^{*T}, y^{*T})^T\| e^{-\gamma t} \quad \forall t > 0, \quad j = 1, 2, \ldots, m,$$

for any solution of system (1.1).

Lemma 1.1 [13]. *Suppose* x *and* y *are two states of system* (1.1), *then we have*

$$\left| \bigwedge_{j=1}^{n} \alpha_{ij}(t) g_j(x) - \bigwedge_{j=1}^{n} \alpha_{ij}(t) g_j(y) \right| \leq \sum_{j=1}^{n} |\alpha_{ij}(t)| |g_j(x) - g_j(y)|$$

and

$$\left| \bigvee_{j=1}^{n} \beta_{ij}(t) g_j(x) - \bigvee_{j=1}^{n} \beta_{ij}(t) g_j(y) \right| \leq \sum_{j=1}^{n} |\beta_{ij}(t)| |g_j(x) - g_j(y)|.$$

The rest of this paper is organized as follows. In Section 2, we will prove the existence of the periodic solution by using the continuation theorem of coincidence degree theory. In Section 3, we establish the result that the periodic solutions are the globally exponentially stable by using Lyapunov function method. In Section 4, an example will be given to illustrate the feasibility and effectiveness of our results. General conclusion is drawn in Section 5.

2. Existence of periodic solution. In this section, based on Mawhin's continuation theorem, we shall study the existence of at least one periodic solution of (1.1). To do so, we shall make some preparations.

Let $X = \{(x^T(t), y^T(t)))^T \in C(R, R^{n+m}) | x(t+\omega) = x(t), y(t+\omega) = y(t)$ for some $\omega > 0\}$ and $\|(x^T(t), y^T(t))^T\| = \sum_{i=1}^{n} \max_{t \in [0,\omega]} |x_i(t)| + \sum_{j=1}^{m} \max_{t \in [0,\omega]} |y_j(t)|$, it can be proved that X is a Banach space.

Consider the following abstract equation in the Banach space X:

$$Lx = \lambda N x \tag{2.1}$$

where $L\colon \operatorname{Dom} L \bigcap X \to X$ is a Fredholm mapping of index zero and $\lambda \in [0, 1]$ is a parameter. There exist two linear and continuous projectors P and Q

$$P\colon X \bigcap \operatorname{Dom} L \to \operatorname{Ker} L, \quad Q\colon X \to X/\operatorname{Im} L$$

such that $\operatorname{Im} P = \operatorname{Ker} L$, $\operatorname{Ker} Q = \operatorname{Im} L$. Due to $\dim \operatorname{Im} Q = \dim \operatorname{Ker} L$, there exists an algebraical and topological isomorphism $J\colon \operatorname{Im} Q \to \operatorname{Ker} L$.

Lemma 2.1 (see [21]). *Let X be a Banach space and L be a Fredholm mapping of index zero. Assume that $N\colon \overline{\Omega} \to X$ is a $L-$ compact on $\overline{\Omega}$ with Ω open and bound in X. Furthermore, suppose that*

(a) *for each* $\lambda \in (0,1)$, $x \in \partial\Omega \bigcap \operatorname{Dom} L$, $Lx \neq \lambda Nx$;

(b) *for each* $x \in \partial\Omega \bigcap \operatorname{Ker} L$, $QNx \neq 0$;

(c) $\deg\{QNx, \Omega \bigcap \operatorname{Ker} L, 0\} \neq 0$,

then the equation $Lx = Nx$ has at least one solution in $\overline{\Omega}$, where $\overline{\Omega}$ is the closure to Ω, $\partial\Omega$ is the boundary of Ω.

Theorem 2.1. *Assume that* (A_1) *and* (A_2) *hold, then system* (1.1) *has at least one ω-periodic solution.*

Proof. In order to use continuation theorem of coincidence degree theory to establish the existence of periodic solution. Let

$$(Nz)_i(t) = -a_i(t)x_i(t) + \bigwedge_{j=1}^{m} \alpha_{ij}(t)f_j(y_j(t)) + \bigwedge_{j=1}^{m} T_{ij}(t)u_j(t) + I_i(t) +$$

$$+ \bigvee_{j=1}^{m} \beta_{ij}(t)f_j(y_j(t)) + \bigvee_{j=1}^{m} H_{ij}(t)u_j(t), \quad i = 1, 2, \ldots, n,$$

$$(Nz)_{n+j}(t) = -b_j(t)y_j(t) + \bigwedge_{i=1}^{n} p_{ji}(t)g_i(x_i(t)) + \bigwedge_{i=1}^{n} K_{ji}(t)u_i(t) + J_j(t) +$$

$$+ \bigvee_{i=1}^{n} q_{ji}(t)g_i(x_i(t)) + \bigvee_{i=1}^{n} N_{ji}(t)u_i(t), \quad j = 1, 2, \ldots, m,$$

$$(Lz)(t) = z'(t), \quad Pz = \frac{1}{\omega}\int_0^{\omega} z(t)dt, \quad QU = \frac{1}{\omega}\int_0^{\omega} U(t)dt$$

for $z(t) = (x^T(t),\, y^T(t))^T \in X \bigcap \operatorname{Dom} L$, $U \in X$. It is easy to prove that L is a Fredholm mapping of index zero, that $P\colon X \bigcap \operatorname{Dom} L \to \operatorname{Ker} L$ and $Q\colon X \to X/\operatorname{Im} L$ are two projector, and N is L compact on $\overline{\Omega}$ for any given open bounded set.

Corresponding to the operator equation $Lz = \lambda Nz$, $\lambda \in (0,1)$, we have

$$x_i'(t) = \lambda \Bigg[-a_i(t)x_i(t) + \bigwedge_{j=1}^{m} \alpha_{ij}(t)f_j(y_j(t)) + \bigwedge_{j=1}^{m} T_{ij}(t)u_j(t) + I_i(t) + $$

$$+ \bigvee_{j=1}^{m} \beta_{ij}(t)f_j(y_j(t)) + \bigvee_{j=1}^{m} H_{ij}(t)u_j(t) \Bigg],$$

$$\tag{2.2}$$

$$y_j'(t) = \lambda \Bigg[-b_j(t)y_j(t) + \bigwedge_{i=1}^{n} p_{ji}(t)g_i(x_i(t)) + \bigwedge_{i=1}^{n} K_{ji}(t)u_i(t) + J_j(t) + $$

$$+ \bigvee_{i=1}^{n} q_{ji}(t)g_i(x_i(t)) + \bigvee_{i=1}^{n} N_{ji}(t)u_i(t) \Bigg].$$

Suppose that $z(t) = (x_1(t), \ldots, x_n(t), y_1(t), \ldots, y_m(t))^T \in X$ is a solution of system (2.2) for a certain $\lambda \in (0,1)$. Integrating (2.2) over $[0,\omega]$, we obtain

$$\int_0^\omega a_i(t)x_i(t)dt = \int_0^\omega \Bigg[\bigwedge_{j=1}^m \alpha_{ij}(t)f_j(y_j(t)) + \bigwedge_{j=1}^m T_{ij}(t)u_j(t) + I_i(t) + \\ + \bigvee_{j=1}^m \beta_{ij}(t)f_j(y_j(t)) + \bigvee_{j=1}^m H_{ij}(t)u_j(t) \Bigg] dt. \tag{2.3}$$

Let $\xi \in [0,\omega]$ such that $x_i(\xi) = \inf_{t\in[0,\omega]} x_i(t)$, $i = 1,2,\ldots,n$. Then by (2.3), we have

$$\omega \bar{a}_i x_i(\xi) \le \int_0^\omega \Bigg[\Bigg| \bigwedge_{j=1}^m \alpha_{ij}(t)f_j(y_j(t)) - \bigwedge_{j=1}^m \alpha_{ij}(t)f_j(0) \Bigg| + \Bigg| \bigwedge_{j=1}^m T_{ij}(t)u_j(t) \Bigg| + |I_i(t)| + \\ + \Bigg| \bigvee_{j=1}^m \beta_{ij}(t)f_j(y_j(t)) - \bigvee_{j=1}^m \beta_{ij}(t)f_j(0) \Bigg| + \Bigg| \bigvee_{j=1}^m H_{ij}(t)u_j(t) \Bigg| \Bigg] dt \le \\ \le \int_0^\omega \Bigg[\sum_{j=1}^m |\alpha_{ij}(t)||f_j(y_j(t))| + \sum_{j=1}^m |\beta_{ij}(t)||f_j(y_j(t))| + \\ + \Bigg| \bigwedge_{j=1}^m T_{ij}(t)u_j(t) \Bigg| + |I_i(t)| + \Bigg| \bigvee_{j=1}^m H_{ij}(t)u_j(t) \Bigg| \Bigg] dt \le \\ \le \omega \Bigg[\sum_{j=1}^m (\alpha_{ij}^+ + \beta_{ij}^+)M_j + (T_{ij}^+ + H_{ij}^+)u_j^+ + I_i^+ \Bigg].$$

Hence

$$x_i(\xi) \le \frac{1}{\bar{a}_i} \left\{ \sum_{j=1}^m (\alpha_{ij}^+ + \beta_{ij}^+)M_j + (T_{ij}^+ + H_{ij}^+)u_j^+ + I_i^+ \right\} := U_i, \quad i = 1,2,\ldots,n. \tag{2.4}$$

Similarly, let $\eta \in [0,\omega]$ such that $y_j(\eta) = \inf_{t\in[0,\omega]}$, $j = 1,2,\ldots,m$, we obtain

$$y_j(\eta) \le \frac{1}{\bar{b}_j} \left\{ \sum_{i=1}^n (p_{ji}^+ + q_{ji}^+)R_j + (K_{ji}^+ + N_{ji}^+)u_i^+ + J_j^+ \right\} := V_j, \quad j = 1,2,\ldots,m. \tag{2.5}$$

Set $t_0 = 0$, $t_{q+1} = \omega$, from (2.2), (2.4) and (2.5), we have

$$\int_0^\omega |x_i'(t)|dt \le \sum_{k=1}^{q+1} \int_{t_{k-1}}^{t_k} |x_i'(t)|dt \le$$

$$\leq \int_0^\omega |a_i(t)||x_i(t)|dt + \int_0^\omega \sum_{j=1}^m (|\alpha_{ij}(t)| + |\beta_{ij}(t)|)|f_j(y_j(t))|dt+$$

$$+ \int_0^\omega \left(\left| \bigwedge_{j=1}^m T_{ij}(t)u_j(t) \right| + \left| \bigvee_{j=1}^m H_{ij}(t)u_j(t) \right| \right) dt + \int_0^\omega |I_i(t)|dt \leq$$

$$\leq \left(\int_0^\omega |a_i(t)|^2 dt \right)^{1/2} \left(\int_0^\omega |x_i(t)|^2 dt \right)^{1/2} +$$

$$+ \sum_{j=1}^m \left(\int_0^\omega |\alpha_{ij}(t)|^2 dt \right)^{1/2} \left(\int_0^\omega |f_j(y_j(t))|^2 dt \right)^{1/2} +$$

$$+ \sum_{j=1}^m \left(\int_0^\omega |\beta_{ij}(t)|^2 dt \right)^{1/2} \left(\int_0^\omega |f_j(y_j(t))|^2 dt \right)^{1/2} + (T_{ij}^+ + H_{ij}^+)u_j^+\omega + I_i^+\omega \leq$$

$$\leq \sqrt{\omega} a_i^+ \|x_i\|_2 + \sum_{j=1}^m \sqrt{\omega}(\alpha_{ij}^+ + \beta_{ij}^+)M_j + (T_{ij}^+ + H_{ij}^+)u_j^+\omega + I_i^+\omega. \qquad (2.6)$$

Multiplying both sides of system (2.2) by $x_i(t)$ and integrating over $[0, \omega]$, we obtain that

$$0 = \int_0^\omega x_i(t)x_i'(t)dt = -\lambda \int_0^\omega a_i(t)x_i^2(t)dt+$$

$$+\lambda \int_0^\omega \left(\bigwedge_{j=1}^m \alpha_{ij}(t)f_j(y_j(t)) + \bigvee_{j=1}^m \beta_{ij}(t)f_j(y_j(t)) \right) \times$$

$$\times x_i(t)dt + \lambda \int_0^\omega \left(\bigwedge_{j=1}^m T_{ij}(t)u_j(t) + \bigvee_{j=1}^m H_{ij}(t)u_j(t) \right) x_i(t)dt + \lambda \int_0^\omega I_i(t)x_i(t)dt. \qquad (2.7)$$

From (2.7) and applying Lemma 1.1, it follows that

$$a_i^- \int_0^\omega |x_i(t)|^2 dt \leq \int_0^\omega \left(\left| \bigwedge_{j=1}^m \alpha_{ij}(t)f_j(y_j(t)) \right| + \left| \bigvee_{j=1}^m \beta_{ij}(t)f_j(y_j(t)) \right| \right) |x_i(t)|dt+$$

$$+ \int_0^\omega \left| \bigwedge_{j=1}^m T_{ij}(t)u_j(t) + \bigvee_{j=1}^m H_{ij}(t)u_j(t) \right| |x_i(t)|dt + \int_0^\omega |I_i(t)||x_i(t)|dt \leq$$

$$\leq \int_0^\omega \sum_{j=1}^m (|\alpha_{ij}| + |\beta_{ij}|)|f_j(y_j(t))||x_i(t)|dt +$$

$$+ \int_0^\omega \left(\bigwedge_{j=1}^m |T_{ij}(t)||u_j(t)| + \bigvee_{j=1}^m |H_{ij}(t)||u_j(t)| \right) |x_i(t)|dt + \int_0^\omega |I_i(t)||x_i(t)|dt \leq$$

$$\leq \left(\sum_{j=1}^m (\alpha_{ij}^+ + \beta_{ij}^+) M_j + T_{ij}^+ u_j^+ + H_{ij}^+ u_j^+ + I_i^+ \right) \sqrt{\omega} \left(\int_0^\omega |x_i(t)|^2 dt \right)^{1/2}. \quad (2.8)$$

It follows from (2.8) that

$$\|x_i\|_2 \leq \frac{1}{a_i^-} \left(\sum_{j=1}^m (\alpha_{ij}^+ + \beta_{ij}^+) M_j + T_{ij}^+ u_j^+ + H_{ij}^+ u_j^+ + I_i^+ \right) \sqrt{\omega} := G_i. \quad (2.9)$$

Substituting (2.9) into (2.6), we obtain that

$$\int_0^\omega |x_i'(t)|dt \leq \sqrt{\omega} a_i^+ G_i + \sum_{j=1}^m \sqrt{\omega} (\alpha_{ij}^+ + \beta_{ij}^+) M_j + (T_{ij}^+ + H_{ij}^+) u_j^+ \omega + I_i^+ \omega. \quad (2.10)$$

From (2.4) and (2.10), there exists positive constant B_i, $i = 1, 2, \ldots, n$, such that for $t \in [0, \omega]$,

$$|x_i(t)| \leq B_i, \quad i = 1, 2, \ldots, n.$$

Similarly, we have

$$|y_j(t)| \leq B_{n+j}, \quad j = 1, 2, \ldots, m.$$

Clearly, B_i, $i = 1, 2, \ldots, n+m$, is independent of λ. Denote $B^* = \sum_{i=1}^{n+m} B_i + \delta$, where $\delta > 0$ is taken sufficiently large such that

$$\min_{1 \leq i \leq n} \bar{a}_i B^* > \max_{1 \leq i \leq n} \left(\sum_{j=1}^m (|\bar{\alpha}_{ij}| + |\bar{\beta}_{ij}|) M_j + T_{ij}^+ u_j^+ + H_{ij}^+ u_j^+ + |\bar{I}_i| \right),$$

$$\min_{1 \leq j \leq m} \bar{b}_i B^* > \max_{1 \leq j \leq m} \left(\sum_{i=1}^n (|\bar{p}_{ji}| + |\bar{q}_{ji}|) R_i + K_{ji}^+ u_i^+ + N_{ji}^+ u_i^+ + |\bar{J}_j| \right).$$

Now we take

$$\Omega = \{ z = (x_1(t), \ldots, x_n(t), y_1(t), \ldots, y_m(t))^T \in R^{n+m} | \|z\| =$$

$$= \|(x_1, \ldots, x_n, y_1, \ldots, y_m)^T\| < B^* \}.$$

Thus the condition (a) of Lemma 2.1 is satisfied.

When $z = (x_1, \ldots, x_n, y_1, \ldots, y_m)^T \in \partial\Omega \bigcap R^{n+m}$, $z = (x_1, \ldots, x_n, y_1, \ldots, y_m)^T$ is a constant vector in R^{n+m} with $|x_1| + \ldots + |x_n| + |y_1| + \ldots + |y_m| = B^*$. Then

$$QNz = QN(x_1, \ldots, x_n, y_1, \ldots, y_m)^T = \begin{pmatrix} \Theta_1 \\ \vdots \\ \Theta_n \\ \Theta_{n+1} \\ \vdots \\ \Theta_{n+m} \end{pmatrix},$$

where

$$\Theta_i = -\overline{a}_i x_i + \bigwedge_{j=1}^{m} \overline{\alpha}_{ij} f_j(y_j) + \bigvee_{j=1}^{m} \overline{\beta}_{ij} f_j(y_j) +$$

$$+ \frac{1}{\omega} \int_0^{\omega} \bigwedge_{j=1}^{m} T_{ij}(t) u_j(t) dt + \frac{1}{\omega} \int_0^{\omega} \bigvee_{j=1}^{m} H_{ij}(t) u_j(t) dt + \overline{I}_i,$$

$$\Theta_{n+j} = -\overline{b}_j y_j + \bigwedge_{i=1}^{n} \overline{p}_{ji} g_i(x_i) + \bigvee_{i=1}^{n} \overline{q}_{ji} g_i(x_i) +$$

$$+ \frac{1}{\omega} \int_0^{\omega} \bigwedge_{i=1}^{n} K_{ji}(t) u_i(t) dt + \frac{1}{\omega} \int_0^{\omega} \bigvee_{i=1}^{n} N_{ji}(t) u_i(t) dt + \overline{J}_j.$$

Therefore,

$$\|QNz\| = \sum_{i=1}^{n} \left| \overline{a}_i x_i - \bigwedge_{j=1}^{m} \overline{\alpha}_{ij} f_j(y_j) - \bigvee_{j=1}^{m} \overline{\beta}_{ij} f_j(y_j) - \frac{1}{\omega} \int_0^{\omega} \bigwedge_{j=1}^{m} T_{ij}(t) u_j(t) dt - \right.$$

$$\left. - \frac{1}{\omega} \int_0^{\omega} \bigvee_{j=1}^{m} H_{ij}(t) u_j(t) dt - \overline{I}_i \right| +$$

$$+ \sum_{j=1}^{m} \left| \overline{b}_j y_j - \bigwedge_{i=1}^{n} \overline{p}_{ji} g_i(x_i) - \bigvee_{i=1}^{n} \overline{q}_{ji} g_i(x_i) - \frac{1}{\omega} \int_0^{\omega} \bigwedge_{i=1}^{n} K_{ji}(t) u_i(t) dt - \right.$$

$$\left. - \frac{1}{\omega} \int_0^{\omega} \bigvee_{i=1}^{n} N_{ji}(t) u_i(t) dt - \overline{J}_j \right| \geq \sum_{i=1}^{n} \overline{a}_i |x_i| - \sum_{i=1}^{n} \left| \bigwedge_{j=1}^{m} \overline{\alpha}_{ij} f_j(y_j) - \bigwedge_{j=1}^{m} \overline{\alpha}_{ij} f_j(0) \right| -$$

$$- \sum_{i=1}^{n} \left| \bigvee_{j=1}^{m} \overline{\beta}_{ij} f_j(y_j) - \bigvee_{j=1}^{m} \overline{\beta}_{ij} f_j(0) \right| - \sum_{i=1}^{n} T_{ij}^{+} u_j^{+} - \sum_{i=1}^{n} H_{ij}^{+} u_j^{+} - \sum_{i=1}^{n} |\overline{I}_i| +$$

$$+ \sum_{j=1}^{m} \overline{b}_i |y_j| - \sum_{j=1}^{m} \left| \bigwedge_{i=1}^{n} \overline{p}_{ji} g_i(x_i) - \bigwedge_{i=1}^{n} \overline{p}_{ji} g_i(0) \right| - \sum_{j=1}^{m} \left| \bigvee_{i=1}^{n} \overline{q}_{ji} g_i(x_i) - \bigvee_{i=1}^{n} \overline{q}_{ji} g_i(0) \right| -$$

$$-\sum_{j=1}^{m} K_{ji}^{+}u_i^{+} - \sum_{j=1}^{m} N_{ji}^{+}u_i^{+} - \sum_{j=1}^{m} |\overline{J}_j| \geq$$

$$\geq \sum_{i=1}^{n} \overline{a}_i|x_i| - \sum_{i=1}^{n} \left(\sum_{j=1}^{m} (|\overline{\alpha}_{ij}| + |\overline{\beta}_{ij}|)M_j + T_{ij}^{+}u_j^{+} + H_{ij}^{+}u_j^{+} + |\overline{I}_i| \right) +$$

$$+ \sum_{j=1}^{m} \overline{b}_j|y_j| - \sum_{j=1}^{m} \left(\sum_{i=1}^{n} (|\overline{p}_{ji}| + |\overline{q}_{ji}|)R_j i + K_{ji}^{+}u_i^{+} + N_{ji}^{+}u_i^{+} + |\overline{J}_j| \right) \geq$$

$$\geq \min_{1\leq i\leq n} \overline{a}_i|x_i| - \max_{1\leq i\leq n} \left(\sum_{j=1}^{m} (|\overline{\alpha}_{ij}| + |\overline{\beta}_{ij}|)M_j + T_{ij}^{+}u_j^{+} + H_{ij}^{+}u_j^{+} + |\overline{I}_i| \right) +$$

$$+ \min_{1\leq j\leq m} \overline{b}_j|y_j| - \max_{1\leq j\leq m} \left(\sum_{i=1}^{n} (|\overline{p}_{ji}| + |\overline{q}_{ji}|)R_i + K_{ji}^{+}u_i^{+} + N_{ji}^{+}u_i^{+} + |\overline{J}_j| \right) > 0.$$

Consequently, $QNz = QN(x_1, \ldots, x_n, y_1, \ldots, y_m)^T \neq (0, 0, \ldots, 0)^T$, for $(x_1, \ldots, x_n, y_1, \ldots, y_m)^T \in \partial\Omega \bigcap \operatorname{Ker} L$. This satisfies condition (b) of Lemma 2.1.

Define $\Phi\colon \operatorname{Dom} L \times [0, 1] \to X$ by

$$\Phi(x_1, \ldots, x_n, y_1, \ldots, y_m, \mu)^T =$$

$$= -\mu(x_1, \ldots, x_n, y_1, \ldots, y_m)^T + (1 - \mu)QN(x_1, \ldots, x_n, y_1, \ldots, y_m)^T.$$

When $(x_1, \ldots, x_n, y_1, \ldots, y_m)^T \in \partial\Omega \bigcap \operatorname{Ker} L$, $(x_1, \ldots, x_n, y_1, \ldots, y_m)^T \in \partial\Omega \bigcap \bigcap \operatorname{Ker} L$ is a constant vector satisfying $\sum_{i=1}^{n} |x_i| + \sum_{j=1}^{m} |y_j| = B^*$. It easily follows that

$$\Phi(x_1, \ldots, x_n, y_1, \ldots, y_m, \mu)^T \neq (0, 0, \ldots, 0)^T.$$

Hence

$$\deg\left(QN(x_1, \ldots, x_n, y_1, \ldots, y_m)^T, \Omega \bigcap \operatorname{Ker} L, (0, 0, \ldots, 0)^T\right)$$

$$= \deg\left((-x_1, \ldots, -x_n, -y_1, \ldots, -y_m)^T, \Omega \bigcap \operatorname{Ker} L, (0, 0, \ldots, 0)^T\right) \neq 0.$$

This satisfies condition (c) of Lemma 2.1. Thus by Lemma 2.1 it follows that $Lx = Nx$ has at least one solution in X, namely, system (1.1) has at least one ω-periodic solution.

Theorem 2.1 is proved.

3. Global exponential stability of the periodic solution. In this section, we will construct some suitable Lyapunov function to study the global exponential stability of the periodic solution of system (1.1).

Theorem 3.1. *If assumptions* $(\mathrm{A}_1), (\mathrm{A}_2)$ *hold, and furthermore assume that*

(A_3) *The following inequalities hold*:

$$a_i^{-} - \sum_{j=1}^{m} (p_{ji}^{+} + q_{ji}^{+})L_i^g > 0, \quad i = 1, 2, \ldots, n,$$

$$b_j^- - \sum_{i=1}^{n}(\alpha_{ij}^+ + \beta_{ij}^+)L_j^f > 0, \quad j = 1, 2, \ldots, m.$$

Then the periodic solution of system (1.1) is globally exponentially stable.

Proof. According to Theorem 2.1, we know that system (1.1) has an ω-periodic solution $z^*(t) = (x_1^*(t), \ldots, x_n^*(t), y_1^*(t), \ldots, y_m^*(t))^T$. Suppose that $z(t) = (x_1(t), \ldots$ $\ldots, x_n(t), y_1(t), \ldots, y_m(t))^T$ is an arbitrary solution of system (1.1), then it follows from system (1.1) that

$$\frac{d}{dt}(x_i(t) - x_i^*(t)) = -a_i(t)(x_i(t) - x_i^*(t)) +$$

$$+ \bigwedge_{j=1}^{m} \alpha_{ij}(t)f_j(y_j(t)) - \bigwedge_{j=1}^{m} \alpha_{ij}(t)f_j(y_j^*(t)) +$$

$$+ \bigvee_{j=1}^{m} \beta_{ij}(t)f_j(y_j(t)) - \bigvee_{j=1}^{m} \beta_{ij}(t)f_j(y_j^*(t)), \quad i = 1, 2, \ldots, n,$$

$$\frac{d}{dt}(y_j(t) - y_j^*(t)) = -b_j(t)(y_j(t) - y_j^*(t)) + \bigwedge_{i=1}^{n} p_{ji}(t)g_i(x_i(t)) - \bigwedge_{i=1}^{n} p_{ji}(t)g_i(x_i^*(t)) +$$

$$+ \bigvee_{i=1}^{n} q_{ji}(t)g_i(x_i(t)) - \bigvee_{i=1}^{n} q_{ji}(t)g_i(x_i^*(t)), \quad j = 1, 2, \ldots, m.$$

By (A_1) and Lemma 2.1, we have

$$\frac{d^+}{dt}|x_i(t) - x_i^*(t)| \le -a_i(t)|x_i(t) - x_i^*(t)| +$$

$$+ \left| \bigwedge_{j=1}^{m} \alpha_{ij}(t)f_j(y_j(t)) - \bigwedge_{j=1}^{m} \alpha_{ij}(t)f_j(y_j^*(t)) \right| +$$

$$+ \left| \bigvee_{j=1}^{m} \beta_{ij}(t)f_j(y_j(t)) - \bigvee_{j=1}^{m} \beta_{ij}(t)f_j(y_j^*(t)) \right| \le$$

$$\le -a_i^-|x_i(t) - x_i^*(t)| + \sum_{j=1}^{m}(\alpha_{ij}^+ + \beta_{ij}^+)L_j^f|y_j(t) - y_j^*(t)|, \tag{3.1}$$

$$\frac{d^+}{dt}|y_j(t) - y_j^*(t)| \le -b_j(t)|y_j(t) - y_j^*(t)| +$$

$$+ \left| \bigwedge_{i=1}^{m} p_{ji}(t)g_i(x_i(t)) - \bigwedge_{i=1}^{n} p_{ji}(t)g_i(x_i^*(t)) \right| +$$

$$+ \left| \bigvee_{i=1}^{n} q_{ji}(t)g_i(x_i(t)) - \bigvee_{i=1}^{n} q_{ji}(t)g_i(x_i^*(t)) \right| \le$$

$$\leq -b_j^- |y_j(t) - y_j^*(t)| + \sum_{i=1}^{n} (p_{ji}^+ + q_{ji}^+) L_i^g |x_i(t) - x_i^*(t)|, \tag{3.2}$$

where d^+/dt denotes the upper right derivative.

Define a Lyapunov function $V(\cdot)$ by

$$V(t) = \sum_{i=1}^{n} |x_i(t) - x_i^*(t)| + \sum_{j=1}^{m} |y_j(t) - y_j^*(t)|$$

for $t \geq 0$, by virtue of (3.1) and (3.2), we have

$$\frac{d^+V(t)}{dt} = \sum_{i=1}^{n} \frac{d^+}{dt} |x_i(t) - x_i^*(t)| + \sum_{j=1}^{m} \frac{d^+}{dt} |y_j(t) - y_j^*(t)| \leq$$

$$\leq \sum_{i=1}^{n} \left(-a_i^- |x_i(t) - x_i^*(t)| + \sum_{j=1}^{m} (\alpha_{ij}^+ + \beta_{ij}^+) L_j^f |y_j(t) - y_j^*(t)| \right) +$$

$$+ \sum_{j=1}^{m} \left(-b_j^- |y_j(t) - y_j^*(t)| + \sum_{i=1}^{n} (p_{ji}^+ + q_{ji}^+) L_i^g |x_i(t) - x_i^*(t)| \right) =$$

$$= - \sum_{i=1}^{n} \left(a_i^- - \sum_{j=1}^{m} (p_{ji}^+ + q_{ji}^+) L_i^g \right) |x_i(t) - x_i^*(t)| -$$

$$- \sum_{j=1}^{m} \left(b_j^- - \sum_{i=1}^{n} (\alpha_{ij}^+ + \beta_{ij}^+) L_j^f \right) |y_j(t) - y_j^*(t)|.$$

Since (A_3) hold, there exists a real number $\gamma > 0$ such that

$$a_i^- - \sum_{j=1}^{m} (p_{ji}^+ + q_{ji}^+) L_i^g \geq \gamma, \quad b_j^- - \sum_{i=1}^{n} (\alpha_{ij}^+ + \beta_{ij}^+) L_j^f \geq \gamma.$$

It follows that

$$\frac{d^+V(t)}{dt} \leq -\gamma V(t) \quad \text{for} \quad t \geq 0. \tag{3.3}$$

Using exponential stability theorem [23], (3.3) implies that

$$V(t) \leq e^{-\gamma t} V(0) \quad \forall t \geq 0.$$

That is

$$\sum_{i=1}^{n} |x_i(t) - x_i^*(t)| + \sum_{j=1}^{m} |y_j(t) - y_j^*(t)| \leq$$

$$\leq e^{-\gamma t} \left(\sum_{i=1}^{n} |x_i(0) - x_i^*(0)| + \sum_{j=1}^{m} |y_j(0) - y_j^*(0)| \right),$$

therefore the periodic solution of system (1.1) is globally exponentially stable.

Theorem 3.1 is proved.

4. Example. In this section, we consider the following fuzzy BAM neural networks with periodic coefficient

$$x_i'(t) = -a_i(t)x_i(t) + \bigwedge_{j=1}^{2} \alpha_{ij} f_j(y_j(t)) + \bigwedge_{j=1}^{2} T_{ij}(t)u_j(t) + I_i(t) +$$
$$+ \bigvee_{j=1}^{2} \beta_{ij}(t) f_j(y_j(t)) + \bigvee_{j=1}^{2} H_{ij}(t)u_j(t), \quad i = 1, 2,$$
$$y_j'(t) = -b_j(t)y_j(t) + \bigwedge_{i=1}^{2} p_{ji}(t) g_i(x_i(t)) + \bigwedge_{i=1}^{2} K_{ji}(t)u_i(t) + J_j(t) +$$
$$+ \bigvee_{i=1}^{2} q_{ji} g_i(x_i(t)) + \bigvee_{i=1}^{2} N_{ji}(t)u_i(t), \quad j = 1, 2, \tag{4.1}$$

where $a_1(t) = 12 - \cos 2t$, $a_2(t) = 13 - 2\cos 2t$, $b_1(t) = 13 + \sin 2t$, $b_2(t) = 13 - - 2\sin 2t$, $\alpha_{11}(t) = \alpha_{21}(t) = 1 + \sin 2t$, $\alpha_{12}(t) = \alpha_{22}(t) = 2 + \sin 2t$, $\beta_{11}(t) = = \beta_{21}(t) = 1 - \sin 2t$, $\beta_{12}(t) = \beta_{22}(t) = 2 - \sin 2t$, $p_{11}(t) = p_{21}(t) = 1 + \cos 2t$, $p_{12}(t) = p_{22}(t) = 2 + \cos 2t$, $q_{11}(t) = q_{21}(t) = 1 - \cos 2t$, $q_{12}(t) = q_{22}(t) = 2 - \cos 2t$, $T_{ij}(t) = H_{ij}(t) = \sin 2t$, $K_{ji}(t) = N_{ji}(t) = \cos 2t$, $u_i(t) = u_j(t) = 2\sin 2t$, $i, j = = 1, 2$, $I_i(t) = J_j(t) = 2\cos 2t$, $i, j = 1, 2$. Take $f_i(x) = g_i(x) = \dfrac{1}{2}(|x+1| - |x-1|)$, $i = 1, 2$, we have $L_i^g = L_j^f = 1$, $i, j = 1, 2$. By simple computation, we have

$$a_1^- = 11, \quad a_2^- = 11, \quad b_1^- = 12, \quad b_2^- = 11,$$
$$\alpha_{11}^+ = \alpha_{21}^+ = 2, \quad \alpha_{12}^+ = \alpha_{22}^+ = 3, \quad \beta_{11}^+ = \beta_{21}^+ = 2,$$
$$\beta_{12} = \beta_{22}^+ = 3, \quad p_{11}^+ = p_{21}^+ = 2, \quad p_{12}^+ = p_{22}^+ = 3,$$
$$q_{11}^+ = q_{21} = 2, \quad q_{12}^+ = q_{22}^+ = 3.$$

Obviously, the following inequalities hold

$$a_i^- - \sum_{j=1}^{2} (p_{ji}^+ + q_{ji}^+) L_i^g > 0, \ i = 1, 2; \quad b_j^- - \sum_{i=1}^{2} (\alpha_{ij}^+ + \beta_{ij}^+) L_i^f > 0, \quad j = 1, 2.$$

Hence, it follows that the assumptions $(\mathrm{A}_1)-(\mathrm{A}_3)$ are satisfied. Therefore, according to Theorems 2.1 and 3.1, system (4.1) has one π-periodic solution which is globally exponentially stable.

5. Conclusion. In this paper, we use the continuation theorem of coincidence degree theory and Lyapunov function to study the existence and global exponential stability of periodic solution for fuzzy BAM neural networks with periodic coefficient. The sufficient conditions of existence and global stability of periodic solution are easily verifiable.

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