

ELEMENTS OF A NON-GAUSSIAN ANALYSIS ON SPACES OF FUNCTIONS OF INFINITELY MANY VARIABLES

We present a review of some results of a non-Gaussian analysis under the biorthogonal approach and consider elements of an analysis associated with the generalized Meixner measure. The main objects of our interest are stochastic integrals, operators of stochastic differentiation, elements of a Wick calculus, and related topics.

Наведено огляд деяких результатів негауссівського аналізу при біортогональному підході та розглянуто елементи аналізу, пов'язаного з узагальненою мірою Майкснера. Основними об'єктами, що розглядаються, є стохастичні інтеграли, оператори стохастичного диференціювання, елементи віківського числення та споріднені питання.

Introduction. The development of modern mathematical branches of science, in particular, of mathematical physics and stochastic analysis, requires to construct an extensive theory of generalized functions of infinitely many variables. One direction of this theory that consists in the study of test and generalized function spaces as infinite tensor products of one-dimensional spaces was triggered by Yu. M. Berezansky, Yu. G. Kondratiev, and Yu. S. Samoilenko (see [21, 85, 86]). Afterwards, independently of one another, Yu. G. Kondratiev [81–83] (see also [87]) and T. Hida (see [54, 55]) constructed a detailed theory of generalized functions of infinitely many variables with special spaces of test and generalized functions such that the pairing between elements of these spaces is generated by integration with respect to the Gaussian measure.

On the other hand, approximately at the same time, Yu. M. Berezansky and disciples were constructing a theory of generalized functions (see [14, 21, 92]) related to product measures. This theory is less detailed than the “Gaussian” one, but is more general because deals with a dual pairing generated by integration with respect to a non-Gaussian measure.

With regard for these researches, it is natural to try to construct a theory of generalized functions of infinitely many variables, which deals with a general, as far as possible, dual pairing, but is similar by detalization of results to the “Gaussian” theory (for this theory we use the term a *non-Gaussian infinite-dimensional analysis*). The first works in this direction are the papers of Y. Ito and I. Kubo [59, 60], in which some results of the Gaussian analysis are transferred to the case where the Gaussian measure is replaced by the Poissonian measure. The posterior development of the above-mentioned theory occurred in different directions. For example, in 1991, Yu. M. Berezansky [15] offered to construct orthogonal decompositions of “base” spaces (these spaces generate dual pairings) using families of commuting self-adjoint operators. Typical examples of works in this direction are [18, 95]. Another direction is based on the idea of Yu. G. Kondratiev to use biorthogonal systems [30] (that consist of generalized Appell polynomials and (generalized) functions dual to these polynomials) as orthogonal bases in spaces of test and generalized functions. This idea was realized by Yu. G. Kondratiev and his colleagues first for the case of so-called smooth twice analytic measures [7, 8, 90], later for more general analytic nondegenerate measures [88, 91]. After that different researches and generalizations were executed by many specialists; in particular, by G. F. Us [116], by Yu. M. Berezansky and Yu. G. Kondratiev [16], by Yu. M. Berezansky [12], by the author, see, e.g., [67–69, 74, 75], by V. A. Tesko [112], by Yu. M. Berezansky and V. A. Tesko

[22–24], by E. Yablonsky [118] etc. Moreover, it was ascertained that insignificant modifications of a biorthogonal approach to the construction of a non-Gaussian analysis give a possibility to extend an area of possible applications. For example, it is possible to use results of this approach in order to construct elements of a so-called coloured noise analysis, see, e.g., [65, 78, 117].

Let us say several words about nonclassical examples of application of the “bi-orthogonal analysis”. In the end of 1990-es, in the papers [84, 89], the infinite-dimensional analog of the so-called gamma-measure was constructed and investigated. One can show (see, e.g., [76]) that this measure satisfies all requirements of the “biorthogonal analysis”. Moreover, the results of the “gamma analysis” that are closely related to a structure and properties of the gamma-measure and cannot be obtained from the “biorthogonal theory” (for example, the construction of the extended stochastic integral on the space of square-integrable functions), are naturally coordinated with general results of the “biorthogonal analysis”. This example is not unique. In 2002, Yu. M. Berezansky [13] constructed and studied an infinite-dimensional analog of the Pascal measure; in 2003, E. W. Lytvynov [96, 97] considered infinite-dimensional analogs of so-called Meixner-type measures (to this type belong the Gaussian, Poissonian, Pascal, Meixner measures and gamma-measures, see [98]) and constructed elements of the corresponding analysis basing on the so-called Jacobi fields theory, see, e.g., [19]; in 2005, I. V. Rodionova [108] considered a wide class of infinite-dimensional Meixner-type measures (these measures are realized as a so-called *generalized Meixner measure*, which subject to parameters can be Gaussian, Poissonian, gamma-measure etc.); her research consists in the generalization of results presented in [97]. It is worth noting that the white noise in [108] is not a Lévy white noise, generally speaking (not time-homogeneous). In the investigations of [96, 97, 108], an important role belongs to the so-called *extended Fock space* [84, 20], which naturally arises in the “Meixner analysis” and, in fact, is the interacting Fock space [3, 58]. All mentioned measures satisfy all requirements of the “biorthogonal analysis”; and specific results related to properties of these measures are naturally coordinated with the results of the “biorthogonal analysis”. Moreover, in order to obtain these specific results it is convenient to use a “tooling” that is developed in the “biorthogonal analysis”.

During recent years, an analysis associated with the Gamma, Pascal and Meixner measures and the corresponding white noises (including stochastic integration theory) became the object of investigation of many authors. In particular, in [1], Lévy processes on the Lie algebra $\mathfrak{sl}(2, \mathbb{R})$ were investigated, components of these processes are classical Lévy processes on \mathbb{R} corresponding to Meixner classes; in [4], the stochastic integral was introduced and studied for a wide class of stochastic processes and it was proved in [2] that the results of [4] can be applied in the “Meixner analysis”; in [52, 53], the stochastic integration theory with applications was constructed for Meixner processes and its generalizations; in [113–115], some properties of gamma-processes were studied; in [5], all Meixner classes within a quantum white noise context were considered from a general point of view.

On the other hand, many specialists study a (non-Gaussian, generally speaking) analysis on the so-called Hida (e.g., [32, 34, 56]), Kondratiev (e.g., [7, 8, 12, 24, 67, 68, 88, 90, 91, 112]) and another similar spaces of test and generalized functions (and on the corresponding weighted Fock spaces). Such an analysis includes a stochastic integration theory, a Wick calculus and different applications (including a theory of normally ordered

white noise equations or, in another terminology, of stochastic equations with Wick-type nonlinearities). Thereupon we refer, in particular, to the papers [8, 27–29, 32, 34, 61, 62, 90, 93, 102–106]. One of tasks in these investigations consists in the study of properties of different operators (including stochastic integrals and stochastic derivatives) and operations (e.g., of a Wick multiplication) subject to the particular spaces under consideration. For example, in [32–34], stochastic integrals with respect to a wide class of Lévy processes on Hida spaces are studied and the corresponding Wick calculus is developed; the constructions in these works are based on the so-called power jump processes [101].

So, the problem of development and improvement of the biorthogonal approach to the construction of a non-Gaussian analysis (in particular, the construction and study of stochastic integrals and operators of stochastic differentiation, of a Wick calculus; improvement of the “tooling” etc.), just as the problem of construction and detailed study of nonclassical examples (in particular, of the “Meixner analysis”), are natural and relevant.

In this paper we review some results of the “biorthogonal analysis” and of the analysis associated with the generalized Meixner measure [108]. In particular, from a general point of view, we consider stochastic integrals, stochastic derivatives, elements of a Wick calculus, and related topics. Note that these objects were not considered in details in the existing surveys [22–24]. As for the recent survey [80], the investigation of in that paper only stochastic integrals in a very specific context is presented there.

The present paper is organized in the following manner. In the first section, we deal with the “biorthogonal analysis”. Namely, we introduce so-called generalized Appell-like polynomials (these polynomials form orthogonal bases in test function spaces), construct test function spaces and describe their properties, introduce pseudodifferential operators on the test function spaces and consider some properties of these operators, introduce a probability measure μ that satisfies certain conditions and consider generalized function spaces that are dual spaces of the test function spaces with respect to $L^2(\mu)$, describe natural orthogonal bases in the generalized function spaces; consider elements of a Wick calculus (an S -transform, a Wick multiplication, Wick versions of holomorphic functions) on the generalized function spaces; introduce and study an analog of the extended stochastic integral on the generalized function spaces, in particular, consider the interconnection between the “extended stochastic integration” and the Wick calculus, consider an example of an equation with Wick-type nonlinearities; introduce and study operators of stochastic differentiation on the generalized function spaces.

The second section is devoted to an analysis that is associated with the generalized Meixner measure μ (more exactly, in this section, we observe results that are associated with peculiarities of μ and can not be obtained from the general “biorthogonal theory”, and describe the interconnection between these results and results of the “biorthogonal analysis”). Namely, following [108], we give a definition of the generalized Meixner measure, consider the space $L^2(\mu)$ and construct a natural orthogonal basis in this space; consider a so-called nonregular rigging of $L^2(\mu)$ (this rigging is the main “chain of spaces” in the “biorthogonal analysis”), describe natural orthogonal bases in spaces of this rigging and consider the interconnection between these bases and the bases that are constructed in the framework of the “biorthogonal analysis”, introduce a so-called parametrized regular rigging of $L^2(\mu)$ and describe natural orthogonal bases

in spaces of this rigging, consider elements of a Wick calculus on the parametrized generalized function spaces, introduce and study the extended stochastic integral with respect to the so-called Meixner process on the parametrized generalized function spaces (in particular, on the space of square integrable with respect to μ functions), consider the interconnection between the extended stochastic integration and the Wick calculus, consider an example of an equation with Wick-type nonlinearities, introduce and study stochastic derivatives and operators of stochastic differentiation on the parametrized generalized function spaces.

1. On a biorthogonal approach to construction of a non-Gaussian infinite-dimensional analysis. 1.1. Generalized Appell-like polynomials. Let $\mathbb{R}_+ := [0, +\infty)$, $T := \{\tau = (\tau_1, \tau_2)\}$, where $\tau_1 \in \mathbb{N}$, $\tau_2: \mathbb{R}_+ \rightarrow [1, +\infty)$ are infinite differentiable functions; $\{\mathcal{H}_\tau\}_{\tau \in T}$ are the Sobolev spaces on \mathbb{R}_+ of order τ_1 weighted by τ_2 , i.e., the scalar product in \mathcal{H}_τ has the form

$$(f, g)_{\mathcal{H}_\tau} = \int_{\mathbb{R}_+} \left(f(u)g(u) + \sum_{k=1}^{\tau_1} f^{(k)}(u)g^{(k)}(u) \right) \tau_2(u) du.$$

We consider the chain

$$\mathcal{D}' = \text{ind} \lim_{\tau' \in T} \mathcal{H}_{-\tau'} \supset \mathcal{H}_{-\tau} \supset \mathcal{H} := L^2(\mathbb{R}_+, du) \supset \mathcal{H}_\tau \supset \mathcal{D} := \text{pr} \lim_{\tau' \in T} \mathcal{H}_{\tau'}, \quad (1.1)$$

where $L^2(\mathbb{R}_+, du)$ is the space of real-valued functions on \mathbb{R}_+ square integrable with respect to the Lebesgue measure, $\text{pr} \lim$ and $\text{ind} \lim$ denote projective and inductive limits with the corresponding topologies, respectively (e.g., [17]), $\mathcal{H}_{-\tau}$ and \mathcal{D}' are the spaces dual of \mathcal{H}_τ and \mathcal{D} with respect to the “zero space” \mathcal{H} , respectively. One can show (e.g., as in [25]) that \mathcal{D} is the Schwartz space of infinite-differentiable functions on \mathbb{R}_+ with compact supports.

Denote by the subindex \mathbb{C} complexifications of spaces, by $Hol_0(\mathcal{D}_{\mathbb{C}})$ and $Hol_0(\mathcal{D}_{\mathbb{C}}, \mathcal{D}_{\mathbb{C}})$ algebras of (germs of) functions on $\mathcal{D}_{\mathbb{C}}$ holomorphic for $0 \in \mathcal{D}_{\mathbb{C}}$ with values in \mathbb{C} and $\mathcal{D}_{\mathbb{C}}$ respectively (see, e.g., [35] for more details), by $\widehat{\otimes}$ a symmetric tensor product. Together with chain (1.1), we consider the chains

$$\mathcal{D}'_{\mathbb{C}} \widehat{\otimes}^n = \text{ind} \lim_{\tau' \in T} \mathcal{H}_{-\tau', \mathbb{C}} \widehat{\otimes}^n \supset \mathcal{H}_{-\tau, \mathbb{C}} \widehat{\otimes}^n \supset \mathcal{H}_{\mathbb{C}} \widehat{\otimes}^n \supset \mathcal{H}_{\tau, \mathbb{C}} \widehat{\otimes}^n \supset \mathcal{D}_{\mathbb{C}} \widehat{\otimes}^n := \text{pr} \lim_{\tau' \in T} \mathcal{H}_{\tau', \mathbb{C}} \widehat{\otimes}^n, \quad (1.2)$$

$n \in \mathbb{N}$, $\mathcal{D}'_{\mathbb{C}} \widehat{\otimes}^0 = \mathcal{H}_{-\tau, \mathbb{C}} \widehat{\otimes}^0 = \mathcal{H}_{\mathbb{C}} \widehat{\otimes}^0 = \mathcal{H}_{\tau, \mathbb{C}} \widehat{\otimes}^0 = \mathcal{D}_{\mathbb{C}} \widehat{\otimes}^0 := \mathbb{C}$. Denote the norms in the spaces $\mathcal{H}_{-\tau, \mathbb{C}} \widehat{\otimes}^n$, $\mathcal{H}_{\mathbb{C}} \widehat{\otimes}^n$ and $\mathcal{H}_{\tau, \mathbb{C}} \widehat{\otimes}^n$ by $|\cdot|_{-\tau}$, $|\cdot|_0$ and $|\cdot|_{\tau}$, respectively, the dual pairings between elements of negative and positive spaces of chains (1.2) by $\langle \cdot, \cdot \rangle$; these pairings are generated by the scalar products in $\mathcal{H}_{\mathbb{C}} \widehat{\otimes}^n$.

Assume that $\gamma \in Hol_0(\mathcal{D}_{\mathbb{C}})$, $\gamma(0) = 1$, $\alpha \in Hol_0(\mathcal{D}_{\mathbb{C}}, \mathcal{D}_{\mathbb{C}})$, $\alpha(0) = 0$. Let there exist the function $\alpha^{-1} \in Hol_0(\mathcal{D}_{\mathbb{C}}, \mathcal{D}_{\mathbb{C}})$ inverse to α and let $\chi: \mathbb{C} \rightarrow \mathbb{C}$ be an entire function such that $\chi(0) = 1$ and in the decomposition

$$\chi(u) = \sum_{n=0}^{\infty} \frac{\chi_n}{n!} u^n, \quad (1.3)$$

$\chi_n \neq 0$ for all $n \in \mathbb{Z}_+$. We consider a function $\chi^{\gamma, \alpha}(\lambda; z) := \gamma(\lambda)\chi(\langle z, \alpha(\lambda) \rangle)$. It is easy to see that $\chi^{\gamma, \alpha}(\cdot; z) \in Hol_0(\mathcal{D}_{\mathbb{C}})$ for each $z \in \mathcal{D}'_{\mathbb{C}}$, therefore, using the Cauchy

inequality (e.g., [35]) and the kernel theorem (e.g., [17]), one can show, that for $z \in \mathcal{D}'_{\mathbb{C}}$ and λ from some (depending on z) neighborhood of $0 \in \mathcal{D}_{\mathbb{C}}$, we have

$$\chi^{\gamma, \alpha}(\lambda; z) = \sum_{n=0}^{\infty} \frac{1}{n!} \langle P_n^{\chi, \gamma, \alpha}(z), \lambda^{\otimes n} \rangle,$$

where $P_n^{\chi, \gamma, \alpha}(z) \in \mathcal{D}'_{\mathbb{C}}^{\widehat{\otimes} n}$ (moreover, if $z \in \mathcal{H}_{-\tau, \mathbb{C}}$, then $P_n^{\chi, \gamma, \alpha}(z) \in \mathcal{H}_{-\tau, \mathbb{C}}^{\widehat{\otimes} n}$). The polynomials $\{\langle P_n^{\chi, \gamma, \alpha}(z), f^{(n)} \rangle, f^{(n)} \in \mathcal{D}_{\mathbb{C}}^{\widehat{\otimes} n}, n \in \mathbb{Z}_+\}$ are called *generalized Appell-like polynomials* (or *Schefer polynomials* in another terminology). The interested reader can find more detailed information about generalized Appell-like polynomials in, e.g., [9, 26] (one-dimensional case), [67, 77, 88, 91] (infinite-dimensional case).

1.2. Test function spaces. Let

$$\mathcal{P}(\mathcal{D}') = \left\{ \sum_{n=0}^N \langle x^{\otimes n}, g^{(n)} \rangle : x \in \mathcal{D}', g^{(n)} \in \mathcal{D}_{\mathbb{C}}^{\widehat{\otimes} n}, N \in \mathbb{Z}_+ \right\}$$

be the set of continuous polynomials on \mathcal{D}' . One can show (e.g., [67]) that it is possible to understand $\mathcal{P}(\mathcal{D}')$ as the set of polynomials

$$f(x) = \sum_{n=0}^{N_f} \langle P_n^{\chi, \gamma, \alpha}(x), f^{(n)} \rangle, \quad x \in \mathcal{D}', \quad f^{(n)} \in \mathcal{D}_{\mathbb{C}}^{\widehat{\otimes} n}, \quad N_f \in \mathbb{Z}_+. \quad (1.4)$$

Let us introduce the family of Hilbert norms $\|\cdot\|_{\tau, q, \chi, \gamma, \alpha}$, $\tau \in T$, $q \in \mathbb{Z}_+$, on $\mathcal{P}(\mathcal{D}')$ by setting, for f of form (1.4), the norm

$$\|f\|_{\tau, q, \chi, \gamma, \alpha}^2 := \sum_{n=0}^{N_f} (n!)^2 2^{qn} |f^{(n)}|_{\tau}^2. \quad (1.5)$$

By $(\mathcal{H}_{\tau})_{q, \chi, \gamma, \alpha}$ denote a Hilbert space that is the closure of $\mathcal{P}(\mathcal{D}')$ with respect to norm (1.5). Let also $(\mathcal{H}_{\tau})_{\chi, \gamma, \alpha} := \text{pr} \lim_{q \in \mathbb{Z}_+} (\mathcal{H}_{\tau})_{q, \chi, \gamma, \alpha}$, $(\mathcal{D})_{\chi, \gamma, \alpha} := \text{pr} \lim_{\tau \in T, q \in \mathbb{Z}_+} (\mathcal{H}_{\tau})_{q, \chi, \gamma, \alpha}$. The spaces $(\mathcal{H}_{\tau})_{q, \chi, \gamma, \alpha}$, $(\mathcal{H}_{\tau})_{\chi, \gamma, \alpha}$, $(\mathcal{D})_{\chi, \gamma, \alpha}$ are called *Kondratiev test function spaces*. It is easy to see that $f \in (\mathcal{H}_{\tau})_{q, \chi, \gamma, \alpha}$ if and only if f can be presented in the form

$$f = \sum_{n=0}^{\infty} \langle P_n^{\chi, \gamma, \alpha}, f^{(n)} \rangle, \quad f^{(n)} \in \mathcal{H}_{\tau, \mathbb{C}}^{\widehat{\otimes} n}, \quad (1.6)$$

with

$$\|f\|_{(\mathcal{H}_{\tau})_{q, \chi, \gamma, \alpha}}^2 = \sum_{n=0}^{\infty} (n!)^2 2^{qn} |f^{(n)}|_{\tau}^2 < \infty.$$

The interested reader can find a more detailed information about Kondratiev spaces in, e.g., [7, 12, 22–24, 67, 68, 88, 90, 91, 112]; here, we note only that

1) for $q \in \mathbb{Z}_+$ sufficiently large, $(\mathcal{H}_{\tau})_{q, \chi, \gamma, \alpha}$ are functional spaces, their elements are restrictions on $\mathcal{H}_{-\tau}$ of entire functions on $\mathcal{H}_{-\tau, \mathbb{C}}$, and for each function $f \in (\mathcal{H}_{\tau})_{q, \chi, \gamma, \alpha}$ decomposition (1.6) is unique;

2) the spaces $(\mathcal{H}_{\tau})_{\chi, \gamma, \alpha}$ and $(\mathcal{D})_{\chi, \gamma, \alpha}$ do not depend on γ and α , therefore below we denote these spaces by $(\mathcal{H}_{\tau})_{\chi}$ and $(\mathcal{D})_{\chi}$, respectively.

1.3. Pseudodifferential operators. Let $\nu \in Hol_0(\mathcal{D}_{\mathbb{C}})$. On $\mathcal{P}(\mathcal{D}')$, we define a pseudodifferential operator $\nu(D_{\chi}) := \sum_{n=0}^{\infty} \frac{1}{n!} \langle \nu_n, D_{\chi}^{\otimes n} \rangle$, where $\nu_n \in \mathcal{D}'_{\mathbb{C}}^{\widehat{\otimes} n}$, $n \in \mathbb{Z}_+$, are the kernels from the decomposition $\nu(\lambda) = \sum_{n=0}^{\infty} \frac{1}{n!} \langle \nu_n, \lambda^{\otimes n} \rangle$;

$$\langle \nu_n, D_{\chi}^{\otimes n} \rangle \langle x^{\otimes m}, f^{(m)} \rangle := 1_{\{m \geq n\}} \frac{m! \chi_{m-n}}{(m-n)! \chi_m} \langle x^{\otimes m-n} \widehat{\otimes} \nu_n, f^{(m)} \rangle,$$

where $\chi_n \in \mathbb{C}$ ($n \in \mathbb{Z}_+$) are the coefficients from decomposition (1.3) for χ ; here and below, 1_A is the indicator of an event A . One can show [67, 77] that

1) for each $\nu_n \in \mathcal{D}'_{\mathbb{C}}^{\widehat{\otimes} n}$,

$$\langle \nu_n, \alpha^{-1}(D_{\chi})^{\otimes n} \rangle \langle P_m^{\chi, \gamma, \alpha}(x), f^{(m)} \rangle = 1_{\{m \geq n\}} \frac{m!}{(m-n)!} \langle P_{m-n}^{\chi, \gamma, \alpha}(x) \widehat{\otimes} \nu_n, f^{(m)} \rangle; \tag{1.7}$$

2) $\nu(\alpha^{-1}(D_{\chi}))$ can be continued to a linear isometric operator acting from $(\mathcal{H}_{\tau})_{q, \chi, \gamma, \alpha}$ to $(\mathcal{H}_{\tau})_{q, \chi, \gamma, \nu, \alpha}$: for $f \in (\mathcal{H}_{\tau})_{q, \chi, \gamma, \alpha}$ of form (1.6), we have

$$\nu(\alpha^{-1}(D_{\chi}))f = \sum_{n=0}^{\infty} \langle P_n^{\chi, \gamma, \nu, \alpha}, f^{(n)} \rangle.$$

1.4. Generalized function spaces. Let μ be a probability measure on $(\mathcal{D}', \mathcal{F}(\mathcal{D}'))$, here and below, \mathcal{F} denotes the σ -algebra generated by cylindrical sets. Denote $(L^2)_{\mu} := L^2(\mathcal{D}', \mu)$ the space of functions $F: \mathcal{D}' \rightarrow \mathbb{C}$ square integrable with respect to μ . Let $\|\cdot\|_{\mu}$ be the norm in this space. In addition, we accept that μ satisfies the following assumptions:

- 1) there exists $\tau \in T$ such that $\mu(\mathcal{H}_{-\tau}) = 1$;
- 2) there exists $K > 1$ such that, for each $n \in \mathbb{Z}_+$, $|\chi_n| \|\cdot\|_{-\tau}^n \leq n! K^n$;
- 3) the set $\mathcal{P}(\mathcal{D}')$ of continuous polynomials on \mathcal{D}' is dense in $(L^2)_{\mu}$;
- 4) there exists a nonempty open set $O \subseteq \mathcal{H}_{-\tau}$ such that μ is positive on nonempty open subsets of O .

Let χ, γ, α and μ satisfy all conditions mentioned above. If we exclude from T some indexes (see [73] for more details), then it is possible to show [79] that, for $q \in \mathbb{Z}_+$ sufficiently large, the spaces $(\mathcal{H}_{\tau})_{q, \chi, \gamma, \alpha}$ are embedded in $(L^2)_{\mu}$, and these embeddings are dense and continuous. Moreover, this result holds true if, inconvenient for verification, condition 4 on μ is substituted by the following one:

4') the polynomials $\{\langle P_n^{\chi, \gamma, \alpha}, f^{(n)} \rangle\}$ are orthogonal in $(L^2)_{\mu}$ and if $\|\langle P_n^{\chi, \gamma, \alpha}, f^{(n)} \rangle\|_{\mu} = 0$, then $|f^{(n)}|_{\tau} = 0$.

We note also that if $\chi = \exp$, then condition 2 is equivalent to holomorphy at zero of the Laplace transform of μ (e.g., [91]), and it follows from this holomorphy that condition 3 is satisfied (e.g., [109]).

We can now consider the chain

$$(\mathcal{D}')_{\chi, \mu} \supset (\mathcal{H}_{-\tau})_{\chi, \mu} \supset (\mathcal{H}_{-\tau})_{-q, \chi, \gamma, \alpha, \mu} \supset (L^2)_{\mu} \supset (\mathcal{H}_{\tau})_{q, \chi, \gamma, \alpha} \supset (\mathcal{H}_{\tau})_{\chi} \supset (\mathcal{D})_{\chi}, \tag{1.8}$$

where $(\mathcal{H}_{-\tau})_{-q, \chi, \gamma, \alpha, \mu}$, $(\mathcal{H}_{-\tau})_{\chi, \mu}$ and $(\mathcal{D}')_{\chi, \mu}$ are the spaces dual of $(\mathcal{H}_{\tau})_{q, \chi, \gamma, \alpha}$, $(\mathcal{H}_{\tau})_{\chi}$ and $(\mathcal{D})_{\chi}$ with respect to $(L^2)_{\mu}$, correspondingly. It will be convenient to understand $(\mathcal{H}_{-\tau})_{\chi, \mu}$ and $(\mathcal{D}')_{\chi, \mu}$ as topological spaces with the inductive limit topologies. The negative spaces of chain (1.8) are called *Kondratiev generalized functions spaces*.

Let us describe natural orthogonal bases in the spaces $(\mathcal{H}_{-\tau})_{-q,\chi,\gamma,\alpha,\mu}$. Let $\langle\langle \cdot, \cdot \rangle\rangle_\mu$ denote the scalar product in $(L^2)_\mu$ (we preserve this notation for dual pairings generated by this scalar product); let

$$\tilde{\gamma}(\lambda) := \frac{1}{\langle\langle 1, \chi(\langle \cdot, \alpha(\lambda) \rangle) \rangle\rangle_\mu}. \quad (1.9)$$

One can show [67] that $\tilde{\gamma} \in Hol_0(\mathcal{D}_\mathbb{C})$, therefore $\frac{\tilde{\gamma}}{\gamma} \in Hol_0(\mathcal{D}_\mathbb{C})$ and one can consider the decomposition

$$\frac{\tilde{\gamma}(\lambda)}{\gamma(\lambda)} = \sum_{n=0}^{\infty} \frac{1}{n!} \langle \rho_n, \lambda^{\otimes n} \rangle, \quad \rho_n \in \mathcal{H}_{-\tau, \mathbb{C}}^{\widehat{\otimes} n}.$$

We set

$$Q_{\mu, m}^{\chi, \gamma, \alpha}(F^{(m)}; \cdot) := \sum_{k=m}^{\infty} \frac{1}{(k-m)!} \left(\langle F^{(m)} \widehat{\otimes} \rho_{k-m}, \alpha^{-1}(D_\chi)^{\otimes k} \rangle^* 1 \right) (\cdot), \quad F^{(m)} \in \mathcal{H}_{-\tau, \mathbb{C}}^{\widehat{\otimes} m},$$

where $\langle F^{(m)} \widehat{\otimes} \rho_{k-m}, \alpha^{-1}(D_\chi)^{\otimes k} \rangle^*$ is the operator adjoint to pseudodifferential operator (1.7). It follows from results of [67] that the system of generalized functions $\{Q_{\mu, m}^{\chi, \gamma, \alpha}(F^{(m)}), F^{(m)} \in \mathcal{H}_{-\tau, \mathbb{C}}^{\widehat{\otimes} m}, m \in \mathbb{Z}_+\}$ form orthogonal bases in $(\mathcal{H}_{-\tau})_{-q,\chi,\gamma,\alpha,\mu}$: $F \in (\mathcal{H}_{-\tau})_{-q,\chi,\gamma,\alpha,\mu}$ if and only if there exists a sequence of kernels $\{F^{(m)} \in \mathcal{H}_{-\tau, \mathbb{C}}^{\widehat{\otimes} m}\}_{m=0}^{\infty}$ such that

$$F = \sum_{m=0}^{\infty} Q_{\mu, m}^{\chi, \gamma, \alpha}(F^{(m)}) \quad (1.10)$$

with

$$\|F\|_{-\tau, -q, \chi, \gamma, \alpha}^2 := \|F\|_{(\mathcal{H}_{-\tau})_{-q, \chi, \gamma, \alpha, \mu}}^2 = \sum_{m=0}^{\infty} 2^{-qm} |F^{(m)}|_{-\tau}^2 < \infty. \quad (1.11)$$

Moreover, the biorthogonality relation

$$\langle\langle Q_{\mu, m}^{\chi, \gamma, \alpha}(F^{(m)}), \langle P_n^{\chi, \gamma, \alpha}, f^{(n)} \rangle \rangle\rangle_\mu = \delta_{mn} n! \langle F^{(n)}, f^{(n)} \rangle$$

is satisfied, here, δ_{mn} is the Kroneker symbol.

Another (based on properties of Fock spaces) approach to the construction of orthogonal bases in generalized function spaces is described in [73].

1.5. Elements of a Wick calculus. Now we consider elements of a so-called Wick calculus on the generalized function spaces. For $F \in (\mathcal{D})'_{\chi, \mu}$, we introduce a so-called S -transform by setting

$$(S_{\chi, \gamma, \alpha, \mu} F)(\lambda) := \langle\langle F, \chi^{\gamma, \alpha}(\lambda; \cdot) \rangle\rangle_\mu = \sum_{m=0}^{\infty} \langle F^{(m)}, \lambda^{\otimes m} \rangle, \quad (1.12)$$

where $F^{(m)} \in \mathcal{D}'_{\mathbb{C}}^{\widehat{\otimes} m}$, $m \in \mathbb{Z}_+$, are the kernels from decomposition (1.10) for F ; $\lambda \in \mathcal{D}_\mathbb{C}$. It is easy to show that the series in the right-hand side of (1.12) converges on

some (depending on F) neighborhood of $0 \in \mathcal{D}_{\mathbb{C}}$. Further, one can show [67] that the S -transform is a one-to-one mapping between $(\mathcal{D})'_{\chi, \mu}$ and $Hol_0(\mathcal{D}_{\mathbb{C}})$.

For $F, G \in (\mathcal{D})'_{\chi, \mu}$ and a function $h: \mathbb{C} \rightarrow \mathbb{C}$, holomorphic for $(S_{\chi, \gamma, \alpha, \mu} F)(0) = F^{(0)}$ (see (1.10)) we define a Wick product $F \diamond_{\mu}^{\chi, \gamma, \alpha} G \in (\mathcal{D})'_{\chi, \mu}$ and a Wick version $h^{\diamond_{\mu}^{\chi, \gamma, \alpha}}(F) \in (\mathcal{D})'_{\chi, \mu}$ by setting

$$F \diamond_{\mu}^{\chi, \gamma, \alpha} G := S_{\chi, \gamma, \alpha, \mu}^{-1} (S_{\chi, \gamma, \alpha, \mu} F \cdot S_{\chi, \gamma, \alpha, \mu} G),$$

$$h^{\diamond_{\mu}^{\chi, \gamma, \alpha}}(F) := S_{\chi, \gamma, \alpha, \mu}^{-1} h(S_{\chi, \gamma, \alpha, \mu} F).$$

Note that if $\gamma(\lambda)$ has form (1.9), then $S_{\chi, \gamma, \alpha, \mu} 1 \equiv 1$, and therefore,

$$h^{\diamond_{\mu}^{\chi, \gamma, \alpha}}(F) = \sum_{n=0}^{\infty} h_n (F - F^{(0)})^{\diamond_{\mu}^{\chi, \gamma, \alpha} n},$$

where $F^{\diamond_{\mu}^{\chi, \gamma, \alpha} n} := \underbrace{F \diamond_{\mu}^{\chi, \gamma, \alpha} \dots \diamond_{\mu}^{\chi, \gamma, \alpha} F}_n = S_{\chi, \gamma, \alpha, \mu}^{-1} [(S_{\chi, \gamma, \alpha, \mu} F)^n]$, $F^{\diamond_{\mu}^{\chi, \gamma, \alpha} 0} := 1$; and

the coefficients h_n from the decomposition

$$h(u) = \sum_{n=0}^{\infty} h_n (u - F^{(0)})^n \tag{1.13}$$

belong to \mathbb{C} .

Let us consider the “coordinate form” of the Wick product and Wick versions of holomorphic functions. It is easy to calculate with the use of (1.12) that, for $F, G \in (\mathcal{D})'_{\chi, \mu}$,

$$F \diamond_{\mu}^{\chi, \gamma, \alpha} G = \sum_{m=0}^{\infty} Q_{\mu, m}^{\chi, \gamma, \alpha} \left(\sum_{k=0}^m F^{(k)} \widehat{\otimes} G^{(m-k)} \right), \tag{1.14}$$

where $F^{(k)}, G^{(k)} \in \mathcal{D}'_{\mathbb{C}} \widehat{\otimes} k$ are the kernels from decompositions (1.10) for F and G . If we apply the induction to this formula, then we obtain

$$F_1 \diamond_{\mu}^{\chi, \gamma, \alpha} \dots \diamond_{\mu}^{\chi, \gamma, \alpha} F_n = \sum_{m=0}^{\infty} Q_{\mu, m}^{\chi, \gamma, \alpha} \left(\sum_{\substack{k_1, \dots, k_n \in \mathbb{Z}_+, \\ k_1 + \dots + k_n = m}} F_1^{(k_1)} \widehat{\otimes} \dots \widehat{\otimes} F_n^{(k_n)} \right), \tag{1.15}$$

where $F_1, \dots, F_n \in (\mathcal{D})'_{\chi, \mu}$, $F_1^{(k)}, \dots, F_n^{(k)} \in \mathcal{D}'_{\mathbb{C}} \widehat{\otimes} k$ are the kernels from decompositions (1.10) for F_1, \dots, F_n , correspondingly. Further, substituting (1.12) in (1.13) and applying $S_{\chi, \gamma, \alpha, \mu}^{-1}$, we obtain

$$h^{\diamond_{\mu}^{\chi, \gamma, \alpha}}(F) = Q_{\mu, 0}^{\chi, \gamma, \alpha}(h_0) + \sum_{m=1}^{\infty} Q_{\mu, m}^{\chi, \gamma, \alpha} \left(\sum_{n=1}^m h_n \sum_{\substack{k_1, \dots, k_n \in \mathbb{N}, \\ k_1 + \dots + k_n = m}} F^{(k_1)} \widehat{\otimes} \dots \widehat{\otimes} F^{(k_n)} \right). \tag{1.16}$$

In particular, note that,

$$Q_{\mu, n}^{\chi, \gamma, \alpha}(F^{(n)}) \diamond_{\mu}^{\chi, \gamma, \alpha} Q_{\mu, m}^{\chi, \gamma, \alpha}(G^{(m)}) = Q_{\mu, n+m}^{\chi, \gamma, \alpha}(F^{(n)} \widehat{\otimes} G^{(m)}).$$

Consider the space $(\mathcal{H}_{-\tau})_{\chi, \mu} \subset (\mathcal{D})'_{\chi, \mu}$ instead of $(\mathcal{D})'_{\chi, \mu}$. The following statement [70] is true.

Theorem 1.1. Let $F, G \in (\mathcal{H}_{-\tau})_{\chi, \mu}$ and $h: \mathbb{C} \rightarrow \mathbb{C}$ be a function holomorphic at $(S_{\chi, \gamma, \alpha, \mu} F)(0)$. Then $F \diamond_{\mu}^{\chi, \gamma, \alpha} G \in (\mathcal{H}_{-\tau})_{\chi, \mu}$, $h \diamond_{\mu}^{\chi, \gamma, \alpha} (F) \in (\mathcal{H}_{-\tau})_{\chi, \mu}$, and coordinate formulas (1.14)–(1.16) hold true. Moreover, the Wick product is continuous in the topology of $(\mathcal{H}_{-\tau})_{\chi, \mu}$: for any $F_1, \dots, F_n \in (\mathcal{H}_{-\tau})_{\chi, \mu}$,

$$\begin{aligned} & \|F_1 \diamond_{\mu}^{\chi, \gamma, \alpha} \dots \diamond_{\mu}^{\chi, \gamma, \alpha} F_n\|_{-\tau, -q, \chi, \gamma, \alpha} \leq \\ & \leq c(n-1) \|F_1\|_{-\tau, -(q-1), \chi, \gamma, \alpha} \dots \|F_n\|_{-\tau, -(q-1), \chi, \gamma, \alpha} \end{aligned}$$

(see (1.11)), where $c(n) := \sqrt{\max_{m \in \mathbb{Z}_+} [2^{-m} (m+1)^n]}$; $q \in \mathbb{N}$ is such that $F_1, \dots, F_n \in (\mathcal{H}_{-\tau})_{-(q-1), \chi, \gamma, \alpha, \mu}$.

1.6. An analog of the extended stochastic integral. Let us consider now an analog of the extended (Skorohod-type, see [57, 63, 64, 110]) stochastic integral in the “bi-orthogonal analysis” (the reader can find a more detailed presentation in [74], see also [73]). In order to explain the idea of our construction, first we briefly recall the construction of the stochastic integral in the Gaussian analysis.

Let μ be the standard Gaussian measure on $(\mathcal{D}', \mathcal{F}(\mathcal{D}'))$, i.e., a probability measure with the Laplace transform

$$l_{\mu}(\lambda) = \int_{\mathcal{D}'} e^{\langle x, \lambda \rangle} \mu(dx) = e^{\langle \lambda, \lambda \rangle / 2}.$$

As is well known (e.g., [54]), one can consider as an orthogonal basis in $(L^2)_{\mu}$ ($(L^2)_{\mu}$ -limits of) the generalized Hermite polynomials $\langle H_n, F^{(n)} \rangle$; these polynomials are generalized Appell-like polynomials with $\chi = \exp$, $\gamma(\lambda) = \exp \left\{ -\frac{1}{2} \langle \lambda, \lambda \rangle \right\}$, $\alpha(\lambda) = \lambda$.

Let now $F \in (L^2)_{\mu} \otimes \mathcal{H}_{\mathbb{C}}$. Then F can be presented in the form

$$F(\cdot) = \sum_{n=0}^{\infty} \langle H_n, F^{(n)} \rangle, \quad F^{(n)} \in \mathcal{H}_{\mathbb{C}}^{\widehat{\otimes} n} \otimes \mathcal{H}_{\mathbb{C}};$$

and if, in addition, F is integrable in the extended (Skorohod) sense, i.e.,

$$\sum_{n=0}^{\infty} (n+1)! |\widehat{F}^{(n)}|_0^2 < \infty,$$

where, for each $n \in \mathbb{Z}_+$, $\widehat{F}^{(n)} \in \mathcal{H}_{\mathbb{C}}^{\widehat{\otimes} n+1}$ is the projection of $F^{(n)}$ onto $\mathcal{H}_{\mathbb{C}}^{\widehat{\otimes} n+1}$, then the extended stochastic integral of F with respect to a Wiener process W has the form (e.g., [56])

$$\int_0^{\infty} F(u) d\widehat{W}_u = \sum_{n=0}^{\infty} \langle H_{n+1}, \widehat{F}^{(n)} \rangle. \quad (1.17)$$

Remark. The described construction of the extended stochastic integral is not unique. For example, it was shown in [31] that (in the Gaussian case) the extended stochastic integral can be interpreted as the logarithmic derivative of the Gaussian measure in the line of a vector field. By analogy, i.e., using an integration by parts formula, one can

construct stochastic integrals in non-Gaussian cases, see, e.g., [10, 99, 100, 107]. Note that, for groups of nonlinear transformations, the definitions of the logarithmic derivative and of the extended stochastic integral were given in [111]. A very general construction of (the analog of) the extended stochastic integral (that is based on a formal differential rule and on the corresponding integration by parts formula) is presented in [47], see also [36–46, 48–50].

Return now to the “biorthogonal analysis”. Let $F \in (\mathcal{D})'_{\chi,\mu} \otimes \mathcal{H}_{\mathbb{C}}$. Then

$$F(\cdot) = \sum_{m=0}^{\infty} Q_{\mu,m}^{\chi,\gamma,\alpha}(F^{(m)}), \quad F^{(m)} \in \mathcal{D}'_{\mathbb{C}}^{\widehat{\otimes} m} \otimes \mathcal{H}_{\mathbb{C}}.$$

By analogy with (1.17), for any $t_1, t_2 \in [0, +\infty]$, $t_1 < t_2$, one can define an analog of the extended stochastic integral by setting

$$\mathbf{I}_{[t_1,t_2]}^{\chi,\gamma,\alpha,\mu}(F) := \sum_{m=0}^{\infty} Q_{\mu,m+1}^{\chi,\gamma,\alpha}(\widehat{F}_{[t_1,t_2]}^{(m)}) \in (\mathcal{D})'_{\chi,\mu},$$

where $\widehat{F}_{[t_1,t_2]}^{(m)} \in \mathcal{D}'_{\mathbb{C}}^{\widehat{\otimes} m+1}$ ($m \in \mathbb{Z}_+$) are the projections of $F^{(m)} \mathbf{1}_{[t_1,t_2]}(\cdot)$ onto $\mathcal{D}'_{\mathbb{C}}^{\widehat{\otimes} m+1}$. It is easy to show [74] that, for all $t_1, t_2 \in [0, +\infty]$, $t_1 < t_2$, $\mathbf{I}_{[t_1,t_2]}^{\chi,\gamma,\alpha,\mu} : (\mathcal{D})'_{\chi,\mu} \otimes \mathcal{H}_{\mathbb{C}} \rightarrow (\mathcal{D})'_{\chi,\mu}$ is a linear continuous operator.

The forthcoming statement follows from results of [74].

Theorem. For all $t_1, t_2 \in [0, +\infty]$, $t_1 < t_2$, and $F \in (\mathcal{D})'_{\chi,\mu} \otimes \mathcal{H}_{\mathbb{C}}$, the relation

$$\mathbf{I}_{[t_1,t_2]}^{\chi,\gamma,\alpha,\mu}(F) = \int_{t_1}^{t_2} F(u) \diamond_{\mu}^{\chi,\gamma,\alpha} Q_{\mu,1}^{\chi,\gamma,\alpha}(\delta_u) du$$

is satisfied, where δ_u is the Dirac delta-function, and the integral on the right-hand side is a Pettis integral.

By analogy with the classical Gaussian analysis, as an example, we consider a simple “integral equation with a Wick-type nonlinearity”. Let

$$X_t = X_0 + r \int_0^t X_u \diamond_{\mu}^{\chi,\gamma,\alpha}(N - X_u) du + v \mathbf{I}_{[0,t]}^{\chi,\gamma,\alpha,\mu}(X \cdot \diamond_{\mu}^{\chi,\gamma,\alpha}(N - X)), \quad (1.18)$$

where $X_0 \in (\mathcal{D})'_{\chi,\mu}$ (correspondingly $(\mathcal{H}_{-\tau})_{\chi,\mu}$), $N, r, v \in \mathbb{R}$, $N > 0$, $r > 0$, $(S_{\chi,\gamma,\alpha,\mu} X_0)(0) > 0$. Note that, in the Gaussian analysis, (1.18) is the so-called “population growth equation” (see, e.g., [94]). Applying to (1.18) the S -transform, solving the obtained equation and applying the inverse S -transform, we obtain the solution

$$X_t = N \left[S_{\chi,\gamma,\alpha,\mu}^{-1} \mathbf{1} + \left(N X_0^{\diamond_{\mu}^{\chi,\gamma,\alpha}(-1)} - S_{\chi,\gamma,\alpha,\mu}^{-1} \mathbf{1} \right) \right]$$

$$\diamond_{\mu}^{\chi,\gamma,\alpha} \exp^{\diamond_{\mu}^{\chi,\gamma,\alpha}} \left\{ -N \left(r t S_{\chi,\gamma,\alpha,\mu}^{-1} \mathbf{1} + v Q_{\mu,1}^{\chi,\gamma,\alpha}(\mathbf{1}_{[0,t]}) \right) \right\}^{\diamond_{\mu}^{\chi,\gamma,\alpha}(-1)} \in (\mathcal{D})'_{\chi,\mu}$$

(correspondingly $(\mathcal{H}_{-\tau})_{\chi,\mu}$), here $Y^{\diamond_{\mu}^{\chi,\gamma,\alpha}(-1)} := S_{\chi,\gamma,\alpha,\mu}^{-1} \left(\frac{1}{S_{\chi,\gamma,\alpha,\mu} Y} \right)$.

As is well known, in the Gaussian analysis, the extended stochastic integral is the adjoint operator to the stochastic derivative. In the “biorthogonal analysis”, the situation is quite similar; let us explain this in details. We define a *stochastic derivative* $\partial^{\chi,\alpha} : (\mathcal{D})_{\chi} \rightarrow (\mathcal{D})_{\chi} \otimes \mathcal{H}_{\mathbb{C}}$ by setting, for $f \in (\mathcal{D})_{\chi}$,

$$\partial^{\chi,\alpha} f := \langle \delta, \alpha^{-1}(D_{\chi}) \rangle f = \sum_{n=0}^{\infty} (n+1) \langle P_n^{\chi,\gamma,\alpha}, f^{(n+1)}(\cdot) \rangle$$

(see (1.7)), where $f^{(n+1)}(\cdot) \in \mathcal{D}_{\mathbb{C}}^{\widehat{\otimes} n} \otimes \mathcal{H}_{\mathbb{C}}$ ($n \in \mathbb{Z}_+$) are the kernels $f^{(n+1)} \in \mathcal{D}_{\mathbb{C}}^{\widehat{\otimes} n+1} \subset \mathcal{D}_{\mathbb{C}}^{\widehat{\otimes} n} \otimes \mathcal{H}_{\mathbb{C}}$ from decomposition (1.6) for f (these kernels can be considered as elements of $\mathcal{D}_{\mathbb{C}}^{\widehat{\otimes} n} \otimes \mathcal{H}_{\mathbb{C}}$). By using the results of [74, 73], we obtain the following statements.

Theorem 1.2. *For all $t_1, t_2 \in [0, +\infty]$, $t_1 < t_2$, the operators $\mathbf{I}_{[t_1, t_2]}^{\chi,\gamma,\alpha,\mu} : (\mathcal{D})'_{\chi,\mu} \otimes \mathcal{H}_{\mathbb{C}} \rightarrow (\mathcal{D})'_{\chi,\mu}$ and $1_{[t_1, t_2]}(\cdot) \partial^{\chi,\alpha} : (\mathcal{D})_{\chi} \rightarrow (\mathcal{D})_{\chi} \otimes \mathcal{H}_{\mathbb{C}}$ are adjoint one to another:*

$$\left(\mathbf{I}_{[t_1, t_2]}^{\chi,\gamma,\alpha,\mu} \right)^* = 1_{[t_1, t_2]}(\cdot) \partial^{\chi,\alpha}; \quad \left(1_{[t_1, t_2]}(\cdot) \partial^{\chi,\alpha} \right)^* = \mathbf{I}_{[t_1, t_2]}^{\chi,\gamma,\alpha,\mu}.$$

In particular, the operator $\mathbf{I}_{[t_1, t_2]}^{\chi,\gamma,\alpha,\mu}$ does not depend on γ and can be denoted by $\mathbf{I}_{[t_1, t_2]}^{\chi,\alpha,\mu}$.

Corollary 1.1. *The operator $\mathbf{I}_{[t_1, t_2]}^{\chi,\alpha,\mu}$ can be written in the form*

$$\mathbf{I}_{[t_1, t_2]}^{\chi,\alpha,\mu}(F) = \int_{t_1}^{t_2} (\partial_u^{\chi,\alpha})^{\dagger} F(u) du,$$

where $(\partial_u^{\chi,\alpha})^{\dagger} : (\mathcal{D})'_{\chi,\mu} \rightarrow (\mathcal{D})'_{\chi,\mu}$ is the operator adjoint to $\partial_u^{\chi,\alpha}$, $u \in \mathbb{R}_+$; the integral on the right-hand side is a Pettis integral.

Similar results are true if we consider operators $\mathbf{I}_{[t_1, t_2]}^{\chi,\alpha,\mu}$ and $\partial^{\chi,\alpha}$ on “pre-limit” spaces.

1.7. Operators of stochastic differentiation. Unfortunately, the operator $\partial^{\chi,\alpha}$, generally speaking, cannot be continued on the generalized function spaces by a natural way. Nevertheless, one can define and study a natural analog of this operator; the reader can find a more detailed information in [72].

Let us now consider operators of stochastic differentiation on the generalized function spaces. Let $F^{(m)} \in \mathcal{D}'_{\mathbb{C}}^{\widehat{\otimes} m}$, $f^{(n)} \in \mathcal{D}_{\mathbb{C}}^{\widehat{\otimes} n}$, $m > n$. We define a “pairing” $\langle F^{(m)}, f^{(n)} \rangle \in \mathcal{D}'_{\mathbb{C}}^{\widehat{\otimes} m-n}$ by setting, for each $g^{(m-n)} \in \mathcal{D}_{\mathbb{C}}^{\widehat{\otimes} m-n}$, the relation

$$\langle \langle F^{(m)}, f^{(n)} \rangle, g^{(m-n)} \rangle = \langle F^{(m)}, f^{(n)} \widehat{\otimes} g^{(m-n)} \rangle.$$

Further, for arbitrary $f^{(n)} \in \mathcal{D}_{\mathbb{C}}^{\widehat{\otimes} n}$, $n \in \mathbb{Z}_+$, we define a linear operator $(\widetilde{\mathbb{D}}_{\chi,\gamma,\alpha,\mu}^n \circ)(f^{(n)})$ in $(\mathcal{D})'_{\chi,\mu}$ by setting, for $F \in (\mathcal{D})'_{\chi,\mu}$, the relation

$$(\widetilde{\mathbb{D}}_{\chi,\gamma,\alpha,\mu}^n F)(f^{(n)}) := \sum_{m=0}^{\infty} \frac{(m+n)!}{m!} Q_{\mu,m}^{\chi,\gamma,\alpha} \left(\langle F^{(m+n)}, f^{(n)} \rangle \right),$$

where $F^{(m)} \in \mathcal{D}'_{\mathbb{C}}^{\widehat{\otimes} m}$, $m \in \mathbb{Z}_+$, are the kernels from decomposition (1.10) for F . It follows from results of [72] that $(\widetilde{\mathbb{D}}_{\chi,\gamma,\alpha,\mu}^n \circ)(f^{(n)})$ is a *continuous* operator in $(\mathcal{D})'_{\chi,\mu}$, and the following statement is true.

Theorem 1.3. *The operator $\tilde{\mathbb{D}}_{\chi,\gamma,\alpha,\mu}^n$ has the following properties:
for $g_1^{(1)}, \dots, g_n^{(1)} \in \mathcal{D}_{\mathbb{C}}$ and $F \in (\mathcal{D})'_{\chi,\mu}$,*

$$\underbrace{\left(\tilde{\mathbb{D}}_{\chi,\gamma,\alpha,\mu}^1 \left(\dots \left(\tilde{\mathbb{D}}_{\chi,\gamma,\alpha,\mu}^1 \left(\tilde{\mathbb{D}}_{\chi,\gamma,\alpha,\mu}^1 (F)(g_1^{(1)}) \right) \right) (g_2^{(1)}) \dots \right) \right) (g_n^{(1)})}_{n} = \left(\tilde{\mathbb{D}}_{\chi,\gamma,\alpha,\mu}^n F \right) \left(g_1^{(1)} \hat{\otimes} \dots \hat{\otimes} g_n^{(1)} \right);$$

for each $F \in (\mathcal{D})'_{\chi,\mu}$, the kernels $F^{(m)} \in \mathcal{D}'_{\mathbb{C}}^{\hat{\otimes} m}$ from decomposition (1.10) can be presented in the form

$$F^{(m)} = \frac{1}{m!} \mathbf{E}(\tilde{\mathbb{D}}_{\chi,\gamma,\alpha,\mu}^m F),$$

i.e., for each $f^{(m)} \in \mathcal{D}_{\mathbb{C}}^{\hat{\otimes} m}$, we have $\langle F^{(m)}, f^{(m)} \rangle = \frac{1}{m!} \mathbf{E}[(\tilde{\mathbb{D}}_{\chi,\gamma,\alpha,\mu}^m F)(f^{(m)})]$, where \mathbf{E} denotes the expectation: $\mathbf{E} \circ = \langle \circ, 1 \rangle_{\mu}$;

the operator adjoint to $\tilde{\mathbb{D}}_{\chi,\gamma,\alpha,\mu}^n$ has a form

$$(\tilde{\mathbb{D}}_{\chi,\gamma,\alpha,\mu}^n g)(f^{(n)})^* = \sum_{k=0}^{\infty} \left\langle P_{k+n}^{\chi,\gamma,\alpha}, f^{(n)} \hat{\otimes} g^{(k)} \right\rangle, \quad f^{(n)} \in \mathcal{D}_{\mathbb{C}}^{\hat{\otimes} n},$$

where $g^{(k)} \in \mathcal{D}_{\mathbb{C}}^{\hat{\otimes} k}$, $k \in \mathbb{Z}_+$, are the kernels from decomposition (1.4) for $g \in (\mathcal{D})_{\chi}$;

the operator $\tilde{\mathbb{D}}_{\chi,\gamma,\alpha,\mu}^1$ is a pre-image of the directional derivative of $S_{\chi,\gamma,\alpha,\mu} \circ$ under the S -transform, i.e., for any $F \in (\mathcal{D})'_{\chi,\mu}$ and $g \in \mathcal{D}_{\mathbb{C}}$,

$$(\tilde{\mathbb{D}}_{\chi,\gamma,\alpha,\mu}^1 F)(g) = S_{\chi,\gamma,\alpha,\mu}^{-1} D_g(S_{\chi,\gamma,\alpha,\mu} F),$$

where D_g is the directional derivative in the direction g ;

the operator $\tilde{\mathbb{D}}_{\chi,\gamma,\alpha,\mu}^1$ is a differentiation with respect to the Wick product, i.e., for all $F, G \in (\mathcal{D})'_{\chi,\mu}$,

$$\tilde{\mathbb{D}}_{\chi,\gamma,\alpha,\mu}^1 (F \diamond_{\mu}^{\chi,\gamma,\alpha} G) = (\tilde{\mathbb{D}}_{\chi,\gamma,\alpha,\mu}^1 F) \diamond_{\mu}^{\chi,\gamma,\alpha} G + F \diamond_{\mu}^{\chi,\gamma,\alpha} (\tilde{\mathbb{D}}_{\chi,\gamma,\alpha,\mu}^1 G);$$

for any $n \in \mathbb{Z}_+$, $F \in (\mathcal{D})'_{\chi,\mu}$, and a function $h: \mathbb{C} \rightarrow \mathbb{C}$ holomorphic for $(S_{\chi,\gamma,\alpha,\mu} F)(0)$ the relations

$$\tilde{\mathbb{D}}_{\chi,\gamma,\alpha,\mu}^1 F \diamond_{\mu}^{\chi,\gamma,\alpha n} = n F \diamond_{\mu}^{\chi,\gamma,\alpha n-1} \diamond_{\mu}^{\chi,\gamma,\alpha} (\tilde{\mathbb{D}}_{\chi,\gamma,\alpha,\mu}^1 F),$$

$$\tilde{\mathbb{D}}_{\chi,\gamma,\alpha,\mu}^1 h \diamond_{\mu}^{\chi,\gamma,\alpha} (F) = h' \diamond_{\mu}^{\chi,\gamma,\alpha} (F) \diamond_{\mu}^{\chi,\gamma,\alpha} (\tilde{\mathbb{D}}_{\chi,\gamma,\alpha,\mu}^1 F)$$

are satisfied; here, h' is the usual derivative of h ;

for all $t_1, t_2 \in [0, +\infty]$, $t_1 < t_2$, and $F \in (\mathcal{D})'_{\chi,\mu} \otimes \mathcal{H}_{\mathbb{C}}$,

$$(\tilde{\mathbb{D}}_{\chi,\gamma,\alpha,\mu}^1 \mathbf{I}_{[t_1,t_2]}^{\chi,\alpha,\mu} (F))(\circ) = \mathbf{I}_{[t_1,t_2]}^{\chi,\alpha,\mu} ((\tilde{\mathbb{D}}_{\chi,\gamma,\alpha,\mu}^1 F)(\circ)) + \int_{t_1}^{t_2} F(u) \circ (u) du,$$

where the last integral on the right-hand side is a Pettis integral.

An information about possible applications of the operators $\widetilde{\mathbb{D}}_{\chi, \gamma, \alpha, \mu}^n$ and the corresponding examples can be found in, e.g., [11, 72].

2. Elements of the white noise analysis associated with the generalized Meixner measure. *2.1. The generalized Meixner measure.* First, we define the generalized Meixner measure (see [108] for more details and explanations). Let $\rho, \nu: \mathbb{R}_+ \rightarrow \mathbb{C}$ be smooth functions such that

$$\theta \stackrel{\text{df}}{=} \rho - \nu: \mathbb{R}_+ \rightarrow \mathbb{R}, \quad \eta \stackrel{\text{df}}{=} \rho\nu: \mathbb{R}_+ \rightarrow \mathbb{R}_+ \quad (2.1)$$

and, moreover, θ and η are bounded on \mathbb{R}_+ . Further, let, for each $u \in \mathbb{R}_+$, $v_{\rho(u), \nu(u)}(ds)$ be a probability measure on $(\mathbb{R}, \mathcal{B}(\mathbb{R}))$ (here \mathcal{B} denotes the Borel σ -algebra) that is defined by its Fourier transform

$$\int_{\mathbb{R}} e^{i\lambda s} v_{\rho(u), \nu(u)}(ds) = \exp \left\{ -i\lambda(\rho(u) + \nu(u)) + \right. \\ \left. + 2 \sum_{m=2}^{\infty} \frac{(\rho(u)\nu(u))^m}{m} \left[\sum_{n=2}^{\infty} \frac{(-i\lambda)^n}{n!} (\nu^{n-2}(u) + \nu^{n-3}(u)\rho(u) + \dots + \rho^{n-2}(u)) \right]^m \right\}.$$

A probability measure μ on the measurable space $(\mathcal{D}', \mathcal{F}(\mathcal{D}'))$ with the Fourier transform

$$\int_{\mathcal{D}'} e^{i\langle x, \xi \rangle} \mu(dx) = \exp \left\{ \int_{\mathbb{R}_+} du \int_{\mathbb{R}} v_{\rho(u), \nu(u)}(ds) \frac{1}{s^2} \left(e^{is\xi(u)} - 1 - is\xi(u) \right) \right\}$$

is called the *generalized Meixner measure*.

Depending on parameters ρ and ν , μ can be, in particular, the Gaussian, Poissonian, Pascal, Meixner or gamma measure.

It was proved in [108] that the generalized Meixner measure μ is the measure of a generalized random process (in the sense of [51]) with independent values; and the Laplace transform l_μ of μ is a holomorphic for zero function.

2.2. The square integrable function space. Let $(L^2) = L^2(\mathcal{D}', \mu)$ be the space of complex-valued square integrable with respect to the generalized Meixner measure μ functions on \mathcal{D}' . Denote by $\langle \langle \cdot, \cdot \rangle \rangle$ the scalar product in (L^2) , this notation will be preserved for generated by $\langle \langle \cdot, \cdot \rangle \rangle$ dual pairings. We construct now a natural orthogonal basis in (L^2) . For $n \in \mathbb{N}$ denote by $\overline{\mathcal{P}}_n$ the closure in (L^2) of the set of all continuous polynomials on \mathcal{D}' of degree $\leq n$, $\overline{\mathcal{P}}_0 := \mathbb{C}$, let also $(L_n^2) := \overline{\mathcal{P}}_n \ominus \overline{\mathcal{P}}_{n-1}$ —the orthogonal difference in (L^2) , $(L_0^2) := \mathbb{C}$. Since μ has a holomorphic at zero Laplace transform, the set of continuous polynomials on \mathcal{D}' is dense in (L^2) [109], therefore $(L^2) = \bigoplus_{n=0}^{\infty} (L_n^2)$. For each $f^{(n)} \in \mathcal{D}_{\mathbb{C}}^{\otimes n}$, $n \in \mathbb{Z}_+$, we define $:\langle x^{\otimes n}, f^{(n)} \rangle:$ as the orthogonal projection of $\langle x^{\otimes n}, f^{(n)} \rangle$ onto (L_n^2) . It follows from results of [108] that $:\langle x^{\otimes n}, f^{(n)} \rangle: = \langle P_n(x), f^{(n)} \rangle$, where $P_n(x) \in \mathcal{D}'^{\otimes n}$ ($n \in \mathbb{Z}_+$) are the kernels of generalized Appell-like polynomials with $\chi = \exp$,

$$\alpha(\lambda) = \lambda + \sum_{n=2}^{\infty} \frac{\lambda^n}{n} (\rho^{n-1} + \rho^{n-2}\nu + \dots + \nu^{n-1}),$$

$$\gamma(\lambda) = \frac{1}{l_{\mu}(\alpha(\lambda))}$$

$$= \exp\left\{-\int_{\mathbb{R}_+} \left(\frac{\lambda^2(u)}{2} + \sum_{n=3}^{\infty} \frac{\lambda^n(u)}{n} (\rho^{n-2}(u) + \rho^{n-3}(u)\nu(u) + \dots + \nu^{n-2}(u))\right) du\right\}. \tag{2.2}$$

We say that the polynomials $\left\{\langle P_n, f^{(n)} \rangle, f^{(n)} \in \mathcal{D}_{\mathbb{C}}^{\widehat{\otimes} n}, n \in \mathbb{Z}_+\right\}$ are called the *generalized Meixner polynomials*.

Let us define a scalar product $\langle \cdot, \cdot \rangle_{\text{ext}}$ on $\mathcal{D}_{\mathbb{C}}^{\widehat{\otimes} n}, n \in \mathbb{Z}_+$, by setting for $f^{(n)}, g^{(n)} \in \mathcal{D}_{\mathbb{C}}^{\widehat{\otimes} n}$

$$\langle f^{(n)}, g^{(n)} \rangle_{\text{ext}} := \frac{1}{n!} \int_{\mathcal{D}'} \langle P_n, f^{(n)} \rangle \langle P_n, g^{(n)} \rangle \mu(dx).$$

It follows from results of [108] that

$$\begin{aligned} \langle f^{(n)}, g^{(n)} \rangle_{\text{ext}} &= \sum_{\substack{k, l_j, s_j \in \mathbb{N}; j=1, \dots, k, l_1 > l_2 > \dots > l_k, \\ l_1 s_1 + \dots + l_k s_k = n}} \frac{n!}{l_1^{s_1} \dots l_k^{s_k} s_1! \dots s_k!} \times \\ &\times \int_{\mathbb{R}_+^{s_1 + \dots + s_k}} f^{(n)}(\underbrace{u_1, \dots, u_1}_{l_1}, \dots, \underbrace{u_{s_1}, \dots, u_{s_1}}_{l_1}, \dots, \underbrace{u_{s_1 + \dots + s_k}, \dots, u_{s_1 + \dots + s_k}}_{l_k}) \times \\ &\times g^{(n)}(\underbrace{u_1, \dots, u_1}_{l_1}, \dots, \underbrace{u_{s_1}, \dots, u_{s_1}}_{l_1}, \dots, \underbrace{u_{s_1 + \dots + s_k}, \dots, u_{s_1 + \dots + s_k}}_{l_k}) \times \\ &\times \eta^{l_1 - 1}(u_1) \dots \eta^{l_1 - 1}(u_{s_1}) \eta^{l_2 - 1}(u_{s_1 + 1}) \dots \eta^{l_2 - 1}(u_{s_1 + s_2}) \dots \\ &\dots \eta^{l_k - 1}(u_{s_1 + \dots + s_{k-1} + 1}) \dots \eta^{l_k - 1}(u_{s_1 + \dots + s_k}) \\ &\times du_1 \dots du_{s_1 + \dots + s_k}. \end{aligned}$$

Let $|\cdot|_{\text{ext}}$ denote the norm generated by the scalar product $\langle \cdot, \cdot \rangle_{\text{ext}}$, i.e., $|f^{(n)}|_{\text{ext}} := \sqrt{\langle f^{(n)}, f^{(n)} \rangle_{\text{ext}}}$. Denote by $\mathcal{H}_{\text{ext}}^{(n)}$ the closure of $\mathcal{D}_{\mathbb{C}}^{\widehat{\otimes} n}$ with respect to $|\cdot|_{\text{ext}}$.

The space $\mathcal{H}_{\text{ext}}^{(n)}$ can be understood as an extension of $\mathcal{H}_{\mathbb{C}}^{\widehat{\otimes} n}$ in a generalized sense: let $F^{(n)} \in \mathcal{H}_{\mathbb{C}}^{\widehat{\otimes} n}, \dot{F}^{(n)} \in F^{(n)}$ be a representative (a function) from the equivalence class $F^{(n)}$ with a “zero diagonale”, i.e., $\dot{F}^{(n)}(u_1, \dots, u_n) = 0$ if there exist $i, j \in \{1, \dots, n\}$ such that $i \neq j$ but $u_i = u_j$. The function $\dot{F}^{(n)}$ generates an equivalence class in $\mathcal{H}_{\text{ext}}^{(n)}$ that can be identified with $F^{(n)}$ [73].

For $F^{(n)} \in \mathcal{H}_{\text{ext}}^{(n)}, n \in \mathbb{Z}_+$, we define $\langle P_n, F^{(n)} \rangle \in (L^2)$ as an (L^2) -limit

$$\langle P_n, F^{(n)} \rangle := \lim_{k \rightarrow \infty} \langle P_n, f_k^{(n)} \rangle,$$

where $\mathcal{D}_{\mathbb{C}}^{\widehat{\otimes} n} \ni f_k^{(n)} \rightarrow F^{(n)}$ in $\mathcal{H}_{\text{ext}}^{(n)}$ as $k \rightarrow \infty$. The forthcoming statement easily follows from the construction of polynomials $\langle P_n, F^{(n)} \rangle$ (see also [108]).

Theorem 2.1. *A function $F \in (L^2)$ if and only if there exists a sequence of kernels*

$$(F^{(n)} \in \mathcal{H}_{\text{ext}}^{(n)})_{n=0}^{\infty} \quad (2.3)$$

such that F can be presented in the form

$$F = \sum_{n=0}^{\infty} \langle P_n, F^{(n)} \rangle, \quad (2.4)$$

where the series converges in (L^2) , i.e., the (L^2) -norm of F

$$\|F\|_{(L^2)}^2 = \sum_{n=0}^{\infty} n! |F^{(n)}|_{\text{ext}}^2 < \infty.$$

Moreover, the system $\{ \langle P_n, F^{(n)} \rangle, F^{(n)} \in \mathcal{H}_{\text{ext}}^{(n)}, n \in \mathbb{Z}_+ \}$ is an orthogonal basis in (L^2) in the sense that for $F, G \in (L^2)$ of form (2.4)

$$\langle\langle F, G \rangle\rangle = \sum_{n=0}^{\infty} n! \langle F^{(n)}, G^{(n)} \rangle_{\text{ext}}.$$

2.3. A nonregular rigging of (L^2) . One can show [73] that the generalized Meixner measure satisfies conditions 1, 2, 3, 4' of the previous section, therefore one can consider (nonregular) chain (1.8) with the central space (L^2) , $\chi = \exp$, α and γ from (2.2); and all results of the "biorthogonal analysis" hold true in the "Meixner analysis". Note that since the generalized Meixner polynomials are orthogonal in (L^2) , natural orthogonal bases in the generalized function spaces $(\mathcal{H}_{-\tau})_{-q, \exp, \gamma, \alpha}$ consist of generalized functions $\langle P_m, F^{(m)} \rangle := (\mathcal{H}_{-\tau})_{-q, \exp, \gamma, \alpha} - \lim_{k \rightarrow \infty} \langle P_m, f_k^{(m)} \rangle$, $m \in \mathbb{Z}_+$, where $F^{(m)} \in \mathcal{H}_{-\tau, \mathbb{C}}^{(m)}$ —the negative space from the chain

$$\mathcal{D}_{\mathbb{C}}^{(m)} \supset \mathcal{H}_{-\tau, \mathbb{C}}^{(m)} \supset \mathcal{H}_{\text{ext}}^{(m)} \supset \mathcal{H}_{\tau, \mathbb{C}}^{\widehat{\otimes} m} \supset \mathcal{D}_{\mathbb{C}}^{\widehat{\otimes} m}$$

(see [73] for more details), $\mathcal{D}_{\mathbb{C}}^{\widehat{\otimes} m} \ni f_k^{(m)} \rightarrow F^{(m)}$ in $\mathcal{H}_{-\tau, \mathbb{C}}^{(m)}$ as $k \rightarrow \infty$. The interconnection between the generalized functions $\langle P_m, \cdot \rangle$ and $Q_{\mu, m}^{\exp, \gamma, \alpha}$ (see the previous section) is given by the formulas

$$\langle P_m, F^{(m)} \rangle = Q_{\mu, m}^{\exp, \gamma, \alpha} (U_m F^{(m)}),$$

where α and γ from (2.2) and the operators $U_m: \mathcal{H}_{-\tau, \mathbb{C}}^{(m)} \rightarrow \mathcal{H}_{-\tau, \mathbb{C}}^{\widehat{\otimes} m}$, $m \in \mathbb{Z}_+$, are defined as follows:

$$\forall f^{(m)} \in \mathcal{H}_{\tau, \mathbb{C}}^{\widehat{\otimes} m} \quad \langle F^{(m)}, f^{(m)} \rangle_{\text{ext}} = \langle U_m F^{(m)}, f^{(m)} \rangle. \quad (2.5)$$

2.4. A parametrized regular rigging of (L^2) . Now we begin to observe some results of the “Meixner analysis” that can not be obtained as consequences of results of the “biorthogonal theory”. First we introduce a so-called (*regular*) *parametrized rigging* of (L^2) . On the set $\mathcal{P}(\mathcal{D}')$ of all continuous polynomials on \mathcal{D}' , presented as the set of functions (1.4) with $\chi = \exp$, α and γ from (2.2) (i.e., now $P_n^{\chi, \gamma, \alpha}(x) = P_n(x)$), we introduce a family of Hilbert norms $\|\cdot\|_{q, \beta}$, $q \in \mathbb{Z}_+$, $\beta \in [0, 1]$ (in what follows, we accept these conditions on default), by setting for f of form (1.4)

$$\|f\|_{q, \beta}^2 := \sum_{n=0}^{N_f} (n!)^{1+\beta} 2^{qn} |f^{(n)}|_{\text{ext}}^2. \quad (2.6)$$

By $(L^2)_q^\beta$ denote a Hilbert space that is the closure of $\mathcal{P}(\mathcal{D}')$ with respect to norm (2.6). Let also $(L^2)^\beta := \text{pr} \lim_{q \in \mathbb{Z}_+} (L^2)_q^\beta$. The spaces $(L^2)_q^\beta$, $(L^2)^\beta$ are called the *parametrized Kondratiev-type test function spaces*. It is easy to see that $f \in (L^2)_q^\beta$ if and only if f can be presented in the form

$$f = \sum_{n=0}^{\infty} \langle P_n, f^{(n)} \rangle, \quad f^{(n)} \in \mathcal{H}_{\text{ext}}^{(n)}, \quad (2.7)$$

with

$$\|f\|_{(L^2)_q^\beta}^2 = \sum_{n=0}^{\infty} (n!)^{1+\beta} 2^{qn} |f^{(n)}|_{\text{ext}}^2 < \infty.$$

It is easy to show [73] that for arbitrary $q \in \mathbb{Z}_+$ and $\beta \in [0, 1]$ the space $(L^2)_q^\beta$ is densely and continuously embedded into (L^2) , therefore one can consider the chain

$$(L^2)^{-\beta} = \text{ind} \lim_{q' \in \mathbb{Z}_+} (L^2)_{-q'}^{-\beta} \supset (L^2)_{-q}^{-\beta} \supset (L^2) \supset (L^2)_q^\beta \supset (L^2)^\beta,$$

where $(L^2)_{-q}^{-\beta}$, $(L^2)^{-\beta}$ are the spaces dual of $(L^2)_q^\beta$, $(L^2)^\beta$ with respect to (L^2) correspondingly. The spaces $(L^2)_{-q}^{-\beta}$, $(L^2)^{-\beta}$ are called the *parametrized Kondratiev-type (regular) generalized function spaces*. Note that for $\beta = q = 0$ $(L^2)_0^0 = (L^2)_{-0}^0 = (L^2)$.

Since the generalized Meixner polynomials are orthogonal in (L^2) , these polynomials form orthogonal bases in $(L^2)_{-q}^{-\beta}$. More exactly, a function $F \in (L^2)_{-q}^{-\beta}$ if and only if there exists sequence (2.3) such that F can be presented in form (2.4) with

$$\|F\|_{-q, -\beta}^2 := \|F\|_{(L^2)_{-q}^{-\beta}}^2 = \sum_{n=0}^{\infty} (n!)^{1-\beta} 2^{-qn} |F^{(n)}|_{\text{ext}}^2 < \infty.$$

2.5. Elements of a Wick calculus. Now we consider elements of a Wick calculus on the parametrized generalized function spaces. For $F \in (L^2)_{-q}^{-\beta}$ we define the S -transform $(SF)(\lambda)$, $\lambda \in \mathcal{D}_{\mathbb{C}}$, as a *formal series*

$$(SF)(\lambda) := \sum_{n=0}^{\infty} \langle F^{(n)}, \lambda^{\otimes n} \rangle_{\text{ext}},$$

where $F^{(n)} \in \mathcal{H}_{\text{ext}}^{(n)}$, $n \in \mathbb{Z}_+$, are the kernels from decomposition (2.4) for F . It is obvious that, in particular, $(SF)(0) = F^{(0)}$, $S1 \equiv 1$. Further, for $F, G \in (L^2)_{-q}^{-\beta}$ and

a holomorphic at $F^{(0)}$ function $h: \mathbb{C} \rightarrow \mathbb{C}$ by analogy with the “biorthogonal case” we define a Wick product $F \diamond G$ and a Wick version $h^\diamond(F)$ by setting formally

$$F \diamond G := S^{-1}(SF \cdot SG), \quad h^\diamond(F) := S^{-1}h(SF).$$

A formal calculation shows that

$$\begin{aligned} F \diamond G &= \sum_{m=0}^{\infty} \left\langle P_m, \sum_{k=0}^m F^{(k)} \diamond G^{(m-k)} \right\rangle, \\ F_1 \diamond \dots \diamond F_n &= \sum_{m=0}^{\infty} \left\langle P_m, \sum_{\substack{k_1, \dots, k_n \in \mathbb{Z}_+, \\ k_1 + \dots + k_n = m}} F_1^{(k_1)} \diamond \dots \diamond F_n^{(k_n)} \right\rangle, \\ h^\diamond(F) &= h_0 + \sum_{m=1}^{\infty} \left\langle P_m, \sum_{n=1}^m h_n \sum_{\substack{k_1, \dots, k_n \in \mathbb{N}, \\ k_1 + \dots + k_n = m}} F^{(k_1)} \diamond \dots \diamond F^{(k_n)} \right\rangle \end{aligned} \quad (2.8)$$

(cf. (1.14)–(1.16)), where $F^{(k)}, G^{(k)}, F_1^{(k)}, \dots, F_n^{(k)} \in \mathcal{H}_{\text{ext}}^{(k)}$ are the kernels from decompositions (2.4) for F, G, F_1, \dots, F_n correspondingly; $h_n \in \mathbb{C}$, $n \in \mathbb{Z}_+$, are the coefficients from the decomposition $h(u) = \sum_{n=0}^{\infty} h_n(u - F^{(0)})^n$; for $F^{(n)} \in \mathcal{H}_{\text{ext}}^{(n)}$, $G^{(m)} \in \mathcal{H}_{\text{ext}}^{(m)}$

$$F^{(n)} \diamond G^{(m)} := U_{n+m}^{-1} \left((U_n F^{(n)}) \widehat{\otimes} (U_m G^{(m)}) \right) \in \mathcal{H}_{\text{ext}}^{(n+m)}, \quad (2.9)$$

see (2.5). The fact that $F^{(n)} \diamond G^{(m)} \in \mathcal{H}_{\text{ext}}^{(n+m)}$ is proved in [73]; moreover, it is shown therein that, roughly speaking, $F^{(n)} \diamond G^{(m)}$ is a symmetrization with respect to all variables of a function

$$\begin{cases} F^{(n)}(u_1, \dots, u_n) G^{(m)}(u_{n+1}, \dots, u_{n+m}), & \text{if } \begin{matrix} \forall i \in \{1, \dots, n\}, \\ \forall j \in \{n+1, \dots, n+m\} \ u_i \neq u_j, \end{matrix} \\ 0, & \text{in other cases} \end{cases}$$

and $|F^{(n)} \diamond G^{(m)}|_{\text{ext}} \leq |F^{(n)}|_{\text{ext}} |G^{(m)}|_{\text{ext}}$.

The forthcoming theorem from results of [66, 73] follows.

Theorem 2.2. *The following statements are fulfilled:*

Let $F, G \in (L^2)^{-\beta}$. Then $F \diamond G \in (L^2)^{-\beta}$. Moreover, the Wick product is continuous in the topology of $(L^2)^{-\beta}$: for arbitrary $F_1, \dots, F_n \in (L^2)^{-\beta}$ there exist (depending on these elements) $q, q' \in \mathbb{Z}_+$ ($q > q' + (1 - \beta) \log_2 n + 1$) such that

$$\|F_1 \diamond \dots \diamond F_n\|_{-q, -\beta} \leq c(n-1) \|F_1\|_{-q', -\beta} \dots \|F_n\|_{-q', -\beta},$$

where $c(n) := \sqrt{\max_{m \in \mathbb{Z}_+} [2^{-m} (m+1)^n]}$.

For $F \in (L^2)^{-1}$ and a holomorphic at $(SF)(0)$ function $h: \mathbb{C} \rightarrow \mathbb{C}$ $h^\diamond(F) \in (L^2)^{-1}$.

Let $h: \mathbb{C} \rightarrow \mathbb{C}$ be a holomorphic at $u_0 \in \mathbb{C}$ not-polynomial function with nonnegative coefficients h_n from the decomposition $h(u) = \sum_{n=0}^{\infty} h_n(u - u_0)^n$. Then for each $\beta \in [0, 1)$ there exists $F \in (L^2)^{-\beta}$ with $(SF)(0) = u_0$ such that $h^\diamond(F) \notin (L^2)^{-\beta}$.

Let $F = \sum_{m=0}^N \langle P_m, F^{(m)} \rangle$, $F^{(m)} \in \mathcal{H}_{\text{ext}}^{(m)}$; $h: \mathbb{C} \rightarrow \mathbb{C}$ be a holomorphic at $F^{(0)}$ function such that the coefficients h_n from the decomposition $h(u) = \sum_{n=0}^{\infty} h_n(u - F^{(0)})^n$ satisfy the estimates

$$|h_n| \leq \frac{K^n}{n^{nN \frac{1-\beta}{2}}}$$

with some $K > 0$. Then $h^\diamond(F) \in (L^2)^{-\beta}$.

Note that the proof of this theorem is based on “coordinate” representations (2.8).

Finally, if $F, G \in (L^2)^{-\beta} \cap (\mathcal{D})'_{\text{exp}, \mu}$ and α, γ are defined in (2.2) then $F \diamond G = F \diamond_{\mu}^{\text{exp}, \gamma, \alpha} G$, see [73].

2.6. An extended stochastic integral. By analogy with the Gaussian analysis, on the probability triplet $(\mathcal{D}', \mathcal{F}(\mathcal{D}'), \mu)$ we define the Meixner random process M by setting for each $u \in \mathbb{R}_+$ $M_u := \langle P_1, 1_{[0, u]} \rangle \in (L^2)$. Using results of [108] one can show that M is a locally square integrable normal martingale (with respect to the generated by M flow of σ -algebras) with orthogonal independent increments. Note that M is not a Lévy process, generally speaking (not time-homogeneous). Let us construct an extended (Skorohod-type) stochastic integral with respect to M . Let $F \in (L^2)^{-\beta}_q \otimes \mathcal{H}_{\mathbb{C}}$. It follows from above-posed results that F can be presented in the form

$$F(\cdot) = \sum_{m=0}^{\infty} \langle P_m, F^{(m)} \rangle, \quad F^{(m)} \in \mathcal{H}_{\text{ext}}^{(m)} \otimes \mathcal{H}_{\mathbb{C}}, \quad (2.10)$$

with

$$\|F\|_{(L^2)^{-\beta}_q \otimes \mathcal{H}_{\mathbb{C}}}^2 = \sum_{m=0}^{\infty} (m!)^{1-\beta} 2^{-qm} |F^{(m)}|_{\mathcal{H}_{\text{ext}}^{(m)} \otimes \mathcal{H}_{\mathbb{C}}}^2 < \infty.$$

If in addition F is such that the kernels $F^{(m)}$ belong to $\mathcal{H}_{\mathbb{C}}^{\otimes m} \otimes \mathcal{H}_{\mathbb{C}} \subset \mathcal{H}_{\text{ext}}^{(m)} \otimes \mathcal{H}_{\mathbb{C}}$ (the embedding in the generalized sense described above) then one can show [73] that F can be presented in the form

$$F(\cdot) = \sum_{m=0}^{\infty} m! \int_0^{\infty} \int_0^{u_m} \dots \int_0^{u_2} F^{(m)}(u_1, \dots, u_m) dM_{u_1} \dots dM_{u_m},$$

i.e., as a series of repeated Itô stochastic integrals with respect to the Meixner process. In this case for arbitrary $t_1, t_2 \in [0, +\infty]$, $t_1 < t_2$, one can define the extended stochastic integral of F with respect to M on $[t_1, t_2]$ as

$$\begin{aligned} & \int_{t_1}^{t_2} F(u) \widehat{d}M_u := \\ & := \sum_{m=0}^{\infty} (m+1)! \int_0^{\infty} \int_0^u \dots \int_0^{u_2} \widehat{F}_{[t_1, t_2]}^{(m)}(u_1, \dots, u_m, u) dM_{u_1} \dots dM_{u_m} dM_u = \end{aligned}$$

$$= \sum_{m=0}^{\infty} \left\langle P_{m+1}, \widehat{F}_{[t_1, t_2]}^{(m)} \right\rangle \in (L^2)_{-q-1}^{-\beta},$$

where $\widehat{F}_{[t_1, t_2]}^{(m)} \in \mathcal{H}_{\mathbb{C}}^{\otimes m+1} \subset \mathcal{H}_{\text{ext}}^{(m+1)}$ is the projection of $F_{[t_1, t_2]}^{(m)}(\cdot)$ onto $\mathcal{H}_{\mathbb{C}}^{\otimes m+1}$. In a general case such a definition can not be accepted because it is impossible to project elements of $\mathcal{H}_{\text{ext}}^{(m)} \otimes \mathcal{H}_{\mathbb{C}}$ onto $\mathcal{H}_{\text{ext}}^{(m+1)}$, generally speaking. Nevertheless, the following generalization is possible.

Lemma 2.1 ([73]). *For given $F_{[t_1, t_2]}^{(m)} \in \mathcal{H}_{\text{ext}}^{(m)} \otimes \mathcal{H}_{\mathbb{C}}$, $m \in \mathbb{Z}_+$, and $t_1, t_2 \in [0, +\infty]$, $t_1 < t_2$, we construct the element $\widehat{F}_{[t_1, t_2]}^{(m)} \in \mathcal{H}_{\text{ext}}^{(m+1)}$ by the following way. Let $\dot{F}_{[t_1, t_2]}^{(m)} \in F_{[t_1, t_2]}^{(m)}$ be some representative (a function) from the equivalence class $F_{[t_1, t_2]}^{(m)}$. We set*

$$\widetilde{F}_{[t_1, t_2]}^{(m)}(u_1, \dots, u_m, u) := \begin{cases} \dot{F}_{[t_1, t_2]}^{(m)}(u_1, \dots, u_m)1_{[t_1, t_2]}(u), & \text{if } u \neq u_1, \dots, u \neq u_m, \\ 0, & \text{in other cases,} \end{cases}$$

$\widehat{F}_{[t_1, t_2]}^{(m)} := Pr \widetilde{F}_{[t_1, t_2]}^{(m)}$, where Pr is the symmetrization operator. Let $\widehat{F}_{[t_1, t_2]}^{(m)} \in \mathcal{H}_{\text{ext}}^{(m+1)}$ be the equivalence class in $\mathcal{H}_{\text{ext}}^{(m+1)}$ that is generated by $\widehat{F}_{[t_1, t_2]}^{(m)}$. This class is well-defined, does not depend on the representative $\dot{F}_{[t_1, t_2]}^{(m)}$, and

$$\left| \widehat{F}_{[t_1, t_2]}^{(m)} \right|_{\text{ext}} \leq |F_{[t_1, t_2]}^{(m)}(\cdot)|_{\mathcal{H}_{\text{ext}}^{(m)} \otimes \mathcal{H}_{\mathbb{C}}} \leq |F_{[t_1, t_2]}^{(m)}|_{\mathcal{H}_{\text{ext}}^{(m)} \otimes \mathcal{H}_{\mathbb{C}}}.$$

Now for $F \in (L^2)_{-q}^{-\beta} \otimes \mathcal{H}_{\mathbb{C}}$ of form (2.10) we can define the extended stochastic integral on $[t_1, t_2]$ with respect to M by setting [73]

$$\int_{t_1}^{t_2} F(u) \widehat{d}M_u := \sum_{m=0}^{\infty} \left\langle P_{m+1}, \widehat{F}_{[t_1, t_2]}^{(m)} \right\rangle \in (L^2)_{-q-1}^{-\beta}.$$

If we consider the extended stochastic integral as an operator acting from $(L^2)_{-q}^{-\beta} \otimes \mathcal{H}_{\mathbb{C}}$ to $(L^2)_{-q}^{-\beta}$ (for example, if $\beta = q = 0$ then it will be an operator acting from $(L^2) \otimes \mathcal{H}_{\mathbb{C}}$ to (L^2)) then for $\beta < 1$ this operator will be unbounded with the domain

$$\left\{ F \in (L^2)_{-q}^{-\beta} \otimes \mathcal{H}_{\mathbb{C}} : \sum_{m=0}^{\infty} ((m+1)!)^{1-\beta} 2^{-q(m+1)} \left| \widehat{F}_{[t_1, t_2]}^{(m)} \right|_{\text{ext}}^2 < \infty \right\}.$$

Theorem 2.3 ([73]). *Let $F \in (L^2) \otimes \mathcal{H}_{\mathbb{C}}$ be integrable on \mathbb{R}_+ by Itô with respect to M (i.e., be adapted with respect to the generated by M flow of σ -algebras). Then for all $t_1, t_2 \in [0, +\infty]$, $t_1 < t_2$, F is integrable on $[t_1, t_2]$ by Itô and in the extended sense, and $\int_{t_1}^{t_2} F(u) \widehat{d}M_u = \int_{t_1}^{t_2} F(u) dM_u$ (the last integral is the Itô one).*

Let us consider main properties of the extended stochastic integral. First we note that if $t_1, t_2, t_3 \in [0, +\infty]$ and $t_1 < t_2 < t_3$ then $\int_{t_1}^{t_2} \circ(u) \widehat{d}M_u + \int_{t_2}^{t_3} \circ(u) \widehat{d}M_u = \int_{t_1}^{t_3} \circ(u) \widehat{d}M_u$; but applying this formula one has to keep in mind that for $\beta < 1$

the domain of $\int_a^b \circ(u) \widehat{dM}_u : (L^2)_{-q}^{-\beta} \otimes \mathcal{H}_{\mathbb{C}} \rightarrow (L^2)_{-q}^{-\beta}$ depends on the integration interval $[a, b]$. Further, taking into consideration that each $g^{(n)} \in \mathcal{H}_{\text{ext}}^{(n)}$, $n \in \mathbb{N}$, can be considered as $g^{(n)}(\cdot) \in \mathcal{H}_{\text{ext}}^{(n-1)} \otimes \mathcal{H}_{\mathbb{C}}$ with $|g^{(n)}(\cdot)|_{\mathcal{H}_{\text{ext}}^{(n-1)} \otimes \mathcal{H}_{\mathbb{C}}} \leq |g^{(n)}|_{\text{ext}}$ [73], we define the (Hida-type) stochastic derivative $1_{[t_1, t_2]}(\cdot) \partial. : (L^2)_{q+1}^{\beta} \rightarrow (L^2)_q^{\beta} \otimes \mathcal{H}_{\mathbb{C}}$, $t_1, t_2 \in [0, +\infty]$, $t_1 < t_2$, by setting for $g \in (L^2)_{q+1}^{\beta}$ of form (2.7)

$$1_{[t_1, t_2]}(\cdot) \partial. g := \sum_{n=0}^{\infty} (n+1) \langle P_n, g^{(n+1)}(\cdot) 1_{[t_1, t_2]}(\cdot) \rangle. \tag{2.11}$$

Of course, one can understand $1_{[t_1, t_2]}(\cdot) \partial.$ as a linear unbounded operator acting from $(L^2)_q^{\beta}$ to $(L^2)_q^{\beta} \otimes \mathcal{H}_{\mathbb{C}}$ with the domain

$$\left\{ g = \sum_{n=0}^{\infty} \langle P_n, g^{(n)} \rangle \in (L^2)_q^{\beta} : \sum_{n=0}^{\infty} (n!)^{1+\beta} 2^{qn} (n+1)^2 \left| g^{(n+1)}(\cdot) 1_{[t_1, t_2]}(\cdot) \right|_{\mathcal{H}_{\text{ext}}^{(n)} \otimes \mathcal{H}_{\mathbb{C}}}^2 < \infty \right\}.$$

Theorem 2.4 ([73]). *For all $t_1, t_2 \in [0, +\infty]$, $t_1 < t_2$, the operators $\int_{t_1}^{t_2} \circ(u) \widehat{dM}_u : (L^2)_{-q}^{-\beta} \otimes \mathcal{H}_{\mathbb{C}} \rightarrow (L^2)_{-q}^{-\beta}$ and $1_{[t_1, t_2]}(\cdot) \partial. : (L^2)_q^{\beta} \rightarrow (L^2)_q^{\beta} \otimes \mathcal{H}_{\mathbb{C}}$ are adjoint one to another:*

$$\int_{t_1}^{t_2} \circ(u) \widehat{dM}_u = (1_{[t_1, t_2]}(\cdot) \partial.)^* ; \quad 1_{[t_1, t_2]}(\cdot) \partial. = \left(\int_{t_1}^{t_2} \circ \widehat{dM} \right)^*.$$

In particular, these operators are closed.

Of course, the statement of this theorem holds true if we consider $\int_{t_1}^{t_2} \circ(u) \widehat{dM}_u : (L^2)_{-q}^{-\beta} \otimes \mathcal{H}_{\mathbb{C}} \rightarrow (L^2)_{-q-1}^{-\beta}$ and $1_{[t_1, t_2]}(\cdot) \partial. : (L^2)_{q+1}^{\beta} \rightarrow (L^2)_q^{\beta} \otimes \mathcal{H}_{\mathbb{C}}$.

By analogy one can consider the extended stochastic integral as a linear continuous operator acting from $(L^2)^{-\beta} \otimes \mathcal{H}_{\mathbb{C}}$ to $(L^2)^{-\beta}$.

Let us consider now the interconnection between the Wick calculus and the extended stochastic integration. Let $M'_u = \langle P_1, \delta_u \rangle \in (\mathcal{H}_{-\tau})_{-q, \text{exp}, \gamma, \alpha, \mu}$ (see (1.8); now α and γ from (2.2)) be the Meixner white noise [73, 108].

Theorem 2.5 ([73]). *For all $t_1, t_2 \in [0, +\infty]$, $t_1 < t_2$, and $F \in (L^2)^{-\beta} \otimes \mathcal{H}_{\mathbb{C}}$ the formally defined Pettis integral $\int_{t_1}^{t_2} F(u) \diamond M'_u du$ can be considered as a linear continuous functional on $(L^2)^{\beta}$ that coincides with $\int_{t_1}^{t_2} F(u) \widehat{dM}_u$, i.e.,*

$$\int_{t_1}^{t_2} F(u) \diamond M'_u du = \int_{t_1}^{t_2} F(u) \widehat{dM}_u \in (L^2)^{-\beta}.$$

Let us consider an example of solving of an equation with Wick-type nonlinearity. Let

$$X_t = X_0 + \int_0^t X_u \diamond F du + \int_0^t X_u \diamond G \widehat{dM}_u,$$

where $X_0, F, G \in (L^2)^{-\beta}$. Applying the S -transform, solving the obtained nonstochastic equation and applying the inverse S -transform we obtain the solution

$$X_t = X_0 \diamond \exp^\diamond \{Ft + G \diamond M_t\} \in (L^2)^{-1}.$$

In order to have $X_t \in (L^2)^{-\beta}$, $\beta < 1$, we need additional restrictions. For example, if F and G are polynomials and $N := \max\{\deg F, \deg G + 1\}$, where \deg denotes the degree of a polynomial, then $X_t \in (L^2)^{-\beta}$ if $N \leq \frac{2}{1-\beta}$ [66].

Finally we note that if $F \in (L^2)^{-\beta} \otimes \mathcal{H}_{\mathbb{C}} \cap (\mathcal{D})'_{\text{exp}, \mu} \otimes \mathcal{H}_{\mathbb{C}}$ and α is defined in (2.2) then for all $t_1, t_2 \in [0, +\infty]$, $t_1 < t_2$,

$$\int_{t_1}^{t_2} F(u) \widehat{dM}_u = \mathbf{I}_{[t_1, t_2]}^{\text{exp}, \alpha, \mu}(F),$$

this result is proved in [73].

2.7. Operators of stochastic differentiation. In contrast to the general “biorthogonal” case, now the operator $1_{[t_1, t_2]}(\cdot)\partial$, $t_1, t_2 \in [0, +\infty]$, $t_1 < t_2$, can be naturally continued to the generalized function spaces. More exactly, for $F \in (L^2)^{-\beta}_{-q}$ of form (2.4) we define $1_{[t_1, t_2]}(\cdot)\partial.F \in (L^2)^{-\beta}_{-q-1} \otimes \mathcal{H}_{\mathbb{C}}$ by setting (cf. (2.11))

$$1_{[t_1, t_2]}(\cdot)\partial.F := \sum_{m=0}^{\infty} (m+1) \left\langle P_m, F^{(m+1)}(\cdot) 1_{[t_1, t_2]}(\cdot) \right\rangle.$$

This operator can be naturally continued to a linear continuous operator acting from $(L^2)^{-\beta}$ to $(L^2)^{-\beta} \otimes \mathcal{H}_{\mathbb{C}}$; also one can consider $1_{[t_1, t_2]}(\cdot)\partial$ as a linear unbounded operator acting from $(L^2)^{-\beta}_{-q}$ to $(L^2)^{-\beta}_{-q} \otimes \mathcal{H}_{\mathbb{C}}$ with the domain consists of $F \in (L^2)^{-\beta}_{-q}$ such that $\|1_{[t_1, t_2]}(\cdot)\partial.F\|_{(L^2)^{-\beta}_{-q} \otimes \mathcal{H}_{\mathbb{C}}} < \infty$, in this case $1_{[t_1, t_2]}(\cdot)\partial$ is closed [71].

Properties of $1_{[t_1, t_2]}(\cdot)\partial$ on the generalized function spaces are similar to properties of $1_{[t_1, t_2]}(\cdot)\partial$ on the test function spaces. In particular, $1_{[t_1, t_2]}(\cdot)\partial$ is the adjoint operator to the restriction of the extended stochastic integral on the corresponding test function space. The interested reader can find a more detailed information in [71].

Now let us consider operators of stochastic differentiation on the generalized function spaces. For $F^{(m)} \in \mathcal{H}_{\text{ext}}^{(m)}$ and $f^{(n)} \in \mathcal{H}_{\text{ext}}^{(n)}$, $m > n$, we define a “pairing” $\langle F^{(m)}, f^{(n)} \rangle_{\text{ext}} \in \mathcal{H}_{\text{ext}}^{(m-n)}$ by setting for each $g^{(m-n)} \in \mathcal{H}_{\text{ext}}^{(m-n)}$

$$\left\langle \left\langle F^{(m)}, f^{(n)} \right\rangle_{\text{ext}}, g^{(m-n)} \right\rangle_{\text{ext}} = \left\langle F^{(m)}, f^{(n)} \diamond g^{(m-n)} \right\rangle_{\text{ext}}$$

(see (2.9)). One can show [71] that for arbitrary $F^{(m)} \in \mathcal{H}_{\text{ext}}^{(m)}$ and $f^{(1)} \in \mathcal{H}_{\text{ext}}^{(1)} = \mathcal{H}_{\mathbb{C}}$

$$\left\langle F^{(m)}, f^{(1)} \right\rangle_{\text{ext}} = \int_{\mathbb{R}_+} F^{(m)}(u) f^{(1)}(u) du,$$

where the integral in the right-hand side is a Pettis one.

For arbitrary $f^{(n)} \in \mathcal{H}_{\text{ext}}^{(n)}$, $n \in \mathbb{Z}_+$, we define a linear operator $(\mathbb{D}^n \circ)(f^{(n)}) : (L^2)_{-q}^{-\beta} \rightarrow (L^2)_{-q-1}^{-\beta}$ by setting for $F \in (L^2)_{-q}^{-\beta}$

$$(\mathbb{D}^n F)(f^{(n)}) := \sum_{m=0}^{\infty} \frac{(m+n)!}{m!} \left\langle P_m, \left\langle F^{(m+n)}, f^{(n)} \right\rangle_{\text{ext}} \right\rangle,$$

where $F^{(m)} \in \mathcal{H}_{\text{ext}}^{(m)}$, $m \in \mathbb{Z}_+$, are the kernels from decomposition (2.4) for F . It follows from results of [71] that $(\mathbb{D}^n \circ)(f^{(n)})$ is a continuous operator that can be continued to a linear continuous operator in $(L^2)^{-\beta}$, and the following statement is fulfilled.

Theorem 2.6. *The operator \mathbb{D}^n has the following properties:*
for $g_1^{(1)}, \dots, g_n^{(1)} \in \mathcal{H}_{\text{ext}}^{(1)} = \mathcal{H}_{\mathbb{C}}$

$$\underbrace{\left(\mathbb{D}^1 \left(\dots \left(\mathbb{D}^1 \left(\mathbb{D}^1 \circ \right) \left(g_1^{(1)} \right) \right) \right) \left(g_2^{(1)} \right) \dots \right)}_n \left(g_n^{(1)} \right) = (\mathbb{D}^n F)(g_1^{(1)} \diamond \dots \diamond g_n^{(1)});$$

for each $F \in (L^2)_{-q}^{-\beta}$ the kernels $F^{(n)} \in \mathcal{H}_{\text{ext}}^{(n)}$ from decomposition (2.4) can be presented in the form

$$F^{(n)} = \frac{1}{n!} \mathbf{E}(\mathbb{D}^n F),$$

i.e., for each $f^{(n)} \in \mathcal{H}_{\text{ext}}^{(n)}$ $\langle F^{(n)}, f^{(n)} \rangle_{\text{ext}} = \frac{1}{n!} \mathbf{E}[(\mathbb{D}^n F)(f^{(n)})]$, here \mathbf{E} denotes the expectation;

for all $F \in (L^2)_{-q}^{-\beta}$ and $f^{(1)} \in \mathcal{H}_{\text{ext}}^{(1)} = \mathcal{H}_{\mathbb{C}}$

$$\int_{\mathbb{R}_+} \partial_u F \cdot f^{(1)}(u) du = (\mathbb{D}^1 F)(f^{(1)}),$$

here the integral in the left-hand side is a Pettis one;
the adjoint to \mathbb{D}^n operator has a form

$$(\mathbb{D}^n g)(f^{(n)})^* = \sum_{m=0}^{\infty} \langle P_{m+n}, f^{(n)} \diamond g^{(m)} \rangle = g \diamond \langle P_n, f^{(n)} \rangle \in (L^2)_q^\beta,$$

where $g \in (L^2)_{q+1}^\beta$, $f^{(n)} \in \mathcal{H}_{\text{ext}}^{(n)}$, and $g^{(m)} \in \mathcal{H}_{\text{ext}}^{(m)}$ ($m \in \mathbb{Z}_+$) are the kernels from decomposition (2.7) for g ;

for all $F \in (L^2)_{-q}^{-\beta}$, $g \in (L^2)_{q+1}^\beta$ and $f^{(1)} \in \mathcal{H}_{\text{ext}}^{(1)} = \mathcal{H}_{\mathbb{C}}$

$$\begin{aligned} \left\langle \left\langle F, \int_{\mathbb{R}_+} g \cdot f^{(1)}(u) \widehat{dM}_u \right\rangle \right\rangle &= \left\langle \left\langle F, g \diamond \langle P_1, f^{(1)} \rangle \right\rangle \right\rangle = \\ &= \left\langle \left\langle F, (\mathbb{D}^1 g)(f^{(1)})^* \right\rangle \right\rangle = \left\langle \left\langle (\mathbb{D}^1 F)(f^{(1)}), g \right\rangle \right\rangle; \end{aligned}$$

the operator \mathbb{D}^1 is a differentiation with respect to the Wick product, i.e., for all $F, G \in (L^2)^{-\beta}$ and $g \in \mathcal{H}_{\text{ext}}^{(1)} = \mathcal{H}_{\mathbb{C}}$

$$(\mathbb{D}^1(F \diamond G))(g) = (\mathbb{D}^1 F)(g) \diamond G + F \diamond (\mathbb{D}^1 G)(g) \in (L^2)^{-\beta};$$

for any $n \in \mathbb{Z}_+$, $F \in (L^2)^{-\beta}$, $g \in \mathcal{H}_{\text{ext}}^{(1)} = \mathcal{H}_{\mathbb{C}}$, and a holomorphic at $(SF)(0)$ function $h: \mathbb{C} \rightarrow \mathbb{C}$

$$(\mathbb{D}^1 F^{\diamond n})(g) = nF^{\diamond n-1} \diamond (\mathbb{D}^1 F)(g) \in (L^2)^{-\beta},$$

$$(\mathbb{D}^1 h^{\diamond}(F))(g) = h'^{\diamond}(F) \diamond (\mathbb{D}^1 F)(g) \in (L^2)^{-1},$$

where h' is the usual derivative of h ;

for all $t_1, t_2 \in [0, +\infty]$, $t_1 < t_2$, $g \in \mathcal{H}_{\text{ext}}^{(1)} = \mathcal{H}_{\mathbb{C}}$, and $F \in (L^2)^{-\beta} \otimes \mathcal{H}_{\mathbb{C}}$

$$\left(\mathbb{D}^1 \int_{t_1}^{t_2} F(u) \widehat{dM}_u \right) (g) = \int_{t_1}^{t_2} (\mathbb{D}^1 F(u))(g) \widehat{dM}_u + \int_{t_1}^{t_2} F(u) g(u) du \in (L^2)^{-\beta},$$

where the last integral in the right-hand side is a Pettis one.

An information about possible applications of the operators \mathbb{D}^n and the corresponding examples can be found in, e.g., [11, 71].

One can consider the operator $(\mathbb{D}^n \circ)(f^{(n)})$, $f^{(n)} \in \mathcal{H}_{\text{ext}}^{(n)}$, as a linear unbounded one acting in $(L^2)_{-q}^{-\beta}$. This operator is closed and the analog of the previous theorem holds true (see details in [71]).

Finally we note that the operators $\widetilde{\mathbb{D}}_{\text{exp}, \gamma, \alpha, \mu}^n$ (α and γ from (2.2)) and \mathbb{D}^n , $n \in \mathbb{N}$, do not coincide on the set $(L^2)^{-\beta} \cap (\mathcal{D})'_{\text{exp}, \mu}$, generally speaking (if $\eta \neq 0$, see (2.1)); but, as we saw, their properties are quite similar.

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