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STOCHASTIC INTEGRAL OF HITSUDA – SKOROKHOD TYPE ON THE EXTENDED FOCK SPACE

СТОХАСТИЧНИЙ ІНТЕГРАЛ ТИПУ ХІТЦУДИ – СКОРОХОДА НА РОЗШИРЕНОМУ ПРОСТОРІ ФОКА

We review some recent results connected with stochastic integrals of Hitsuda – Skorokhod type acting on the extended Fock space and its riggings.

Наведено огляд деяких останніх результатів, пов'язаних із стохастичними інтегралами типу Хітцуди – Скорохода, що діють на розширеному просторі Фока та його оснащеннях.

1. Introduction. The problem of extension for an Itô stochastic integral is a subject of interest of many researchers. First who proposed such extensions were M. Hitsuda [1], Yu. L. Daletsky [2, 3], A. V. Skorokhod and Yu. M. Kabanov [4 – 6]. The definitions of the extended stochastic integral proposed by M. Hitsuda and A. V. Skorokhod were equivalent and given in terms of the Fock space structure by using the Chaos Representation Property (CRP) of the Wiener process (this property was derived by Itô in [7]). Yu. L. Daletsky used another approach: his extension based on the integration by parts formula. In [6] Yu. M. Kabanov introduced the notion of the Hitsuda – Skorokhod type stochastic integral in the case of integration with respect to a compensated Poisson process (this process also possesses the CRP, see, e.g., [8]). Afterwards it became clear that in the construction of the Hitsuda – Skorokhod type integral, the Gaussian and Poisson character of processes never appears. One uses only the CRP of Wiener or Poisson processes. Thus in [9] (see also [10, 11]) it was shown that the Hitsuda – Skorokhod integral as an operator on the Fock space is an extension of the Itô integral not only in the Wiener and Poisson cases but in the case of any normal martingale with CRP (the reader can find examples and properties of normal martingales with CRP in, e.g., [9, 12 – 16]).

In the present paper *we will explore the Hitsuda – Skorokhod type integral connected with some normal martingales without CRP*. But in order to explain our motivation and make our considerations clear, first we recall the Gaussian case (see, e.g., [17 – 19] for more detailed presentation).

Let μ_G be the Gaussian measure on the Schwartz distributions space $\mathcal{D}' = \mathcal{D}'(\mathbb{R}_+)$ and $L^2(\mathcal{D}', \mu_G)$ be the corresponding L^2-space. By definition the space $\mathcal{D}' = \mathcal{D}'(\mathbb{R}_+)$ is the dual one of the Schwartz space $\mathcal{D} = \mathcal{D}(\mathbb{R}_+)$ of infinite differentiable functions on $\mathbb{R}_+$ with compact supports. Denote by $\langle x, \varphi\rangle$ the action of $x \in \mathcal{D}'$ or $\varphi \in \mathcal{D}$ and construct a Wiener process $\{W_t\}_{t\in\mathbb{R}_+}$ by the formula

$$W_t(x) := \langle x, 1\!\!1_{[0,t)}\rangle := \lim_{n\to\infty} \langle x, \varphi_n\rangle \qquad (\text{limit in } L^2(\mathcal{D}', \mu_G)), \tag{1.1}$$

where $\{\varphi_n\}_{n=0}^{\infty} \subset \mathcal{D}$ is a sequence converging in $L^2(\mathbb{R}_+) = L^2(\mathbb{R}_+, dt)$ to the indicator function $1\!\!1_{[0,t)}$ of the set $[0, t)$. Note that passing to a limit in (1.1) is possible due to properties of the Gaussian measure μ_G, see Section 2 for details.

The CRP of $\{W_t\}_{t\in\mathbb{R}_+}$ implies that for any function $F \in L^2(\mathcal{D}', \mu_G)$ there exists a uniquely defined vector $f = (f_n)_{n=0}^{\infty}$ from the symmetric Fock space

$$\mathcal{F} := \mathbb{C} \oplus \bigoplus_{n=1}^{\infty} L^2_{\mathbb{C}}(\mathbb{R}_+)^{\widehat{\otimes} n} n!$$

such that

$$F = \sum_{n=0}^{\infty} I_n(f_n), \qquad I_n(f_n) := n! \int_{\Delta^n} f_n(t_1, \ldots, t_n)\, dW_{t_1} \ldots dW_{t_n}, \tag{1.2}$$

where $\Delta^n = \{(t_1, \ldots, t_n) \in \mathbb{R}^n_+ | \, t_1 < \ldots < t_n\}$ and $I_n(f_n)$ is a multiple stochastic integral. More exactly, the mapping (the so-called *Wiener – Itô – Segal isomorphism*)

$$I_G \colon \mathcal{F} \to L^2(\mathcal{D}', \mu_G), \qquad f = (f_n)_{n=0}^{\infty} \mapsto I_G f := \sum_{n=0}^{\infty} I_n(f_n), \tag{1.3}$$

is a well-defined isometrically isomorphic (unitary) operator. Note that $W_t(x) = \big(I_G(0, 1\!\!1_{[0,t)}, 0, 0, \ldots)\big)(x)$.

It should be noticed that the isomorphism I_G has a simple and naturale interpretation from the spectral point of view. Namely, it is possible to understand the mapping I_G as the Fourier transform of a certain family (the so-called *free field*) of commuting selfadjoint operators that act in the Fock space $\mathcal{F}$ and have a *Jacobi structure*. This result was obtained by V. D. Koshmanenko and Yu. S. Samoilenko in [20]; see also [21]. Taking into account this fact, we can rewrite representation (1.3) in the form

$$(I_G f)(\cdot) = \sum_{n=0}^{\infty} \langle P_n(\cdot), f_n \rangle \in L^2(\mathcal{D}', \mu_G), \tag{1.4}$$

where each $\langle P_n(\cdot), f_n \rangle$ is a polynomial of the first kind connected with the free field or, in other terminology, $\langle P_n(\cdot), f_n \rangle$ is a generalized Hermite polynomial on $\mathcal{D}'$, see, e.g., [21, 17, 22].

Now we are ready to pass to the definition of the Hitsuda – Skorokhod integral. Let $F \in L^2(\mathbb{R}_+; L^2(\mathcal{D}', \mu_G)) \cong L^2(\mathcal{D}', \mu_G) \otimes L^2(\mathbb{R}_+)$. Then, for almost all $t \in \mathbb{R}_+$, we can apply Wiener – Itô – Segal expansion (1.2) to the function $F(t) = F(\cdot, t) \in L^2(\mathcal{D}', \mu_G)$ and write

$$F(t) = \sum_{n=0}^{\infty} n! \int_{\Delta^n} f_n(t_1, \ldots, t_n; t)\, dW_{t_1} \ldots dW_{t_n}. \tag{1.5}$$

If F is integrable by Itô with respect to W then using term by term integration we obtain

$$\int_{\mathbb{R}_+} F(t)\, dW_t = \sum_{n=0}^{\infty} (n+1)! \int_{\Delta^{n+1}} \widehat{f}_n(t_1, \ldots, t_n, t)\, dW_{t_1} \ldots dW_{t_n} dW_t =$$

$$= \sum_{n=0}^{\infty} I_{n+1}(\widehat{f}_n) \in L^2(\mathcal{D}', \mu_G),$$

where $\widehat{f}_n$ is the symmetrization of $f_n(t_1, \ldots, t_n; t)$ with respect to $n+1$ variables. This representation of the Itô integral suggests us to define its extension by

$$\int_{\mathbb{R}_+} F(t)\, \widehat{d}W_t := \sum_{n=0}^{\infty} I_{n+1}(\widehat{f}_n) \tag{1.6}$$

for all $F \in L^2(\mathbb{R}_+; L^2(\mathcal{D}', \mu_G))$ such that

$$\sum_{n=0}^{\infty} I_{n+1}(\widehat{f}_n) \in L^2(\mathcal{D}', \mu_G) \quad \text{or, equivalently,} \quad (0, \widehat{f}_0, \widehat{f}_1, \dots) \in \mathcal{F}.$$

Note that exactly in such a way the extended stochastic integral was defined by Hitsuda and Skorokhod.

Clearly, one can identify $\int_{\mathbb{R}_+} F(t)\, \widehat{d}W_t$ with the vector $(0, \widehat{f}_0, \widehat{f}_1, \dots)$ from the Fock space $\mathcal{F}$ and consider this integral as an unbounded operator

$$\mathbb{I}_{\text{ext}} \colon L^2(\mathbb{R}_+; \mathcal{F}) \to \mathcal{F}, \qquad f(\cdot) = (f_n(\cdot))_{n=0}^{\infty} \mapsto \mathbb{I}_{\text{ext}}(f) := (0, \widehat{f}_0, \widehat{f}_1, \dots), \tag{1.7}$$

with the domain

$$\text{Dom}\,(\mathbb{I}_{\text{ext}}) := \Big\{ f(\cdot) = (f_n(\cdot))_{n=0}^{\infty} \in L^2(\mathbb{R}_+; \mathcal{F}) \;\Big|\; (0, \widehat{f}_0, \widehat{f}_1, \dots) \in \mathcal{F} \Big\}.$$

Further, one can interpret the Malliavin's gradient (the stochastic derivative, see, e.g., [17, 19]) as an operator acting from $\mathcal{F}$ to $L^2(\mathbb{R}_+; \mathcal{F})$; to formulate "on this language" some properties of the stochastic integral and the stochastic derivative (for example, the stochastic integral and the stochastic derivative are adjoint one to another operators) etc.

If we apply a Wiener – Itô – Segal type isomorphism to the integral $\mathbb{I}_{\text{ext}}$ we obtain a naturale extension of the Itô integral not only in the Wiener case but in the case of any normal martingale with CRP. Moreover, the properties of the extended stochastic integral and the stochastic derivative that can be formulated "on the language of Fock spaces", i.e., with using of the coefficients from (1.2) – (1.6) only, coincide (up to the corresponding Wiener – Itô – Segal type isomorphisms) for all these martingales. Thus this point of view enables us to treat the stochastic analysis of all these processes in the one framework (as the analysis on the Fock space $\mathcal{F}$).

In view of this it is natural to ask: "*is it possible to construct an analog of the Hitsuda – Skorokhod integral for processes without the CRP?* ". Recently it became clear (see [23 – 25]) that this is possible at least for the cases of stochastic integration with respect to Gamma, Pascal and Meixner processes (the processes of Meixner type). In spite of the fact that these processes are normal martingales without the CRP, they are connected with Jacobi fields (generalizations of the free field), which act in the so-called extended Fock space

$$\mathcal{F}_{\text{ext}} := \mathbb{C} \oplus \bigoplus_{n=1}^{\infty} \mathcal{F}_{n,\text{ext}} n!. \tag{1.8}$$

Here each $\mathcal{F}_{n,\text{ext}}$ consists of symmetric square integrable with respect to some measure ρ_n functions (see, e.g., [26, 27]). It should be noticed that the theory of Jacobi fields in the Fock space was created by Yu. M. Berezansky in [28] and carried out by him and his collaborators, see survey [29] for a more complete bibliography.

So, it follows from results of [30] (see also [31 – 38]) that for each process of Meixner type there exists a Jacobi field in the space $\mathcal{F}_{\text{ext}}$ that is a certain family $A = \{A(\varphi)\}_{\varphi \in \mathcal{D}}$ of commuting selfadjoint operators. These operators have a Jacobi structure and are connected with the orthogonal decomposition in (1.8). Applying the projection spectral theorem to this field we can construct a Fourier transform I with respect to the generalized joint eigenvectors of the family A. This transform has a form similar to (1.4), but the operator I is a unitary between the spaces $\mathcal{F}_{\mathrm{ext}}$ and $L^2(\mathcal{D}', \mu)$,

$$I\colon \mathcal{F}_{\mathrm{ext}} \to L^2(\mathcal{D}', \mu), \qquad f = (f_n)_{n=0}^{\infty} \mapsto (If)(\cdot) := \sum_{n=0}^{\infty} \langle P_n(\cdot), f_n \rangle, \tag{1.9}$$

where μ is a measure of Meixner type and $\{P_n(x)\}_{n=0}^{\infty} = P(x)$ is a generalized joint eigenvector of the operator $A(\varphi)$ corresponding to the eigenvalue $\langle P_1(x), \varphi \rangle \equiv \equiv \langle x, \varphi \rangle$. The sequence $\{P_n(x)\}_{n=0}^{\infty}$ is called the sequence of polynomials of the first kind connected with the family A.

In this case, the process of Meixner type $\{M_t\}_{t \in \mathbb{R}_+}$ is defined by the formula

$$M_t(x) := \langle x, 1\!\!1_{[0,t)} \rangle := \big(I(0, 1\!\!1_{[0,t)}, 0, 0, \ldots) \big)(x).$$

Note also that $\langle P_n(\cdot), f_n \rangle$, $n \in \mathbb{Z}_+$, as well as the generalized Hermite polynomials, are Schefer polynomials, that is orthogonal polynomials with a generating function of exponential type, see [36, 37, 30] for more details.

In the present paper, using Fourier transform (1.9), we introduce and study extended stochastic integrals connected with processes of Meixner type. We define these integrals by analogy with (1.7), but using instead of the Fock space $\mathcal{F}$ the extended Fock space $\mathcal{F}_{\mathrm{ext}}$. Note that related results to this topic have been established in [39–41].

The paper is organized in the following manner. In the forthcoming section we give a brief introduction in the Gaussian white noise analysis and recall the construction of stochastic integrals on a Fock space and its riggings in the framework of this analysis, this section serves as a model example. In Section 3 we give a necessary information about the generalized Meixner measure and the extended Fock space. Section 4 is devoted to the construction and study of the Itô stochastic integral on the extended Fock space. Finally, in Section 5 we give definitions and establish main properties of extended stochastic integrals on the extended Fock space and its riggings.

2. Stochastic integrals in the Gaussian white noise analysis. In this section we recall some basic concepts of the Gaussian white noise analysis (see, e.g., [17, 21, 18] for more details) and describe a general approach to construction of the extended stochastic integral.

2.1. Elements of the Gaussian white noise analysis. Denote by $\mathcal{D} := C_0^{\infty}(\mathbb{R}_+)$ the set of all real-valued infinite differentiable functions on $\mathbb{R}_+$ with compact supports. This set can be naturally endowed with a projective limit topology

$$\mathcal{D} = \operatorname*{pr\,lim}_{\tau \in T} D_{\tau},$$

where T denotes the set of all pairs $\tau = (\tau_1, \tau_2)$ such that $\tau_1 \in \mathbb{N}$ and τ_2 is an infinite differentiable function on $\mathbb{R}_+$ such that $\tau_2(t) \geq 1$ for all $t \in \mathbb{R}_+$; D_{τ} is the closure of $C_0^{\infty}(\mathbb{R}_+)$ in the norm $|\cdot|_{D_{\tau}}$ generated by the scalar product

$$(\varphi, \psi)_{D_{\tau}} = \int_{\mathbb{R}_+} \left(\sum_{k=0}^{\tau_1} \varphi^{(k)}(t) \psi^{(k)}(t) \right) \tau_2(t)\, dt,$$

i.e., D_{τ} denotes the Sobolev space of order τ_1 weighted by the function τ_2. Henceforth we will regard $\mathcal{D}$ as the corresponding *topological space*.

As is known (see, e.g., [21, 42]), D_τ are densely and continuously embedded into the space $L^2(\mathbb{R}_+)$ of square integrable with respect to the Lebesgue measure real-valued functions on $\mathbb{R}_+$. Therefore one can consider the chain (the rigging of $L^2(\mathbb{R}_+)$)

$$\mathcal{D}' \supset D_{-\tau} \supset L^2(\mathbb{R}_+) \supset D_\tau \supset \mathcal{D}, \tag{2.1}$$

where $D_{-\tau}$, $\mathcal{D}' = \operatorname{ind}\lim_{\tau\in T} D_{-\tau}$ are the dual of D_τ, $\mathcal{D}$ with respect to $L^2(\mathbb{R}_+)$ spaces respectively. We denote by $\langle\cdot\,,\cdot\rangle$ the dual pairing between elements of $\mathcal{D}'$ and $\mathcal{D}$ (and also $D_{-\tau}$ and D_τ) inducted by the scalar product $(\cdot\,,\cdot)_{L^2(\mathbb{R}_+)}$ in $L^2(\mathbb{R}_+)$, i.e., we set

$$\langle f,\varphi\rangle := (f,\varphi)_{L^2(\mathbb{R}_+)}, \qquad f\in L^2(\mathbb{R}_+), \quad \varphi\in\mathcal{D},$$

and then extend this definition by continuity. We preserve the notation $\langle\cdot\,,\cdot\rangle$ for the dual pairings in tensor powers and complexifications of chain (2.1).

We denote by $\mathcal{C}(\mathcal{D}')$ the generated by cylinder sets σ-algebra on $\mathcal{D}'$. Let μ_G be the *Gaussian measure* on $\mathcal{C}(\mathcal{D}')$, i.e., a probability measure with the Fourier transform

$$\int_{\mathcal{D}'} e^{i\langle x,\varphi\rangle}\mu_G(dx) = e^{-\frac{1}{2}|\varphi|^2_{L^2(\mathbb{R}_+)}}, \quad \varphi\in\mathcal{D}. \tag{2.2}$$

Denote by $L^2(\mathcal{D}',\mu_G)$ the space of square integrable with respect to μ_G complex-valued functions on $\mathcal{D}'$. It follows from (2.2) that

$$\int_{\mathcal{D}'} \langle x,\varphi\rangle^2\mu_G(dx) = |\varphi|^2_{L^2(\mathbb{R}_+)}, \quad \varphi\in\mathcal{D}.$$

Therefore, extending the mapping

$$L^2(\mathbb{R}_+)\supset\mathcal{D}\ni\varphi\mapsto\langle\cdot\,,\varphi\rangle\in L^2(\mathcal{D}',\mu_G)$$

by continuity, we obtain a random variable $\langle\cdot\,,f\rangle\in L^2(\mathcal{D}',\mu_G)$ for each $f\in L^2(\mathbb{R}_+)$. Thus we can define a random process $\{W_t\}_{t\in\mathbb{R}_+}$ as

$$W_t(\cdot) := \langle\cdot\,,1\!\!1_{[0,t]}\rangle$$

(here and below $1\!\!1_A$ denotes the indicator of a set A). It is easy to see that finite-dimensional distributions of a random process $W_\cdot$ coincide with those of a Wiener one. Namely, for all $N\in\mathbb{N}$, $u_1,\ldots,u_N\in\mathbb{R}$ and $t_1,\ldots,t_N\in\mathbb{R}_+$

$$\int_{\mathcal{D}'}\exp\left(i\sum_{k=1}^N u_k W_{t_k}(x)\right)\mu_G(dx) = \int_{\mathcal{D}'}\exp\left(i\left\langle x,\sum_{k=1}^N u_k 1\!\!1_{[0,t_k]}\right\rangle\right)\mu_G(dx) =$$

$$= \exp\left(-\frac{1}{2}\left|\sum_{k=1}^N u_k 1\!\!1_{[0,t_k]}\right|^2_{L^2(\mathbb{R}_+)}\right) = \exp\left(-\frac{1}{2}\sum_{k,j=1}^N u_k u_j \min\{t_k,t_j\}\right).$$

Hence $\{W_t\}_{t\in\mathbb{R}_+}$ can be interpreted as a **Wiener process**.

An important technical tool in the Gaussian white noise analysis is the **Wiener – Itô – Segal isomorphism**

$$I_G:\mathcal{F}\to L^2(\mathcal{D}',\mu_G)$$

between the **symmetric Fock space** $\mathcal{F} := \mathcal{F}(L^2(\mathbb{R}_+))$ over $L^2(\mathbb{R}_+)$ and the complex Hilbert space $L^2(\mathcal{D}', \mu_G)$. Let us recall that the symmetric Fock space $\mathcal{F}$ is defined as

$$\mathcal{F} = \bigoplus_{n=0}^{\infty} L^2_{\mathbb{C}}(\mathbb{R}_+)^{\widehat{\otimes} n} n!, \qquad L^2_{\mathbb{C}}(\mathbb{R}_+)^{\widehat{\otimes} 0} := \mathbb{C},$$

i.e., $\mathcal{F}$ is a complex Hilbert space of sequences $f = (f_n)_{n=0}^{\infty}$, $f_n \in L^2_{\mathbb{C}}(\mathbb{R}_+)^{\widehat{\otimes} n}$, such that

$$\|f\|^2_{\mathcal{F}} = \sum_{n=0}^{\infty} |f_n|^2_{L^2_{\mathbb{C}}(\mathbb{R}_+)^{\widehat{\otimes} n}} n! < \infty.$$

Here and below $\widehat{\otimes}$ denotes a symmetric tensor product ($\otimes$ denotes an ordinary tensor product), the subindex $\mathbb{C}$ denotes the complexification of a real space.

In what follows, we always identify in the natural way the space $L^2_{\mathbb{C}}(\mathbb{R}_+)^{\widehat{\otimes} n}$ with the space $L^2_{\mathbb{C},\mathrm{sym}}(\mathbb{R}^n_+)$ of all complex-valued symmetric functions from $L^2_{\mathbb{C}}(\mathbb{R}^n_+)$. Namely, we identify each element $g_1 \widehat{\otimes} \ldots \widehat{\otimes} g_n \in L^2_{\mathbb{C}}(\mathbb{R}_+)^{\widehat{\otimes} n}$ with the symmetric function

$$\frac{1}{n!} \sum_{\sigma} g_1(t_{\sigma(1)}) \ldots g_n(t_{\sigma(n)}) \in L^2_{\mathbb{C},\mathrm{sym}}(\mathbb{R}^n_+)$$

(σ running over all permutations of $\{1, \ldots, n\}$) and extend this procedure to all elements of $L^2_{\mathbb{C}}(\mathbb{R}_+)^{\widehat{\otimes} n}$. This is possible because the mapping

$$g_1 \widehat{\otimes} \ldots \widehat{\otimes} g_n \mapsto \frac{1}{n!} \sum_{\sigma} g_1(t_{\sigma(1)}) \ldots g_n(t_{\sigma(n)})$$

after being extended by linearity and continuity to the whole space $L^2_{\mathbb{C}}(\mathbb{R}_+)^{\widehat{\otimes} n}$ is a unitary operator acting from $L^2_{\mathbb{C}}(\mathbb{R}_+)^{\widehat{\otimes} n}$ to $L^2_{\mathbb{C},\mathrm{sym}}(\mathbb{R}^n_+)$. Naturally

$$|f_n|^2_{L^2_{\mathbb{C}}(\mathbb{R}_+)^{\widehat{\otimes} n}} = \int_{\mathbb{R}^n_+} |f_n(t_1, \ldots, t_n)|^2 \, dt_1 \ldots dt_n =$$

$$= n! \int_{\Delta^n} |f_n(t_1, \ldots, t_n)|^2 \, dt_1 \ldots dt_n$$

for all $f_n \in L^2_{\mathbb{C}}(\mathbb{R}_+)^{\widehat{\otimes} n} \cong L^2_{\mathbb{C},\mathrm{sym}}(\mathbb{R}^n_+)$, where $\Delta^n = \{(t_1, \ldots, t_n) \in \mathbb{R}^n_+ \,|\, t_1 < \ldots \ldots < t_n\}$.

Return to the Wiener – Itô – Segal isomorphism I_G. There are several equivalent ways of construction of I_G: using multiple stochastic integrals either the Jacobi fields approach or the system of infinite-dimensional Hermite polynomials. We do not discuss this in details (see, e.g., [17, 19 – 22] and references therein), but we note that I_G is completely characterized by the following properties:

(i) $I_G : \mathcal{F} \to L^2(\mathcal{D}', \mu_G)$ is a unitary operator (an isometrical isomorphism);

(ii) $I_G(f_0, 0, 0, \ldots) = f_0$ for all $f_0 \in \mathbb{C}$;

(iii) for each $n \in \mathbb{N}$ and any disjoint Borel sets $\alpha_1, \ldots, \alpha_n$ of finite Lebesgue measure

$$\Big(I_G\Big(\underbrace{0, \ldots, 0}_{n \text{ times}}, 1\!\!1_{\alpha_1} \widehat{\otimes} \ldots \widehat{\otimes} 1\!\!1_{\alpha_n}, 0, 0, \ldots\Big)\Big)(\cdot) = W_{\alpha_1}(\cdot) \ldots W_{\alpha_n}(\cdot),$$

where $W_{\alpha_k}(\cdot) := \langle \cdot \,, 1\!\!1_{\alpha_k} \rangle$ for all $k \in \{1, \ldots, n\}$.

It should be noted that properties (i) – (iii) of I_G and the fact that the set

$$\mathbb{C}\bigoplus \operatorname{span}\left\{\Big(\underbrace{0,\ldots,0}_{n\text{ times}},\mathbb{1}_{\alpha_1}\widehat{\otimes}\ldots\widehat{\otimes}\mathbb{1}_{\alpha_n},0,0,\ldots\Big)\ \middle|\ n\in\mathbb{N};\ \alpha_i\in\mathcal{B}(\mathbb{R}_+);\ \alpha_i\cap\alpha_j=\varnothing, i\neq j\right\}$$

is dense in the Fock space $\mathcal{F}$ play a fundamental role in the construction of the Itô integral and its extensions in terms of the Fock space structure, see below for more details.

Let us construct a convenient for our considerations rigging of $\mathcal{F}$. For $\tau\in T$ and $q\in\mathbb{N}$ we set

$$\mathcal{F}(\tau,q):=\bigoplus_{n=0}^{\infty}D_{\tau,\mathbb{C}}^{\widehat{\otimes}n}(n!)^2 2^{qn},\qquad \mathcal{F}_+:=\operatorname*{pr\,lim}_{\tau\in T,q\in\mathbb{N}}\mathcal{F}(\tau,q),$$

where $\mathcal{F}(\tau,q)$ denotes a complex Hilbert space of sequences $f=(f_n)_{n=0}^{\infty}$ such that $f_n\in D_{\tau,\mathbb{C}}^{\widehat{\otimes}n}$ ($D_{\tau,\mathbb{C}}^{\widehat{\otimes}0}:=\mathbb{C}$) and

$$\|f\|^2_{\mathcal{F}(\tau,q)}:=\sum_{n=0}^{\infty}|f_n|^2_{D_{\tau,\mathbb{C}}^{\widehat{\otimes}n}}(n!)^2 2^{qn}<\infty.$$

It can be shown that for all $q\in\mathbb{N}$ and $\tau\in T$ the dense and continuous embedding $\mathcal{F}(\tau,q)\hookrightarrow\mathcal{F}$ takes place. Thus one can construct a rigging of the Fock space $\mathcal{F}$

$$\mathcal{F}_-\supset\mathcal{F}(-\tau,-q)\supset\mathcal{F}\supset\mathcal{F}(\tau,q)\supset\mathcal{F}_+, \tag{2.3}$$

where the spaces

$$\mathcal{F}(-\tau,-q)=\bigoplus_{n=0}^{\infty}D_{-\tau,\mathbb{C}}^{\widehat{\otimes}n}2^{-qn},\qquad \mathcal{F}_-=\operatorname*{ind\,lim}_{\tau\in T,q\in\mathbb{N}}\mathcal{F}(-\tau,-q) \tag{2.4}$$

are dual ones of $\mathcal{F}(\tau,q)$ and $\mathcal{F}_+$ with respect to the zero space $\mathcal{F}$ respectively. The (generated by the scalar product in $\mathcal{F}$) pairing between elements of $\mathcal{F}_-$ and $\mathcal{F}_+$ (and also $\mathcal{F}(-\tau,-q)$ and $\mathcal{F}(\tau,q)$) will be denoted by $\langle\langle\cdot,\cdot\rangle\rangle_{\mathcal{F}}$.

Using rigging (2.3) and the isomorphism I_G one can construct the rigging

$$\begin{array}{ccccc}\mathcal{F}_- & \supset & \mathcal{F} & \supset & \mathcal{F}_+\\ & & \downarrow I_G & & \downarrow I_G\\ (\mathcal{D}')_- & \supset & L^2(\mathcal{D}',\mu_G) & \supset & (\mathcal{D})_+,\end{array}$$

where the space of test functions $(\mathcal{D})_+:=I_G\mathcal{F}_+$ is the I_G-image of the Fock space $\mathcal{F}_+$ with the topology that is inducted by the topology of $\mathcal{F}_+$, the space of generalized functions $(\mathcal{D}')_-$ is the dual one of $(\mathcal{D})_+$ with respect to $L^2(\mathcal{D}',\mu_G)$. Note that I_G can be extended to an isomorphism between $\mathcal{F}_-$ and $(\mathcal{D}')_-$. We keep the same notation I_G for the corresponding extension.

Now we recall definitions of some important operators on Fock spaces.

For each $\xi = (\xi_n)_{n=0}^{\infty} \in \mathcal{F}_-$ we define the S-transform by the formula

$$(S\xi)(\lambda) := \sum_{n=0}^{\infty} \langle \xi_n, \lambda^{\otimes n} \rangle, \quad \lambda \in \mathcal{D}_{\mathbb{C}},$$

where the series converges absolutely in a (depending on ξ) neighborhood of $0 \in \mathcal{D}_{\mathbb{C}}$. Each vector ξ from $\mathcal{F}_-$ is uniquely determined by its S-transform. More exactly, let $\mathrm{Hol}_0(\mathcal{D}_{\mathbb{C}})$ be the set of all (germs of) functions that are holomorphic at $0 \in \mathcal{D}_{\mathbb{C}}$. It follows from [43] that the S-transform is a one-to-one map between $\mathcal{F}_-$ and $\mathrm{Hol}_0(\mathcal{D}_{\mathbb{C}})$.

Taking into account that $\mathrm{Hol}_0(\mathcal{D}_{\mathbb{C}})$ is an algebra with ordinary algebraic operations we can define a **Wick product** $\xi \Diamond \zeta$ of $\xi, \zeta \in \mathcal{F}_-$ by the formula

$$\xi \Diamond \zeta := S^{-1}(S\xi \cdot S\zeta) \in \mathcal{F}_-.$$

It is easy to calculate that for all $\xi = (\xi_n)_{n=0}^{\infty}, \zeta = (\zeta_n)_{n=0}^{\infty} \in \mathcal{F}_-$

$$\xi \Diamond \zeta = \left(\sum_{m=0}^{n} \xi_m \widehat{\otimes} \zeta_{n-m} \right)_{n=0}^{\infty}. \tag{2.5}$$

Furthermore, if $\xi = (\xi_n)_{n=0}^{\infty} \in \mathcal{F}_-$ and

$$h(\cdot) = \sum_{n=0}^{\infty} h_n (\cdot - \xi_0)^n \colon \mathbb{C} \to \mathbb{C}$$

is a holomorphic at $(S\xi)(0) = \xi_0$ function then one defines the Wick version of h by

$$h^{\Diamond}(\xi) := S^{-1} h(S\xi) \in \mathcal{F}_-.$$

As is easy to see,

$$h^{\Diamond}(\xi) = \sum_{n=0}^{\infty} h_n (0, \xi_1, \xi_2, \dots)^{\Diamond n}, \tag{2.6}$$

where $\xi^{\Diamond n} := \xi \Diamond \dots \Diamond \xi$ (n times) and $\xi^{\Diamond 0} := 1$.

Using the isomorphism I_G, all the above definitions and results can be reformulated in terms of the generalized functions space $(\mathcal{D}')_-$. In particular, a Wick product and Wick versions of holomorphic functions can be defined on $(\mathcal{D}')_-$ and used in order to study so-called stochastic equations with Wick-type nonlinearities (see, e.g., [44, 17]).

For each $t \in \mathbb{R}_+$ we define the *annihilation operator* $a_-(\delta_t)$ on $\mathcal{F}_+$ and the *creation operator* $a_+(\delta_t)$ on $\mathcal{F}_-$ (here δ_t denotes the delta function at t) by setting "on coordinates"

$$\begin{gathered}(a_-(\delta_t)\varphi_n)(t_1, \dots, t_{n-1}) := n\varphi_n(t_1, \dots, t_{n-1}, t), \quad \varphi_n \in \mathcal{D}_{\mathbb{C}}^{\widehat{\otimes} n}; \\ (a_+(\delta_t)\xi_n) := \delta_t \widehat{\otimes} \xi_n, \quad \xi_n \in \mathcal{D}'^{\widehat{\otimes} n}_{\mathbb{C}}.\end{gathered} \tag{2.7}$$

It is easy to show (see, e.g., [22]) that the operators $a_-(\delta_t)$ and $a_+(\delta_t)$ can be extended to linear continuous operators on $\mathcal{F}(\tau, q)$ and $\mathcal{F}(-\tau, -q)$ respectively, and $a_+(\delta_t)$ is the dual operator of $a_-(\delta_t)$ in the sense that for all $\xi \in \mathcal{F}(-\tau, -q)$ and $\varphi \in \mathcal{F}(\tau, q)$

$$\langle\langle a_+(\delta_t)\xi, \varphi \rangle\rangle_{\mathcal{F}} = \langle\langle \xi, a_-(\delta_t)\varphi \rangle\rangle_{\mathcal{F}}.$$

It is obvious that

$$a_+(\delta_t)\xi = \xi\Diamond(0, \delta_t, 0, 0, \ldots), \quad \xi \in \mathcal{F}(-\tau, -q). \tag{2.8}$$

Also we note that

$$\partial(\delta_t) := I_G a_-(\delta_t) I_G^{-1} \colon (\mathcal{D})_+ \to (\mathcal{D})_+$$

is the Gateaux derivative in the direction δ_t:

$$(\partial(\delta_t)F)(x) = \lim_{\varepsilon \to 0} \frac{F(x + \varepsilon\delta_t) - F(x)}{\varepsilon}, \qquad F \in (\mathcal{D})_+, \quad x \in \mathcal{D}',$$

see, e.g., [17]. The operator $\partial(\delta_t)$ is called the *Hida derivative*.

2.2. Stochastic integrals on a Fock space and its riggings. In this subsection we recall the construction of stochastic integrals on a Fock space and its riggings in the framework of the Gaussian white noise analysis. Namely, starting from the classical Itô integral with respect to a Wiener process, we define the Itô integral on the Fock space $\mathcal{F}$ and construct its generalization — the extended (Hitsuda – Skorokhod type) stochastic integral on $\mathcal{F}$ and its riggings. We stress that this approach enables us to define the extended stochastic integral not only with respect to a Wiener process but also with respect to any normal martingale with the Chaos Representation Property (CRP).

We start from a definition of the classical Itô integral (we refer, e.g., to the books [45, 46] for details). Let $\{\mathcal{A}_t\}_{t\in\mathbb{R}_+}$ be a natural filtration of σ-algebras $\mathcal{A}_t = \sigma\{W_s \mid s \leq \leq t\}$ generated by a Wiener process $\{W_t\}_{t\in\mathbb{R}_+}$ (this filtration is made complete and right continuous). We denote by $L^2_a(\mathbb{R}_+ \times \mathcal{D}')$ the set of all adapted with respect to $\{\mathcal{A}_t\}_{t\in\mathbb{R}_+}$ functions from the space

$$L^2(\mathcal{D}' \times \mathbb{R}_+) := L^2(\mathcal{D}' \times \mathbb{R}_+, \mathcal{C}(\mathcal{D}') \times \mathcal{B}(\mathbb{R}_+), \mu_G \times dt) \cong L^2(\mathcal{D}', \mu_G) \otimes L^2(\mathbb{R}_+),$$

where $\mathcal{B}$ denotes the Borel σ-algebra. It can be shown that $L^2_a(\mathbb{R}_+ \times \mathcal{D}')$ is a subspace of $L^2(\mathbb{R}_+ \times \mathcal{D}')$ (a linear closed subset of $L^2(\mathbb{R}_+ \times \mathcal{D}')$). We will refer to $L^2_a(\mathbb{R}_+ \times \mathcal{D}')$ as to the *space of Itô integrable functions*.

Let us recall that a function $F \in L^2(\mathcal{D}' \times \mathbb{R}_+)$ is adapted (or nonanticipative) with respect to the filtration $\{\mathcal{A}_t\}_{t\in\mathbb{R}_+}$ if for almost all $t \in \mathbb{R}_+$ the function $F(\cdot, t)$ is $\mathcal{A}_t$-measurable. In other words, $F \in L^2(\mathcal{D}' \times \mathbb{R}_+)$ is adapted with respect to $\{\mathcal{A}_t\}_{t\in\mathbb{R}_+}$ if $F(\cdot, t) = \mathbb{E}[F(\cdot, t)\,|\mathcal{A}_t]$ for almost all $t \in \mathbb{R}_+$, where $\mathbb{E}[\,\cdot\,|\mathcal{A}_t]$ denotes the conditional expectation with respect to the σ-algebra $\mathcal{A}_t$. We note that $\mathbb{E}[\,\cdot\,|\mathcal{A}_t]$ is the *orthogonal projection* in $L^2(\mathcal{D}', \mu_G)$ onto the subspace of all $\mathcal{A}_t$-measurable functions.

Let $F(t) = F(x, t)$ be a *simple Itô integrable function*. That is, $F \in L^2_a(\mathbb{R}_+ \times \mathcal{D}')$ can be written as

$$F(\cdot) = \sum_{k=0}^{n-1} F_{(k)} 1\!\!1_{(t_k, t_{k+1}]}(\cdot) \in L^2_a(\mathcal{D}' \times \mathbb{R}_+),$$

where $0 \leq t_0 < t_1 < \ldots < t_n < \infty$ (evidently, every $F_{(k)}$ is an $\mathcal{A}_{t_k}$-measurable function from the space $L^2(\mathcal{D}', \mu_G)$). The **Itô integral** of F with respect to a Wiener process $W_\cdot$ is defined by the formula

$$\int\limits_{\mathbb{R}_+} F(t)\, dW_t := \sum_{k=0}^{n-1} F_{(k)}(W_{t_{k+1}} - W_{t_k}) \in L^2(\mathcal{D}', \mu_G) \tag{2.9}$$

and has the isometry property

$$\left\| \int_{\mathbb{R}_+} F(t)\, dW_t \right\|^2_{L^2(\mathcal{D}', \mu_G)} = \|F\|^2_{L^2(\mathcal{D}' \times \mathbb{R}_+)} \equiv \int_{\mathbb{R}_+} \|F(t)\|^2_{L^2(\mathcal{D}', \mu_G)}\, dt. \qquad (2.10)$$

Since the set $L^2_{a,s}(\mathcal{D}' \times \mathbb{R}_+)$ of all simple Itô integrable functions is dense in $L^2_a(\mathcal{D}' \times \mathbb{R}_+)$ (with respect to the topology of $L^2(\mathcal{D}' \times \mathbb{R}_+)$), isometry property (2.10) allows us to extend the Itô integral to the set $L^2_a(\mathbb{R}_+ \times \mathcal{D}')$ of Itô integrable functions, and (2.10) still holds on this set. Namely, extending the mapping

$$L^2_{a,s}(\mathcal{D}' \times \mathbb{R}_+) \ni F \mapsto \int_{\mathbb{R}_+} F(t)\, dW_t \in L^2(\mathcal{D}', \mu_G)$$

by continuity we obtain a definition of the Itô integral on $L^2_a(\mathcal{D}' \times \mathbb{R}_+)$.

Let us now turn from the Itô integral on the space $L^2(\mathcal{D}', \mu_G)$ to one on the Fock space $\mathcal{F}$. This integral will be defined in the simplest possible way as the I_G^{-1}-image of the Itô integral $\int_{\mathbb{R}_+} F(t)\, dW_t$. To be precise, denote by $L^2(\mathbb{R}_+; \mathcal{F})$ the Hilbert space of $\mathcal{F}$-valued functions (more exactly, of equivalence classes)

$$\mathbb{R}_+ \ni t \mapsto f(t) \in \mathcal{F}, \qquad \|f\|^2_{L^2(\mathbb{R}_+; \mathcal{F})} := \int_{\mathbb{R}_+} \|f(t)\|^2_{\mathcal{F}}\, dt < \infty$$

with the corresponding scalar product. It is clear that any function f from the space $L^2(\mathbb{R}_+; \mathcal{F})$ has a form $f(t) = (f_n(t))_{n=0}^{\infty}$, where each $f_n(t_1, \ldots, t_n; t)$ belongs to the space $L^2_{\mathbb{C}}(\mathbb{R}_+)^{\widehat{\otimes} n} \otimes L^2(\mathbb{R}_+)$. This means that f_n belongs to $L^2_{\mathbb{C}}(\mathbb{R}_+^{n+1})$ and f_n is symmetric with respect to first n variables.

Since the spaces $L^2(\mathcal{D}' \times \mathbb{R}_+)$ and $L^2(\mathbb{R}_+; \mathcal{F})$ can be interpreted as tensor products $L^2(\mathcal{D}', \mu_G) \otimes L^2(\mathbb{R}_+)$ and $\mathcal{F} \otimes L^2(\mathbb{R}_+)$ respectively, we conclude that

$$I_G \otimes 1 \colon L^2(\mathbb{R}_+; \mathcal{F}) \to L^2(\mathcal{D}' \times \mathbb{R}_+)$$

is a well-definite unitary operator.

Definition 2.1. *We say that a function $f \in L^2(\mathbb{R}_+; \mathcal{F})$ is Itô integrable if $(I_G \otimes 1) f$ belongs to $L^2_a(\mathcal{D}' \times \mathbb{R}_+)$, i.e., if*

$$f \in L^2_a(\mathbb{R}_+; \mathcal{F}) := (I_G \otimes 1)^{-1} L^2_a(\mathcal{D}' \times \mathbb{R}_+).$$

The Itô integral of $f \in L^2_a(\mathbb{R}_+; \mathcal{F})$, denoted by $\mathbb{I}(f)$, is defined by

$$\mathbb{I}(f) := I_G^{-1} \left(\int_{\mathbb{R}_+} I_G(f(t))\, dW_t \right) \in \mathcal{F}.$$

***Remark* 2.1.** It follows from definition of $\mathbb{I}$ and equality (2.10) that, for all $f \in L^2_a(\mathbb{R}_+; \mathcal{F})$,

$$\|\mathbb{I}(f)\|^2_{\mathcal{F}} = \int_{\mathbb{R}_+} \|f(t)\|^2_{\mathcal{F}}\, dt.$$

As a consequence, the operator $\mathbb{I}$ acts isometrically from the subspace $L^2_a(\mathbb{R}_+; \mathcal{F})$ of $L^2(\mathbb{R}_+; \mathcal{F})$ into the Fock space $\mathcal{F}$.

It is natural now to ask: "How to verify that a function $f(\cdot) = (f_n(\cdot))_{n=0}^{\infty} \in L^2(\mathbb{R}_+;\mathcal{F})$ is Itô integrable and how to express the corresponding Itô integral in terms of the Fock space structure?" The answer is following.

Theorem 2.1. *The following statements are fulfilled*:

(I) *A function $\mathbb{R}_+ \ni t \mapsto f(t) = (f_n(t))_{n=0}^{\infty} \in \mathcal{F}$ is Itô integrable* (*i.e., f belongs to $L_a^2(\mathbb{R}_+;\mathcal{F})$*) *if and only if $f \in L^2(\mathbb{R}_+;\mathcal{F})$ and for almost all $t \in \mathbb{R}_+$*

$$f(t) = (f_0(t), f_1(t)\mathbb{1}_{[0,t)}, f_2(t)\mathbb{1}_{[0,t)^2}, \ldots).$$

(II) *For each $f(\cdot) = (f_n(\cdot))_{n=0}^{\infty} \in L_a^2(\mathbb{R}_+;\mathcal{F})$*

$$\mathbb{I}(f) = (0, \widehat{f}_0, \widehat{f}_1, \ldots) \in \mathcal{F}, \tag{2.11}$$

where $\widehat{f}_n \in L^2_{\mathbb{C}}(\mathbb{R}_+)^{\widehat{\otimes} n+1}$ denotes the symmetrization of $f_n(t_1,\ldots,t_n;t)$ with respect to all variables, or, equivalently, $\widehat{f}_n$ is the projection of $f_n \in L^2_{\mathbb{C}}(\mathbb{R}_+)^{\widehat{\otimes} n} \otimes L^2(\mathbb{R}_+)$ onto $L^2_{\mathbb{C}}(\mathbb{R}_+)^{\widehat{\otimes} n+1}$. Since the function $f_n(t_1,\ldots,t_n;t)$ is symmetric with respect to first n variables, its symmetrization $\widehat{f}_n$ is given by

$$\widehat{f}_n(t_1,\ldots,t_{n+1}) := \frac{1}{n+1}\sum_{k=1}^{n+1} f_n(t_1,\ldots,\not{t_k},\ldots,t_{n+1};t_k).$$

Although this theorem easily follows from the results of, e.g., [19, 9, 11], for the reader's convenience we present here a proof.

Proof. In order to prove (I), it is sufficient to show that

$$\mathbb{E}[I_G f_n|\mathcal{A}_t] = I_G(\mathbb{1}_{[0,t)^n} f_n) \tag{2.12}$$

for any $f_n \in L^2_{\mathbb{C}}(\mathbb{R}_+)^{\widehat{\otimes} n}$ (here and below in this proof we identify f_n with $(0,\ldots,0,f_n,0,0,\ldots) \in \mathcal{F}$, where f_n standing at the n-th position). Moreover, since functions

$$f_n = \mathbb{1}_{\alpha_1}\widehat{\otimes}\ldots\widehat{\otimes}\mathbb{1}_{\alpha_n}, \qquad \alpha_i \in \mathcal{B}(\mathbb{R}_+), \quad \alpha_i \cap \alpha_j = \varnothing, \quad i \neq j,$$

form a total set in $L^2_{\mathbb{C}}(\mathbb{R}_+)^{\widehat{\otimes} n}$, it is sufficient to check (2.12) for these functions. Using property (iii) of I_G, properties of a conditional expectation and the fact that

$$\mathbb{E}[W_s|\mathcal{A}_t] = W_t, \quad t \leq s,$$

since the Wiener process W is a martingale with respect to $\{\mathcal{A}_t\}_{t\in\mathbb{R}_+}$, we get

$$\mathbb{E}[I_G f_n|\mathcal{A}_t] = \mathbb{E}[I_G(\mathbb{1}_{\alpha_1}\widehat{\otimes}\ldots\widehat{\otimes}\mathbb{1}_{\alpha_n})|\mathcal{A}_t] = \mathbb{E}[W_{\alpha_1}\ldots W_{\alpha_n}|\mathcal{A}_t] =$$

$$= \mathbb{E}\left[\prod_{i=1}^{n}\left(W_{\alpha_i\cap[0,t)} + W_{\alpha_i\cap[t,\infty)}\right) \bigm| \mathcal{A}_t\right] =$$

$$= W_{\alpha_1\cap[0,t)}\ldots W_{\alpha_n\cap[0,t)} =$$

$$= I_G(\mathbb{1}_{\alpha_1\cap[0,t)}\widehat{\otimes}\ldots\widehat{\otimes}\mathbb{1}_{\alpha_n\cap[0,t)}) = I_G(\mathbb{1}_{[0,t)^n} f_n).$$

The first part of the theorem is proved.

Let us establish the second part of the theorem. First of all we note that according to Remark 2.1 and Theorem 3.2 from [11] we have

$$\|\mathbb{I}(f)\|_{\mathcal{F}}^2 = \int_{\mathbb{R}_+} \|f(t)\|_{\mathcal{F}}^2 \, dt \quad \text{and} \quad \|(0, \widehat{f}_0, \widehat{f}_1, \ldots)\|_{\mathcal{F}}^2 = \int_{\mathbb{R}_+} \|f(t)\|_{\mathcal{F}}^2 \, dt$$

for all $f \in L_a^2(\mathbb{R}_+; \mathcal{F})$. Hence, the linear mappings

$$f \mapsto \mathbb{I}(f) \quad \text{and} \quad f \mapsto (0, \widehat{f}_0, \widehat{f}_1, \ldots)$$

act continuously (more exactly, isometrically) from $L_a^2(\mathbb{R}_+; \mathcal{F})$ to $\mathcal{F}$. Therefore, it is sufficient to check (2.11) for simple functions $f \in L_a^2(\mathbb{R}_+; \mathcal{F})$ of form

$$f(t) = g\mathbb{1}_{(s_1, s_2]}(t), \qquad g = \mathbb{1}_{\alpha_1} \widehat{\otimes} \ldots \widehat{\otimes} \mathbb{1}_{\alpha_n}, \tag{2.13}$$

where $n \in \mathbb{N}$, Borel sets $\alpha_i \in \mathcal{B}(\mathbb{R}_+)$, $i \in \{1, \ldots, n\}$, are disjoint and $(s_1, s_2] \subset \mathbb{R}_+$ (these functions form a total set in $L_a^2(\mathbb{R}_+; \mathcal{F})$). We note that if $f(\cdot) = g\mathbb{1}_{(s_1, s_2]}(\cdot) \in$ $\in L_a^2(\mathbb{R}_+; \mathcal{F})$ has form (2.13) then by assertion (I)

$$g = \mathbb{1}_{\alpha_1} \widehat{\otimes} \ldots \widehat{\otimes} \mathbb{1}_{\alpha_n} = (\mathbb{1}_{\alpha_1} \widehat{\otimes} \ldots \widehat{\otimes} \mathbb{1}_{\alpha_n}) \mathbb{1}_{[0, s_1]^n}.$$

So in this case $\alpha_i \subset [0, s_1]$. In particular $\alpha_i \cap (s_1, s_2] = \varnothing$ for all $i \in \{1, \ldots, n\}$.

Let $f(\cdot) = g\mathbb{1}_{(s_1, s_2]}(\cdot) \in L_a^2(\mathbb{R}_+; \mathcal{F})$ be of form (2.13). Evidently, in this case

$$f(t) = \Big(\underbrace{0, \ldots, 0}_{n \text{ times}}, f_n(t), 0, 0, \ldots\Big), \qquad f_n(t) := (\mathbb{1}_{\alpha_1} \widehat{\otimes} \ldots \widehat{\otimes} \mathbb{1}_{\alpha_n}) \mathbb{1}_{(s_1, s_2]}(t),$$

and

$$F := (I_G \otimes 1) f = I_G(g) \mathbb{1}_{(s_1, s_2]}$$

is a simple Itô integrable function with respect to W, i.e., $F \in L_{a,s}^2(\mathcal{D}' \times \mathbb{R}_+)$. Therefore, using Definition 2.1, equality (2.9), property (iii) of the isomorphism I_G and taking into account that $\alpha_i \in \mathcal{B}(\mathbb{R}_+)$, $i \in \{1, \ldots, n\}$, are disjoint and $\alpha_i \cap (s_1, s_2] = \varnothing$, we get

$$\mathbb{I}(f) = I_G^{-1} \left(\int_{\mathbb{R}_+} I_G(f(t)) \, dW_t \right) = I_G^{-1} \left(\int_{\mathbb{R}_+} I_G(g) \mathbb{1}_{(s_1, s_2]}(t) \, dW_t \right) =$$

$$= I_G^{-1}\big(I_G(g) W_{(s_1, s_2]}\big) = I_G^{-1}\big(I_G(\mathbb{1}_{\alpha_1} \widehat{\otimes} \ldots \widehat{\otimes} \mathbb{1}_{\alpha_n}) W_{(s_1, s_2]}\big) =$$

$$= I_G^{-1}\big((W_{\alpha_1} \cdot \ldots \cdot W_{\alpha_n}) W_{(s_1, s_2]}\big) = \mathbb{1}_{\alpha_1} \widehat{\otimes} \ldots \widehat{\otimes} \mathbb{1}_{\alpha_n} \widehat{\otimes} \mathbb{1}_{(s_1, s_2]} =$$

$$= \Big(\underbrace{0, \ldots, 0}_{n+1 \text{ times}}, \widehat{f}_n, 0, 0, \ldots\Big).$$

The theorem is proved.

Remark **2.2.** This theorem is one of the most useful results for our purpose. Analyzing the proof, we see that it does not depend upon the Gaussian character of the Wiener – Itô – Segal isomorphism I_G. One only makes use the properties (i) – (iii) of the isomorphism I_G and the fact that the Wiener process W is a martingale. This observation plays a crucial role in the construction of the extended stochastic integral with respect to any normal martingale with the CRP (see Remark 2.7).

***Remark* 2.3.** Let $(s_1, s_2] \subset \mathbb{R}_+$ be fixed. Choose a vector $g = (g_n)_{n=0}^{\infty} \in \mathcal{F}$ such that $g = (g_0, g_1 1\!\!1_{[0,s_1)}, g_2 1\!\!1_{[0,s_1)^2}, \ldots)$ and define a simple function $f \in L^2_a(\mathbb{R}_+; \mathcal{F})$ by

$$f(t) := g 1\!\!1_{(s_1,s_2]}(t) = (f_n(t))_{n=0}^{\infty}, \qquad f_n(t) := g_n 1\!\!1_{(s_1,s_2]}(t).$$

Then, according to (2.11) and (2.5) we obtain

$$\mathbb{I}(g 1\!\!1_{(s_1,s_2]}) = \mathbb{I}(f) = (0, \widehat{f}_0, \widehat{f}_1, \ldots) =$$
$$= (0, g_0 \widehat{\otimes} 1\!\!1_{(s_1,s_2]}, g_1 \widehat{\otimes} 1\!\!1_{(s_1,s_2]}, \ldots) = g \Diamond (0, 1\!\!1_{(s_1,s_2]}, 0, 0, \ldots). \tag{2.14}$$

If we compare (2.14) with (2.9) we will see the relationship between the Wick multiplication $\Diamond$ on $\mathcal{F}$ and the ordinary multiplication on $L^2(\mathcal{D}', \mu_G)$. Namely, suppose $t \in \mathbb{R}_+$ and $F \in L^2(\mathcal{D}', \mu_G)$ is an $\mathcal{A}_t$-adapted function. Then for each interval $(s_1, s_2] \subset (t, \infty)$ the function $F(W_{s_2} - W_{s_1})$ belongs to $L^2(\mathcal{D}', \mu_G)$ and the I_G^{-1}-image of $F(W_{s_2} - W_{s_1})$ has the form

$$I_G^{-1}(F(W_{s_2} - W_{s_1})) = I_G^{-1}(F) \Diamond I_G^{-1}(W_{s_2} - W_{s_1}) = I_G^{-1}(F) \Diamond (0, 1\!\!1_{(s_1,s_2]}, 0, 0, \ldots).$$

However it can be shown that in general case the I_G-image of the Wick multiplication $\Diamond$ distinguishes from the ordinary multiplication.

We next turn our attention to generalizations of the Itô integral $\mathbb{I}$. The most naive and natural idea is to define a generalization of $\mathbb{I}$ by formula (2.11) for all functions $f(\cdot) = (f_n(\cdot))_{n=0}^{\infty} \in L^2(\mathbb{R}_+; \mathcal{F})$ such that $(0, \widehat{f}_0, \widehat{f}_1, \ldots) \in \mathcal{F}$. Namely, we accept the following definition.

Definition 2.2. *For a function* $f(\cdot) = (f_n(\cdot))_{n=0}^{\infty} \in L^2(\mathbb{R}_+; \mathcal{F})$ *such that*

$$(0, \widehat{f}_0, \widehat{f}_1, \ldots) \in \mathcal{F} \quad \text{or, equivalently,} \quad \sum_{n=0}^{\infty} |\widehat{f}_n|^2_{L^2_{\mathbb{C}}(\mathbb{R}_+)^{\widehat{\otimes} n+1}} (n+1)! < \infty \tag{2.15}$$

we define its **extended stochastic integral** *by the formula*

$$\mathbb{I}_{\mathrm{ext}}(f) := (0, \widehat{f}_0, \widehat{f}_1, \ldots).$$

Applying the Wiener – Itô – Segal isomorphism to $\mathbb{I}_{\mathrm{ext}}$, we obtain the extended stochastic integral introduced by Hitsuda and Skorokhod.

Properties of $\mathbb{I}_{\mathrm{ext}}$ can be easily obtained from the corresponding properties of the extended stochastic integral on $L^2(\mathcal{D}' \times \mathbb{R}_+)$. In particular, let us consider the annihilation operator $a_-(\delta_t)$ (see (2.7)) as an unbounded one

$$a_-(\delta_\cdot) \colon \mathcal{F} \to L^2(\mathbb{R}_+; \mathcal{F}), \qquad g = (g_n)_{n=0}^{\infty} \mapsto a_-(\delta_\cdot) g = ((n+1) g_{n+1}(\cdot))_{n=0}^{\infty} \tag{2.16}$$

with the dense in $\mathcal{F}$ domain

$$\mathrm{Dom}(a_-(\delta_\cdot)) := \left\{ g = (g_n)_{n=0}^{\infty} \in \mathcal{F} \,\middle|\, \sum_{n=0}^{\infty} |g_n|^2_{L^2_{\mathbb{C}}(\mathbb{R}_+)^{\widehat{\otimes} n}} n! n < \infty \right\}.$$

Note that the I_G-image of $a_-(\delta_\cdot)$ is the so-called *Malliavin's gradient*, see, e.g., [19].

The following statement follows from, e.g., [47] (see also [17, 19]).

Theorem 2.2. *The extended stochastic integral* $\mathbb{I}_{\mathrm{ext}} \colon L^2(\mathbb{R}_+; \mathcal{F}) \to \mathcal{F}$ *and the annihilation operator* $a_-(\delta_\cdot) \colon \mathcal{F} \to L^2(\mathbb{R}_+; \mathcal{F})$ *are adjoint one to another. In particular, these operators are closed.*

Remark **2.4.** The fact that in the Gaussian case the Skorokhod integral is adjoint to the stochastic derivative (the Malliavin gradient) was proved for the first time in [47]. This result is a starting point in developing of a stochastic calculus for nonadapted processes (the so-called anticipating stochastic calculus). We refer here to the book [19] and reference therein for an exhaustive presentation of results, techniques and applications of the anticipating stochastic calculus.

Remark **2.5.** Note that one can get rid of restriction (2.15) and introduce elements of a Wick calculus considering stochastic integrals on the I_G-pre-image of a so-called *regular rigging of* $L^2(\mathcal{D}', \mu_G)$, see, e.g., [25] for details.

We will now show that the extended stochastic integral $\mathbb{I}_{\text{ext}}$ can be regarded as an ordinary Bochner one. Before establishing the corresponding result, let as first look at the following heuristic argumentation.

According to Remark 2.3 for a simple Itô integrable function

$$f(\cdot) = \sum_{k=0}^{n-1} f_{(k)} 1\!\!1_{(t_k, t_{k+1}]}(\cdot) \in L^2_a(\mathbb{R}_+; \mathcal{F}), \quad f_{(k)} \in \mathcal{F},$$

we have

$$\mathbb{I}(f) = \sum_{k=0}^{n-1} f_{(k)} \lozenge (0, 1\!\!1_{(t_k, t_{k+1}]}, 0, 0, \ldots).$$

Using this equality, (2.8) and the formal representation

$$(0, 1\!\!1_{(t_k, t_{k+1}]}, 0, 0, \ldots) = \int\limits_{(t_k, t_{k+1}]} (0, \delta_t, 0, 0, \ldots)\, dt$$

we obtain (at least formally)

$$\mathbb{I}(f) = \sum_{k=0}^{n-1} f_{(k)} \lozenge (0, 1\!\!1_{(t_k, t_{k+1}]}, 0, 0, \ldots) =$$

$$= \sum_{k=0}^{n-1} f_{(k)} \lozenge \int\limits_{(t_k, t_{k+1}]} (0, \delta_t, 0, 0, \ldots)\, dt =$$

$$= \sum_{k=0}^{n-1} \int\limits_{(t_k, t_{k+1}]} f_{(k)} \lozenge (0, \delta_t, 0, 0, \ldots)\, dt =$$

$$= \int\limits_{\mathbb{R}_+} \left(\sum_{k=0}^{n-1} f_{(k)} 1\!\!1_{(t_k, t_{k+1}]}(t) \right) \lozenge (0, \delta_t, 0, 0, \ldots)\, dt =$$

$$= \int\limits_{\mathbb{R}_+} f(t) \lozenge (0, \delta_t, 0, 0, \ldots)\, dt = \int\limits_{\mathbb{R}_+} a_+(\delta_t) f(t)\, dt.$$

Since the delta-function δ_t is not a square integrable one, the last formula can not be accepted as a definition of the extended stochastic integral on $L^2(\mathbb{R}_+; \mathcal{F})$. However from results of [17, 11] the correctness of the following definition follows.

Definition 2.3. *The extended stochastic integral of a function*

$$\xi(\cdot) = (\xi_n(\cdot))_{n=0}^{\infty} \in L^2(\mathbb{R}_+; \mathcal{F}(-\tau, -q))$$

is defined as a Bochner one in the space $\mathcal{F}(-\tau, -q)$ *(see* (2.4)*),* τ *is such that* $\int_{\mathbb{R}_+} |\delta_t|^2_{D_{-\tau}} dt < \infty$, *by the formula*

$$\widehat{\mathbb{I}}_{\text{ext}}(\xi) := \int_{\mathbb{R}_+} a_+(\delta_t)\xi(t)\,dt = \int_{\mathbb{R}_+} \xi(t)\Diamond(0, \delta_t, 0, 0, \ldots)\,dt \in \mathcal{F}(-\tau, -q). \tag{2.17}$$

Not complicated direct calculation shows that

$$\widehat{\mathbb{I}}_{\text{ext}}(\xi) = (0, \widehat{\xi}_0, \widehat{\xi}_1, \ldots),$$

where each $\widehat{\xi}_n \in D_{-\tau,\mathbb{C}}^{\widehat{\otimes} n+1}$ is the projection of $\xi_n(\cdot) \in D_{-\tau,\mathbb{C}}^{\widehat{\otimes} n+1} \otimes L^2(\mathbb{R}_+)$ onto $D_{-\tau,\mathbb{C}}^{\widehat{\otimes} n+1}$. This property means in particular that $\widehat{\mathbb{I}}_{\text{ext}}$ is an extension of $\mathbb{I}_{\text{ext}}$, i.e.,

$$\mathbb{I}_{\text{ext}}(f) = \widehat{\mathbb{I}}_{\text{ext}}(f) = \int_{\mathbb{R}_+} a_+(\delta_t)f(t)\,dt, \quad f \in \text{Dom}\,(\mathbb{I}_{\text{ext}}).$$

This result explains the same name for the integrals $\mathbb{I}_{\text{ext}}$ and $\widehat{\mathbb{I}}_{\text{ext}}$.

It can be easily shown that the analog of Theorem 2.2 holds true for operators

$$\widehat{\mathbb{I}}_{\text{ext}} \colon L^2(\mathbb{R}_+; \mathcal{F}(-\tau, -q)) \to \mathcal{F}(-\tau, -q), \qquad a_-(\delta.) \colon \mathcal{F}(\tau, q) \to L^2(\mathbb{R}_+; \mathcal{F}(\tau, q)),$$

where $a_-(\delta.)$ is the restriction of operator (2.16) on $\mathcal{F}(\tau, q)$. Moreover, now $\widehat{\mathbb{I}}_{\text{ext}}$ and $a_-(\delta.)$ are continuous operators.

Remark **2.6.** The I_G-image of integral $\widehat{\mathbb{I}}_{\text{ext}}$ has the form

$$I_G\big(\widehat{\mathbb{I}}_{\text{ext}}(\xi)\big) = \int_{\mathbb{R}_+} \partial^+(\delta_t)\Psi(t)\,dt = \int_{\mathbb{R}_+} \Psi(t)\Diamond_D \dot{W}_t\,dt,$$

where $\Psi(t) := I_G\xi(t)$, $\partial^+(\delta_t) := I_G a_+(\delta_t) I_G^{-1} \colon (\mathcal{D}')_- \to (\mathcal{D}')_-$ is an adjoint operator to the Hida derivative $\partial(\delta_t)$, $\dot{W}_t := \langle \cdot, \delta_t \rangle = I_G(0, \delta_t, 0, 0, \ldots) \in (\mathcal{D}')_-$ is the so-called Gaussian white noise and $\Diamond_D$ denotes the Wick product in $(\mathcal{D}')_-$, i.e.,

$$\Psi \Diamond_D \Phi := I_G(I_G^{-1}\Psi \Diamond I_G^{-1}\Phi), \quad \Psi,\ \Phi \in (\mathcal{D}')_-.$$

Thus in such a way we obtain the well-known presentation

$$\int_{\mathbb{R}_+} \Psi(t)\widehat{d}W_t = \int_{\mathbb{R}_+} \partial^+(\delta_t)\Psi(t)\,dt = \int_{\mathbb{R}_+} \Psi(t)\Diamond_D \dot{W}_t\,dt,$$

where $\int_{\mathbb{R}_+} \circ(t)\widehat{d}W_t$ denotes the extended (Hitsuda – Skorokhod) integral with respect to W (see, e.g., [17, 48, 44] and reference therein for more details).

Remark **2.7.** Let $(\Omega, \mathcal{A}, P)$ be a complete probability space with a right continuous filtration $\{\mathcal{A}_t\}_{t \in \mathbb{R}_+}$, i.e., $\mathcal{A}_s \subset \mathcal{A}_t$ if $s \le t$ and $\mathcal{A}_t = \bigcap_{s>t} \mathcal{A}_s$ for all $t \in \mathbb{R}_+$. Suppose that $\mathcal{A}$ coincides with the smallest σ-algebra generated by $\bigcup_{t \in \mathbb{R}_+} \mathcal{A}_t$ and $\mathcal{A}_0$ contains all P-zero sets of $\mathcal{A}$. In addition, suppose that $\mathcal{A}_0$ is trivial, that is for each $\alpha \in \mathcal{A}_0$ we have $P(\alpha) = 0$ or $P(\alpha) = 1$.

By definition a process $N = \{N_t\}_{t\in\mathbb{R}_+}$, $N_0 = 0$, is a **normal martingale** on $(\Omega, \mathcal{A}, P)$ with respect to $\{\mathcal{A}_t\}_{t\in\mathbb{R}_+}$ if $\{N_t\}_{t\in\mathbb{R}_+}$ and $\{N_t^2 - t\}_{t\in\mathbb{R}_+}$ are martingales with respect to $\{\mathcal{A}_t\}_{t\in\mathbb{R}_+}$. This means that for all $s, t \in \mathbb{R}_+$ such that $s \le t$

$$\mathbb{E}[N_t - N_s|\mathcal{A}_s] = 0, \qquad \mathbb{E}[(N_t - N_s)^2|\mathcal{A}_s] = t - s,$$

where as before $\mathbb{E}[\,\cdot\,|\mathcal{A}_s]$ denotes the conditional expectation with respect to $\mathcal{A}_s$.

It is known (see, for example, [13, 15]) that a mapping

$$I_N\colon \mathcal{F} \to L^2(\Omega, \mathcal{A}, P), \qquad f = (f_n)_{n=0}^{\infty} \mapsto I_N f := \sum_{n=0}^{\infty} I_{N,n}(f_n),$$

is a well-defined isometry. Here $I_{N,0}(f_0) := f_0$ and, for each $n \in \mathbb{N}$,

$$I_{N,n}(f_n) := n! \int_{\Delta^n} f_n(t_1, \ldots, t_n)\, dN_{t_1} \ldots dN_{t_n},$$

$$\Delta^n = \big\{(t_1, \ldots, t_n) \in \mathbb{R}_+^n \,\big|\, t_1 < \ldots < t_n\big\},$$

is an iterated stochastic integral with respect to N. The integrals $I_{N,n}(f_n)$ have the isometry property

$$\big\|I_{N,n}(f_n)\big\|^2_{L^2(\Omega,\mathcal{A},P)} = (n!)^2 \int_{\Delta^n} |f_n(t_1, \ldots, t_n)|^2\, dt_1 \ldots dt_n = |f_n|^2_{L^2_{\mathbb{C}}(\mathbb{R}_+)^{\widehat{\otimes} n}} n!,$$

and, moreover, the orthogonality property

$$(I_{N,n}(f_n), I_{N,m}(f_m))_{L^2(\Omega,\mathcal{A},P)} = \begin{cases} 0, & n \ne m, \\ |f_n|^2_{L^2_{\mathbb{C}}(\mathbb{R}_+)^{\widehat{\otimes} n}} n!, & n = m. \end{cases}$$

When $I_N\colon \mathcal{F} \to L^2(\Omega, \mathcal{A}, P)$ is a unitary operator (i.e., I_N isometrically maps the whole space $\mathcal{F}$ onto whole $L^2(\Omega, \mathcal{A}, P)$) one says that N possesses the **Chaotic Representation Property** (CRP). The unique decomposition of $F \in L^2(\Omega, \mathcal{A}, P)$ as $F = \sum_{n=0}^{\infty} I_{N,n}(f_n)$ is called the chaotic expansion of F. We observe that the standard Wiener process W, the compensated Poisson process and some Azéma martingales are examples of normal martingales, which possess the CRP. We refer to [13, 15, 9, 16, 49] for more information about normal martingales and their properties.

Let N be a normal martingale with CRP. Then as in the Gaussian case the mapping I_N is completely characterized by the following properties:

(i) $I_N\colon \mathcal{F} \to L^2(\Omega, \mathcal{A}, P)$ is a unitary operator;

(ii) $I_{N,0}(f_0) = f_0$ for all $f_0 \in \mathbb{C}$;

(iii) for each $n \in \mathbb{N}$ and any disjoint Borel sets $\alpha_1, \ldots, \alpha_n$ of finite Lebesgue measure,

$$I_{N,n}(1\!\!1_{\alpha_1} \widehat{\otimes} \ldots \widehat{\otimes} 1\!\!1_{\alpha_n}) = N(\alpha_1) \cdot \ldots \cdot N(\alpha_n),$$

where $\mathcal{B}(\mathbb{R}_+) \ni \alpha \mapsto N(\alpha) \in L^2(\Omega, \mathcal{A}, P)$ is a vector-valued measure generated by the normal martingale N, i.e., we set

$$N((s_1,s_2]) = N_{s_2} - N_{s_1}, \qquad N(\{0\}) := N_0 = 0, \qquad N(\varnothing) := 0,$$

and extend this definition to all Borel subsets of $\mathbb{R}_+$.

Since I_N has properties (i) – (iii) and the proof of Theorem 2.1 is based on the corresponding properties of the Wiener – Itô – Segal isomorphism only, we can conclude that the I_N-image of $\mathbb{I}$ is the Itô integral with respect to the normal martingale N and as a consequence the I_N-image of $\mathbb{I}_{\mathrm{ext}}$ gives an extension of this Itô integral. We refer to [9] for properties and applications of extended stochastic integrals connected with normal martingales.

3. The generalized Meixner measure and the extended Fock space. Recall the definition of the generalized Meixner measure on $\mathcal{D}^{'}$, see [30].

Let us fix arbitrary functions

$$\alpha\colon \mathbb{R}_+ \to \mathbb{C}, \qquad \beta\colon \mathbb{R}_+ \to \mathbb{C}$$

that are smooth and satisfy the conditions

$$\theta(s) := -\alpha(s) - \beta(s) \in \mathbb{R}, \qquad \eta(s) := \alpha(s)\beta(s) \in \mathbb{R}_+$$

for each $s \in \mathbb{R}_+$. We also assume that the functions θ and η are bounded on $\mathbb{R}_+$. Note that in a certain sense η is a *“key parameter”*, which will be used often below.

For each $s \in \mathbb{R}_+$ denote by $\nu_{\alpha(s),\beta(s)}$ a probability measure on $\mathbb{R}$ that is defined by its Fourier transform

$$\int_{\mathbb{R}} e^{i\lambda t}\, \nu_{\alpha(s),\beta(s)}(dt) = \exp\Bigg(-i\lambda\big(\alpha(s)+\beta(s)\big) +$$

$$+\, 2\sum_{m=1}^{\infty} \frac{(\alpha(s)\beta(s))^m}{m} \left[\sum_{n=2}^{\infty} \frac{(-i\lambda)^n}{n!}\big(\beta(s)^{n-2} + \beta(s)^{n-3}\alpha(s) + \ldots + \alpha(s)^{n-2}\big)\right]^m\Bigg).$$

Definition 3.1. *We say that a probability measure μ on the measurable space $(\mathcal{D}',\mathcal{C}(\mathcal{D}'))$ with the Fourier transform*

$$\int_{\mathcal{D}'} e^{i\langle x,\varphi\rangle}\, \mu(dx) = \exp\left[\int_{\mathbb{R}_+}\int_{\mathbb{R}} \big(e^{it\varphi(s)} - 1 - it\varphi(s)\big)\frac{1}{t^2}\, \nu_{\alpha(s),\beta(s)}(dt)\, ds\right], \quad \varphi \in \mathcal{D},$$

is called the generalized Meixner measure.

Theorem 3.1 [30]. *The measure μ is a generalized stochastic process with independent values in the sense of* [50]. *The Laplace transform of μ is a holomorphic at $0 \in \mathcal{D}_{\mathbb{C}}$ function.*

Let α and β be constants. Accordingly to the classical classification [51] (see also [36, 37, 30]) μ is the Gaussian measure for $\alpha = \beta = 0$; μ is the centered Poissonian measure for $\alpha \neq 0$, $\beta = 0$; μ is the centered Gamma measure [26, 31] for $\alpha = \beta \neq 0$; μ is the centered Pascal measure [33] for $\alpha \neq \beta$, $\alpha\beta \neq 0$, $\alpha,\beta \in \mathbb{R}$; μ is the centered Meixner measure for $\alpha = \overline{\beta}$, $\mathrm{Im}(\alpha) \neq 0$. Thus the *“key parameter”* $\eta = 0$ *if and only if μ is the Gaussian or Poissonian measure.*

Denote by $(L^2) := L^2(\mathcal{D}',\mu)$ the space of complex-valued square integrable with respect to μ functions on $\mathcal{D}'$. A function

$$\mathcal{D}' \ni x \mapsto F(x) = \sum_{k=0}^{n} \langle x^{\otimes k}, \varphi_k \rangle \in \mathbb{C}, \qquad \varphi_k \in \mathcal{D}_{\mathbb{C}}^{\widehat{\otimes} k}, \quad \varphi_n \neq 0,$$

is called a continuous polynomial on $\mathcal{D}'$ of order n. Since the measure μ has a holomorphic at $0 \in \mathcal{D}_{\mathbb{C}}$ Laplace transform (Theorem 3.1), the set of all continuous polynomials on $\mathcal{D}'$ is dense in (L^2) [52]. Due to this fact, using the procedure of orthogonalization of polynomials (see, e.g., [21] for details) one can construct an orthogonal decomposition of the space (L^2). Namely, for $n \in \mathbb{Z}_+$ let $\mathcal{P}_n$ be the set of all continuous polynomials on $\mathcal{D}'$ of order $\leq n$, $\widetilde{\mathcal{P}}_n$ be the closure of $\mathcal{P}_n$ in (L^2) and $(L^2_n) := \widetilde{\mathcal{P}}_n \ominus \widetilde{\mathcal{P}}_{n-1}$, where $\ominus$ denotes the orthogonal difference in (L^2), $(L^2_0) := \mathbb{C}$. Thus we can regard (L^2) as the orthogonal direct sum of subspaces (L^2_n), i.e.,

$$(L^2) = \bigoplus_{n=0}^{\infty} (L^2_n).$$

We pass now to the construction of the extended Fock space. To this end, for each $\varphi_n \in \mathcal{D}_{\mathbb{C}}^{\widehat{\otimes} n}$ we define $:\langle x^{\otimes n}, \varphi_n \rangle:$ as the orthogonal projection of $\langle x^{\otimes n}, \varphi_n \rangle$ onto (L^2_n). It follows from results of [30] that $:\langle x^{\otimes n}, \varphi_n \rangle: = \langle P_n(x), \varphi_n \rangle$, where $P_n(x) \in \mathcal{D}'^{\widehat{\otimes} n}$ and for μ-almost all $x \in \mathcal{D}'$

$$P_0(x) = 1, \qquad P_1(x) = x,$$

and for all $\varphi_n \in \mathcal{D}_{\mathbb{C}}^{\widehat{\otimes} n}$, $\psi \in \mathcal{D}_{\mathbb{C}}$

$$\langle P_{n+1}(x), \varphi_n \widehat{\otimes} \psi \rangle = \langle P_n(x), \varphi_n \rangle \langle P_1(x), \psi \rangle -$$
$$- n \langle P_n(x), \Pr\left[\theta(\cdot)\psi(\cdot)\varphi_n(\cdot, \cdot_2, \ldots, \cdot_n)\right] \rangle -$$
$$- n \langle P_{n-1}(x), \varphi_n^{\psi} \rangle - n(n-1) \langle P_{n-1}(x), \Pr\left[\eta(\cdot)\psi(\cdot)\varphi_n(\cdot, \cdot, \cdot_3 \ldots, \cdot_n)\right] \rangle. \tag{3.1}$$

Here $\Pr$ denotes the symmetrization operator and

$$\varphi_n^{\psi}(\cdot_1, \ldots, \cdot_{n-1}) := \int_{\mathbb{R}_+} \varphi_n(\cdot_1, \ldots, \cdot_{n-1}, t)\psi(t)\, dt \in \mathcal{D}_{\mathbb{C}}^{\widehat{\otimes} n-1}.$$

It should be noticed that $\langle P_n(\cdot), \varphi_n \rangle$, $n \in \mathbb{Z}_+$, are Schefer polynomials, i.e., orthogonal polynomials with a generating function of exponential type, see [37, 36, 30].

Let $\mathcal{F}_{n,\mathrm{ext}}$ be a Hilbert space that is obtained as the closure of $\mathcal{D}_{\mathbb{C}}^{\widehat{\otimes} n}$ with respect to the norm $|\cdot|_{\mathcal{F}_{n,\mathrm{ext}}}$ generated by the scalar product

$$(\varphi_n, \psi_n)_{\mathcal{F}_{n,\mathrm{ext}}} := \frac{1}{n!} \int_{\mathcal{D}'} \langle P_n(x), \varphi_n \rangle \langle P_n(x), \psi_n \rangle \, \mu(dx), \quad \varphi_n, \ \psi_n \in \mathcal{D}_{\mathbb{C}}^{\widehat{\otimes} n}$$

(note that since this scalar product is real, $|\cdot|_{\mathcal{F}_{n,\mathrm{ext}}} = \sqrt{(\cdot, \bar{\cdot})_{\mathcal{F}_{n,\mathrm{ext}}}}$). Note that $\mathcal{F}_{n,\mathrm{ext}}$ depends on a parameter η; but we omit this parameter for simplification of notation. In the situations when the dependence on η is significant we specify this.

It is possible to give an inner description of the scalar product in the space $\mathcal{F}_{n,\mathrm{ext}}$. Namely, according to [25] (see also [30]) we have

$$(\varphi_n, \psi_n)_{\mathcal{F}_{n,\mathrm{ext}}} = \sum_{\substack{k, l_j, s_j \in \mathbb{N}:\ j=1,\ldots,k,\ l_1 > l_2 > \ldots > l_k,\\ l_1 s_1 + \ldots + l_k s_k = n}} \frac{n!}{l_1^{s_1} \ldots l_k^{s_k} s_1! \ldots s_k!} \times$$

$$\times \int_{\mathbb{R}_+^{s_1+\ldots+s_k}} \varphi_n \Big(\underbrace{t_1, \ldots, t_1}_{l_1}, \ldots, \underbrace{t_{s_1}, \ldots, t_{s_1}}_{l_1}, \ldots, \underbrace{t_{s_1+\ldots+s_k}, \ldots, t_{s_1+\ldots+s_k}}_{l_k} \Big) \times$$

$$\times \psi_n \Big(\underbrace{t_1, \ldots, t_1}_{l_1}, \ldots, \underbrace{t_{s_1}, \ldots, t_{s_1}}_{l_1}, \ldots, \underbrace{t_{s_1+\ldots+s_k}, \ldots, t_{s_1+\ldots+s_k}}_{l_k} \Big) \times$$

$$\times \eta(t_1)^{l_1-1} \ldots \eta(t_{s_1})^{l_1-1} \eta(t_{s_1+1})^{l_2-1} \ldots \eta(t_{s_1+s_2})^{l_2-1} \ldots \eta(t_{s_1+\ldots+s_{k-1}+1})^{l_k-1} \ldots$$

$$\ldots \eta(t_{s_1+\ldots+s_k})^{l_k-1}\, dt_1 \ldots dt_{s_1+\ldots+s_k}. \qquad (3.2)$$

It follows from (3.2) that actually $\mathcal{F}_{n,\mathrm{ext}}$ is not connected directly with the measure μ and depends on the function η only. Moreover, it can be shown that for all $n \in \mathbb{N}$

$$\mathcal{F}_{n,\mathrm{ext}} \subseteq \widehat{L}^2(\mathbb{R}_+^n, \rho_n) := \big\{ f_n \in L^2(\mathbb{R}_+^n, \rho_n) \colon f_n \text{ is symmetric in all variables} \big\}, \quad (3.3)$$

where the Borel measure ρ_n is constructed by using (3.2). In particular, ρ_1 is the Lebesgue measure on $\mathbb{R}_+$. If μ is the Gaussian or Poissonian measure then $\eta = 0$ and therefore ρ_n is the Lebesgue measure on $\mathcal{B}(\mathbb{R}_+^n)$. We refer a reader to [32] for a more detailed discussion of spaces like $\mathcal{F}_{n,\mathrm{ext}}$.

Definition 3.2. *We define the* **extended Fock space** $\mathcal{F}_{\mathrm{ext}}$ *by the formula*

$$\mathcal{F}_{\mathrm{ext}} := \bigoplus_{n=0}^{\infty} \mathcal{F}_{n,\mathrm{ext}} n!, \qquad \mathcal{F}_{0,\mathrm{ext}} := \mathbb{C}.$$

Thus $\mathcal{F}_{\mathrm{ext}}$ *is a complex Hilbert space of sequences* $f = (f_n)_{n=0}^{\infty}$, $f_n \in \mathcal{F}_{n,\mathrm{ext}}$ *such that*

$$\|f\|_{\mathcal{F}_{\mathrm{ext}}}^2 = \sum_{n=0}^{\infty} |f_n|_{\mathcal{F}_{n,\mathrm{ext}}}^2 n! < \infty.$$

***Remark* 3.1.** Let us explain the term the *extended* Fock space. It is not difficult to show by analogy with [32] that the space $\mathcal{F}_{n,\mathrm{ext}}$ is, generally speaking, the orthogonal sum of $L^2_{\mathbb{C}}(\mathbb{R}_+)^{\widehat{\otimes} n}$ and some another Hilbert spaces. In this sense $\mathcal{F}_{n,\mathrm{ext}}$ *is an extension of* $L^2_{\mathbb{C}}(\mathbb{R}_+)^{\widehat{\otimes} n}$ and therefore $\mathcal{F}_{\mathrm{ext}}$ is an extension of $\mathcal{F}$.

One can give another explanation of the fact that $\mathcal{F}_{n,\mathrm{ext}}$ is a more wide space than $L^2_{\mathbb{C}}(\mathbb{R}_+)^{\widehat{\otimes} n}$. Namely, let $f_n \in L^2_{\mathbb{C}}(\mathbb{R}_+)^{\widehat{\otimes} n}$ (f_n is an equivalence class in $L^2_{\mathbb{C}}(\mathbb{R}_+)^{\widehat{\otimes} n}$). We select a representative (a function) $\dot{f}_n \in f_n$ with a "zero diagonal", i.e., $\dot{f}_n(t_1, \ldots, t_n) = 0$ if there exist $i, j \in \{1, \ldots, n\}$, $i \neq j$ such that $t_i = t_j$. This function generates the equivalence class $\widetilde{f}_n$ in $\mathcal{F}_{n,\mathrm{ext}}$ that can be identified with f_n (see [25] for details).

Let $\mathcal{F}_{\mathrm{fin}}$ denote the set of all finite sequences $(\varphi_n)_{n=0}^{\infty}$, $\varphi_n \in \mathcal{D}_{\mathbb{C}}^{\widehat{\otimes} n}$. It is clear that $\mathcal{F}_{\mathrm{fin}}$ is a dense subset of $\mathcal{F}_{\mathrm{ext}}$ and the mapping

$$\mathcal{F}_{\mathrm{ext}} \supset \mathcal{F}_{\mathrm{fin}} \ni \varphi = (\varphi_n)_{n=0}^{\infty} \mapsto (I\varphi)(\cdot) := \sum_{n=0}^{\infty} \langle P_n(\cdot), \varphi_n \rangle \in (L^2)$$

(the series, in fact, finite) is isometric. Extending this mapping by continuity to the whole space $\mathcal{F}_{\mathrm{ext}}$ we obtain a unitary operator acting between $\mathcal{F}_{\mathrm{ext}}$ and (L^2). We keep the notation I for the extension, and we will refer to the operator $I\colon \mathcal{F}_{\rm ext}\to (L^2)$ as to the **generalized Wiener – Itô – Segal isomorphism**. Note that I is the Fourier transform of a *Jacobi field* that act in the extended Fock space $\mathcal{F}_{\rm ext}$, see [30] and also [31 – 38].

In what follows, for each $f_n\in\mathcal{F}_{n,\rm ext}$ we set

$$\langle P_n, f_n\rangle := I\biggl(\underbrace{0,\dots,0}_{n\ \text{times}}, f_n, 0,\dots\biggr).$$

Then for each $f=(f_n)_{n=0}^\infty\in\mathcal{F}_{\rm ext}$ we have

$$If=\sum_{n=0}^\infty\langle P_n, f_n\rangle\in (L^2). \tag{3.4}$$

4. The Itô integral on the extended Fock space. We consider the *Meixner random process*

$$\bigl\{M_t(\cdot):=\langle\cdot\,,1\!\!1_{[0,t)}\rangle\bigr\}_{t\in\mathbb{R}_+}$$

on the probability space $\bigl(\mathcal{D}',\mathcal{C}(\mathcal{D}'),\mu\bigr)$. It follows from Theorem 3.1 that this process has orthogonal independent increments. Since in addition $M.$ is locally square integrable, this process is a normal martingale with respect to the natural filtration of σ-algebras $\mathcal{A}_t:=\sigma\{M_s\,|\,s\le t\}$ (the filtration is made complete and right continuous).

***Remark* 4.1.** Note that if the parameter η from the definition of the measure μ is not a constant then $M.$ is not a Lévy process because in this case $M.$ is not a time homogeneous one.

We will define the Itô integral on the extended Fock space as the I^{-1}-image of the classical Itô stochastic integral with respect to the Meixner process. Namely, denote by $(L^2)\otimes L^2(\mathbb{R}_+)^a$ the set of all adapted with respect to the filtration $\{\mathcal{A}_t\}_{t\in\mathbb{R}_+}$ functions from the space $L^2(\mathcal{D}'\times\mathbb{R}_+,\mathcal{C}(\mathcal{D}')\times\mathcal{B}(\mathbb{R}_+),\mu\times dt)\cong(L^2)\otimes L^2(\mathbb{R}_+)$, i.e.,

$$(L^2)\otimes L^2(\mathbb{R}_+)^a:=\Bigl\{F\in(L^2)\otimes L^2(\mathbb{R}_+)\,\Bigm|\,F(\cdot,t)=\mathbb{E}[F(\cdot,t)\,|_{\mathcal{A}_t}]\ \text{for a.a.}\ t\in\mathbb{R}_+\Bigr\}. \tag{4.1}$$

Definition 4.1. *We say that a function* $f\in L^2(\mathbb{R}_+;\mathcal{F}_{\rm ext})\cong\mathcal{F}_{\rm ext}\otimes L^2(\mathbb{R}_+)$ *is Itô integrable if* $(I\otimes 1)f$ *belongs to* $(L^2)\otimes L^2(\mathbb{R}_+)^a$, *i.e., if*

$$f\in L^2_a(\mathbb{R}_+;\mathcal{F}_{\rm ext})\cong\mathcal{F}_{\rm ext}\otimes L^2(\mathbb{R}_+)^a:=(I\otimes 1)^{-1}(L^2)\otimes L^2(\mathbb{R}_+)^a.$$

On this case the **Itô integral** *of* $f\in L^2_a(\mathbb{R}_+;\mathcal{F}_{\rm ext})$ *is defined as an element of* $\mathcal{F}_{\rm ext}$ *given by*

$$\mathbb{I}(f):=I^{-1}\left(\int_{\mathbb{R}_+}I(f(t))\,dM_t\right).$$

Before giving an inner description of the set $L^2_a(\mathbb{R}_+;\mathcal{F}_{\rm ext})$ and express the Itô integral in terms of the extended Fock space $\mathcal{F}_{\rm ext}$, let us look at the generalized Wiener – Itô – Segal isomorphism I more carefully. First of all we note that this isomorphism has analogs of properties (i) – (iii) of the Wiener – Itô – Segal isomorphism I_G, i.e.,

(i) $I\colon\mathcal{F}_{\rm ext}\to(L^2)$ is a unitary operator;

(ii) $I(f_0,0,0,\dots)=f_0$ for all $f_0\in\mathbb{C}$;

(iii) for each $n \in \mathbb{N}$ and any disjoint Borel sets $\alpha_1, \ldots, \alpha_n$ of finite Lebesgue measure

$$\Big(I(\underbrace{0, \ldots, 0}_{n \text{ times}}, 1\!\!1_{\alpha_1} \widehat{\otimes} \ldots \widehat{\otimes} 1\!\!1_{\alpha_n}, 0, 0, \ldots)\Big)(\cdot) = M_{\alpha_1}(\cdot) \ldots M_{\alpha_n}(\cdot),$$

where $M_{\alpha_k}(\cdot) := \langle \cdot \,, 1\!\!1_{\alpha_k}\rangle$ for all $k \in \{1, \ldots, n\}$.

However in contrast to the Gaussian case if $\eta \neq 0$ then the isomorphism I is not uniquely determined by its properties (i) – (iii) because the set

$$\mathbb{C} \bigoplus \operatorname{span} \Bigg\{ \Big(\underbrace{0, \ldots, 0}_{n \text{ times}}, 1\!\!1_{\alpha_1} \widehat{\otimes} \ldots \widehat{\otimes} 1\!\!1_{\alpha_n}, 0, 0, \ldots \Big) \Bigg| \; n \in \mathbb{N};\; \alpha_i \in \mathcal{B}(\mathbb{R}_+);\; \alpha_i \cap \alpha_j = \varnothing,\; i \neq j \Bigg\},$$

is not dense in the extended Fock space $\mathcal{F}_{\text{ext}}$. Therefore in order to give a description of the set $L^2_a(\mathbb{R}_+; \mathcal{F}_{\text{ext}})$ and to express the Itô integral in terms of the extended Fock space structure one can not use the (based on (i) – (iii)) scheme of the proof of Theorem 2.1. Somehow one must use another properties of the isomorphism I. As it follows from [25] an appropriate property of I is recurrence relation (3.1), see details below.

We have the following result (cf. Theorem 2.1).

Theorem 4.1. *A function $f(\cdot) = (f_n(\cdot))_{n=0}^{\infty} \in L^2(\mathbb{R}_+; \mathcal{F}_{\text{ext}})$ belongs to $L^2_a(\mathbb{R}_+; \mathcal{F}_{\text{ext}})$ if and only if for almost all $t \in \mathbb{R}_+$*

$$f(t) = \big(f_0(t), f_1(t) 1\!\!1_{[0,t)}, \ldots, f_n(t) 1\!\!1_{[0,t)^n}, \ldots\big).$$

Taking into account (4.1) and the definition of the space $L^2_a(\mathbb{R}_+; \mathcal{F}_{\text{ext}})$, Theorem 4.1 is an immediate consequence of the equality

$$\mathbb{E}\big[\langle P_0, f_0\rangle | \mathcal{A}_t\big] = \langle P_0, f_0\rangle = f_0, \qquad t \in \mathbb{R}_+, \quad f_0 \in \mathbb{C},$$

and the following statement:

Theorem 4.2. *Let $f_n \in \mathcal{F}_{n,\text{ext}}$, $n \in \mathbb{N}$. Then for all $t \in \mathbb{R}_+$*

$$\mathbb{E}\big[\langle P_n, f_n\rangle | \mathcal{A}_t\big] = \langle P_n, f_n 1\!\!1_{[0,t)^n}\rangle. \tag{4.2}$$

Proof. Let us fix $t \in \mathbb{R}_+$. Since a conditional expectation $\mathbb{E}[\,\cdot\,|\mathcal{A}_t]$ is an orthogonal projection in (L^2), it is sufficient to prove (4.2) on a total in $\mathcal{F}_{n,\text{ext}}$ set. We use the induction with respect to n. For $n = 1$ equality (4.2) is fulfilled because

$$\mathbb{E}[\langle P_1, 1\!\!1_{[a,b)}\rangle | \mathcal{A}_t] = \mathbb{E}[\langle P_1, 1\!\!1_{[0,b)}\rangle | \mathcal{A}_t] - \mathbb{E}[\langle P_1, 1\!\!1_{[0,a)}\rangle | \mathcal{A}_t] =$$

$$= \mathbb{E}[M_b | _{\mathcal{A}_t}] - \mathbb{E}[M_a | \mathcal{A}_t] = M_{\min\{b,t\}} - M_{\min\{a,t\}} = \langle P_1, 1\!\!1_{[a,b)} 1\!\!1_{[0,t)}\rangle$$

and the set of indicators $1\!\!1_{[a,b)}$ of intervals $[a, b) \subset \mathbb{R}_+$ is total in $\mathcal{F}_{1,\text{ext}} = L^2_{\mathbb{C}}(\mathbb{R}_+)$. Assume (4.2) is fulfilled for $n \in \{1, 2, \ldots, m\}$ and let us prove this statement for $n = m + 1$. To this end, we need the following technical result.

Lemma 4.1. *Let $t \in \mathbb{R}_+$ and $n \in \mathbb{N}$. The set*

$$\Big\{ \varphi^{\otimes k} \widehat{\otimes} \psi^{\otimes n-k} \;\big|\; \varphi, \psi \in \mathcal{D}_{\mathbb{C}},\ \operatorname{supp}\varphi \subset [0, t),\ \operatorname{supp}\psi \subset [t, \infty),\ k \in \{0, 1, \ldots, n\} \Big\} \tag{4.3}$$

is total in the space $\mathcal{F}_{n,\text{ext}}$.

Proof. Let $t \in \mathbb{R}_+$ and $n \in \mathbb{N}$. For $\phi \in \mathcal{D}_{\mathbb{C}}$ we set $\phi_1 := \phi 1\!\!1_{[0,t)}$, $\phi_2 := \phi 1\!\!1_{[t,\infty)}$. Then

$$\phi^{\otimes n} = (\phi_1 + \phi_2)^{\otimes n} = \sum_{k=0}^{n} C_n^k \phi_1^{\otimes k} \widehat{\otimes} \phi_2^{\otimes n-k}, \tag{4.4}$$

where $\phi_1^{\otimes k} \widehat{\otimes} \phi_2^{\otimes n-k}$ denotes the symmetrization with respect to all variables of the function

$$\phi_1(\cdot_1) \ldots \phi_1(\cdot_k)\phi_2(\cdot_{k+1}) \ldots \phi_2(\cdot_n)$$

(note that $\phi_1^{\otimes k} \widehat{\otimes} \phi_2^{\otimes n-k}$ is a symmetric function, but not necessary from $\mathcal{D}_{\mathbb{C}}^{\widehat{\otimes} n}$). One can show by direct calculation that each $\phi_1^{\otimes k} \widehat{\otimes} \phi_2^{\otimes n-k}$ belongs to $\mathcal{F}_{n,\mathrm{ext}}$ and can be approximated in this space by a sequence

$$\left\{ \varphi_l^{\otimes k} \widehat{\otimes} \psi_l^{\otimes n-k} \,\middle|\, \varphi_l, \psi_l \in \mathcal{D}_{\mathbb{C}},\ \operatorname{supp} \varphi_l \subset [0,t),\ \operatorname{supp} \psi_l \subset [t,\infty) \} \right\}_{l=0}^{\infty} \tag{4.5}$$

(one can select $\varphi_l \to \phi_1$, $\psi_l \to \phi_2$ pointwisely as $l \to \infty$, then $\varphi_l^{\otimes k} \widehat{\otimes} \psi_l^{\otimes n-k} \to$ $\to \phi_1^{\otimes k} \widehat{\otimes} \phi_2^{\otimes n-k}$ in $\mathcal{F}_{n,\mathrm{ext}}$ as $l \to \infty$ by the Lebesgue theorem).

Let now $f_n \in \mathcal{F}_{n,\mathrm{ext}}$ be fixed. In order to prove the lemma, it is sufficient to check that f_n can be approximated by linear combinations of elements of set (4.3). Since the set $\{\phi^{\otimes n} \,|\, \phi \in \mathcal{D}_{\mathbb{C}}\}$ is total in $\mathcal{F}_{n,\mathrm{ext}}$, for arbitrary $\varepsilon > 0$ there exist $N \in \mathbb{N}$, constants $c_1, \ldots, c_N$ and functions $\phi_{(1)}, \ldots, \phi_{(N)} \in \mathcal{D}_{\mathbb{C}}$ such that

$$\left| f_n - \sum_{s=1}^{N} c_s \phi_{(s)}^{\otimes n} \right|_{\mathcal{F}_{n,\mathrm{ext}}} < \frac{\varepsilon}{2}.$$

Decomposing each $\phi_{(s)}$ in the sum $\phi_{(s)1} + \phi_{(s)2}$ as above and using (4.4) we get

$$\sum_{s=1}^{N} c_s \phi_{(s)}^{\otimes n} = \sum_{s=1}^{N} \sum_{k=0}^{n} c_s C_n^k \phi_{(s)1}^{\otimes k} \widehat{\otimes} \phi_{(s)2}^{\otimes n-k}.$$

Let $\{\varphi_{(s),l}^{\otimes k} \widehat{\otimes} \psi_{(s),l}^{\otimes n-k}\}_{l=0}^{\infty}$ be sequence (4.5) for $\phi_{(s)1}^{\otimes k} \widehat{\otimes} \phi_{(s)2}^{\otimes n-k}$. Then

$$\left| f_n - \sum_{s=1}^{N} \sum_{k=0}^{n} c_s C_n^k \varphi_{(s),l}^{\otimes k} \widehat{\otimes} \psi_{(s),l}^{\otimes n-k} \right|_{\mathcal{F}_{n,\mathrm{ext}}} \leq$$

$$\leq \left| f_n - \sum_{s=1}^{N} c_s \phi_{(s)}^{\otimes n} \right|_{\mathcal{F}_{n,\mathrm{ext}}} +$$

$$+ \left| \sum_{s=1}^{N} \sum_{k=0}^{n} c_s C_n^k \big(\phi_{(s)1}^{\otimes k} \widehat{\otimes} \phi_{(s)2}^{\otimes n-k} - \varphi_{(s),l}^{\otimes k} \widehat{\otimes} \psi_{(s),l}^{\otimes n-k} \big) \right|_{\mathcal{F}_{n,\mathrm{ext}}} < \varepsilon,$$

if l is sufficiently large.

Thus the lemma is proved.

We return now to the proof of the theorem. Taking into account the result of this lemma it is sufficient to prove (4.2) for arbitrary f_{m+1} of the form

$$f_{m+1} = \varphi^{\otimes k} \widehat{\otimes} \psi^{\otimes m+1-k},$$

where $\varphi, \psi \in \mathcal{D}_{\mathbb{C}}$, $\operatorname{supp}\varphi \subset [0,t)$, $\operatorname{supp}\psi \subset [t,\infty)$ and $k \in \{0,1,\ldots,m+1\}$. We will use recurrent relations (3.1) and the induction hypothesis. The following cases are possible.

1. Let $k = m+1$, i.e., $f_{m+1} = \varphi^{\otimes m+1}$. Since $\operatorname{supp}\varphi \subset [0,t)$, we see that

$$\big\langle P_{m+1}(\cdot), \varphi^{\otimes m+1}\big\rangle = \big\langle P_{m+1}(\cdot), 1\!\!1_{[0,t)^{m+1}}\varphi^{\otimes m+1}\big\rangle$$

and the function $\langle P_1(\cdot), \varphi\rangle$ is $\mathcal{A}_t$-measurable. Hence using (3.1) we get

$$\mathbb{E}\Big[\big\langle P_{m+1}, \varphi^{\otimes m+1}\big\rangle \,\Big|\, \mathcal{A}_t\Big] = \mathbb{E}\Big[\langle P_1, \varphi\rangle\langle P_m, \varphi^{\otimes m}\rangle \,\Big|\, \mathcal{A}_t\Big] -$$

$$-m\mathbb{E}\Big[\langle P_m, \Pr\left[\theta(\cdot)\varphi^2(\cdot)\varphi^{\otimes m-1}(\cdot_1,\ldots,\cdot_{m-1})\right]\rangle \,\Big|\, \mathcal{A}_t\Big] -$$

$$-m\mathbb{E}\Big[\langle P_{m-1}, \langle\varphi,\varphi\rangle\varphi^{\otimes m-1}\rangle \,\Big|\, \mathcal{A}_t\Big] -$$

$$-m(m-1)\mathbb{E}\Big[\big\langle P_{m-1}, \Pr\big[\eta(\cdot)\varphi^3(\cdot)\varphi^{\otimes m-2}(\cdot_1,\ldots,\cdot_{m-2})\big]\big\rangle \,\Big|\, \mathcal{A}_t\Big] =$$

$$= \big\langle P_1, \varphi\big\rangle\big\langle P_m, \varphi^{\otimes m}\big\rangle - m\big\langle P_m, \Pr\left[\theta(\cdot)\varphi^2(\cdot)\varphi^{\otimes m-1}(\cdot_1,\ldots,\cdot_{m-1})\right]\big\rangle -$$

$$-m\big\langle P_{m-1}, \langle\varphi,\varphi\rangle\varphi^{\otimes m-1}\big\rangle - m(m-1)\big\langle P_{m-1}, \Pr\left[\eta(\cdot)\varphi^3(\cdot)\varphi^{\otimes m-2}(\cdot_1,\ldots,\cdot_{m-2})\right]\big\rangle =$$

$$= \big\langle P_{m+1}, \varphi^{\otimes m+1}\big\rangle = \big\langle P_{m+1}, 1\!\!1_{[0,t)^{m+1}}\varphi^{\otimes m+1}\big\rangle.$$

2. Let $k = 0$, i.e., $f_{m+1} = \psi^{\otimes m+1}$. Since $\operatorname{supp}\psi \subset [t,\infty)$, we conclude that

$$\big\langle P_{m+1}(\cdot), 1\!\!1_{[0,t)^{m+1}}\psi^{\otimes m+1}\big\rangle = 0.$$

Hence

$$\mathbb{E}\big[\langle P_{m+1}, \psi^{\otimes m+1}\rangle \,\big|\, \mathcal{A}_t\big] = \mathbb{E}\big[\langle P_1, \psi\rangle\langle P_m, \psi^{\otimes m}\rangle \,\big|\, \mathcal{A}_t\big] -$$

$$-m\mathbb{E}\big[\langle P_m, \Pr\left[\theta(\cdot)\psi^2(\cdot)\psi^{\otimes m-1}(\cdot_1,\ldots,\cdot_{m-1})\right]\rangle \,\big|\, \mathcal{A}_t\big] -$$

$$-m\mathbb{E}\big[\big\langle P_{m-1}, \langle\psi,\psi\rangle\psi^{\otimes m-1}\big\rangle \,\big|\, \mathcal{A}_t\big] -$$

$$-m(m-1)\mathbb{E}\big[\big\langle P_{m-1}, \Pr\left[\eta(\cdot)\psi^3(\cdot)\psi^{\otimes m-2}(\cdot_1,\ldots,\cdot_{m-2})\right]\big\rangle \,\big|\, \mathcal{A}_t\big] =$$

$$= \mathbb{E}\big[\langle P_1, \psi\rangle\langle P_m, \psi^{\otimes m}\rangle\big] = 0 = \big\langle P_{m+1}, 1\!\!1_{[0,t)^{m+1}}\psi^{\otimes m+1}\big\rangle$$

if $m > 1$, and

$$\mathbb{E}\big[\langle P_2, \psi^{\otimes 2}\rangle \,|\mathcal{A}_t\big] =$$

$$= \mathbb{E}\big[\langle P_1, \psi\rangle^2 \,\big|\, \mathcal{A}_t\big] - \mathbb{E}\big[\langle P_1, \theta\psi^2\rangle \,\big|\, \mathcal{A}_t\big] - \mathbb{E}\big[\langle P_0, \langle\psi,\psi\rangle\rangle \,\big|\, \mathcal{A}_t\big] =$$

$$= \mathbb{E}\big[\langle P_1, \psi\rangle^2\big] - \langle\psi,\psi\rangle = 0 = \langle P_2, 1\!\!1_{[0,t)^2}\psi^{\otimes 2}\rangle$$

if $m = 1$ (here $\mathbb{E}[\cdot]$ denotes an expectation).

3. Let $k \in \{1,\ldots,m\}$, i.e., $f_{m+1} = \varphi^{\otimes k}\widehat{\otimes}\psi^{\otimes m+1-k}$. Since $\operatorname{supp}\psi \subset [t,\infty)$, we see that

$$\Big\langle P_{m+1}(\cdot), 1\!\!1_{[0,t)^{m+1}}\varphi^{\otimes k}\widehat{\otimes}\psi^{\otimes m+1-k}\Big\rangle = 0.$$

In view of the latter and $\mathcal{A}_t$-measurability of $\langle P_1(\cdot), \varphi\rangle$ (because $\operatorname{supp}\varphi \subset [0,t)$) we obtain

$$\mathbb{E}\big[\langle P_{m+1},\varphi^{\otimes k}\widehat{\otimes}\psi^{\otimes m+1-k}\rangle\,\big|\,\mathcal{A}_t\big]=\mathbb{E}\big[\langle P_1,\varphi\rangle\langle P_m,\varphi^{\otimes k-1}\widehat{\otimes}\psi^{\otimes m+1-k}\rangle\,\big|\,\mathcal{A}_t\big]-$$
$$-m\mathbb{E}\big[\langle P_m,\Pr\left[\theta(\cdot)\varphi(\cdot)\big(\varphi^{\otimes k-1}\widehat{\otimes}\psi^{\otimes m+1-k}\big)(\cdot,\cdot_2,\ldots,\cdot_m)\right]\rangle\,\big|\,\mathcal{A}_t\big]-$$
$$-m\mathbb{E}\big[\langle P_{m-1},(\varphi^{\otimes k-1}\widehat{\otimes}\psi^{\otimes m+1-k})^{\varphi}\rangle\,|\mathcal{A}_t\big]-$$
$$-m(m-1)\mathbb{E}\big[\langle P_{m-1},\Pr\left[\eta(\cdot)\varphi(\cdot)\big(\varphi^{\otimes k-1}\widehat{\otimes}\psi^{\otimes m+1-k}\big)(\cdot,\cdot,\cdot_3,\ldots,\cdot_m)\right]\rangle\,|\mathcal{A}_t\big]=$$
$$=0=\langle P_{m+1},1\!\!1_{[0,t)^{m+1}}\varphi^{\otimes k}\widehat{\otimes}\psi^{\otimes m+1-k}\rangle.$$

The theorem is proved.

In order to express the Itô integral $\mathbb{I}(f)$ in terms of the extended Fock space $\mathcal{F}_{\mathrm{ext}}$ we accept the following convention.

Convention 4.1. When we consider elements of the space $L^2(\mathbb{R}_+;\mathcal{F}_{n,\mathrm{ext}})\cong\cong\mathcal{F}_{n,\mathrm{ext}}\otimes L^2(\mathbb{R}_+)$ we always select a representative that vanishes on the set

$$d_{n+1}:=\Big\{(t_1,\ldots,t_n;t)\subset\mathbb{R}_+^{n+1}\,\Big|\,\exists t_j=t\Big\}.$$

Such a choice of representative will not affect our discussion because in compliance with (3.3) we have $\mathcal{F}_{n,\mathrm{ext}}\otimes L^2(\mathbb{R}_+)\subseteq\widehat{L}^2(\mathbb{R}_+^n,\rho_n)\otimes L^2(\mathbb{R}_+)$ and $(\rho_n\otimes m)(d_{n+1})=0$, where m denotes the Lebesgue measure on $\mathbb{R}_+$.

Now we have the following statement (cf. (2.11)).

Theorem 4.3. *For each* $f(\cdot)=(f_n(\cdot))_{n=0}^{\infty}\in L_a^2(\mathbb{R}_+;\mathcal{F}_{\mathrm{ext}})$,

$$\mathbb{I}(f)=(0,\widehat{f}_0,\widehat{f}_1,\ldots)\in\mathcal{F}_{\mathrm{ext}},\tag{4.6}$$

where $\widehat{f}_n\in\mathcal{F}_{n+1,\mathrm{ext}}$ *is the symmetrizations of* $f_n(t_1,\ldots,t_n;t)$ *with respect to* $n+1$ *variables.*

Proof. The correctness of the definition of $\widehat{f}_n$ was proved in [25], Lemma 3.2. Equality (4.6) is based on (3.1) and easily follow from Theorem 4.1 and [25] (see the proof of Theorem 3.1 therein).

5. Extended stochastic integrals on the extended Fock space and its riggings. In this section we define and study generalizations of Itô integral (4.6). These generalizations are constructed by analogy with the case of Fock spaces (see Definitions 2.2 and 2.3).

5.1. An extended stochastic integral on $\mathcal{F}_{\mathrm{ext}}$. Taking into account Theorem 4.3 the simplest way to define an extended stochastic integral on $\mathcal{F}_{\mathrm{ext}}$ is the following.

Definition 5.1. *For a function* $f(\cdot)=\big(f_n(\cdot)\big)_{n=0}^{\infty}\in L^2\big(\mathbb{R}_+;\mathcal{F}_{\mathrm{ext}}\big)$ *such that*

$$(0,\widehat{f}_0,\widehat{f}_1,\ldots)\in\mathcal{F}_{\mathrm{ext}}\quad\textit{or, equivalently,}\quad\sum_{n=0}^{\infty}|\widehat{f}_n|^2_{\mathcal{F}_{n+1,\mathrm{ext}}}(n+1)!<\infty$$

we define its extended stochastic integral by the formula

$$\mathbb{I}_{\mathrm{ext}}(f):=(0,\widehat{f}_0,\widehat{f}_1,\ldots)\in\mathcal{F}_{\mathrm{ext}}.\tag{5.1}$$

Thus the extended stochastic integral $\mathbb{I}_{\mathrm{ext}}$ is defined as an unbounded operator

$$\mathbb{I}_{\mathrm{ext}}\colon L^2(\mathbb{R}_+;\mathcal{F}_{\mathrm{ext}})\to\mathcal{F}_{\mathrm{ext}},\qquad f(\cdot)=(f_n(\cdot))_{n=0}^{\infty}\mapsto\mathbb{I}_{\mathrm{ext}}(f):=(0,\widehat{f}_0,\widehat{f}_1,\ldots)$$

with the dense in $L^2(\mathbb{R}_+;\mathcal{F}_{\mathrm{ext}})$ domain

$$\mathrm{Dom}\,(\mathbb{I}_{\mathrm{ext}}) := \Big\{f(\cdot) = (f_n(\cdot))_{n=0}^{\infty} \in L^2(\mathbb{R}_+;\mathcal{F}_{\mathrm{ext}}) \;\Big|\; (0, \widehat{f}_0, \widehat{f}_1, \dots) \in \mathcal{F}_{\mathrm{ext}}\Big\}.$$

It follows immediately from Theorem 4.3 and Definition 5.1 that if f is integrable by Itô (i.e., $f \in L^2_a(\mathbb{R}_+;\mathcal{F}_{\mathrm{ext}})$) then f is integrable in the extended sense (i.e., $f \in \in \mathrm{Dom}\,(\mathbb{I}_{\mathrm{ext}})$) and $\mathbb{I}_{\mathrm{ext}}(f) = \mathbb{I}(f)$. Hence $\mathbb{I}_{\mathrm{ext}}$ is an extension of the Itô integral $\mathbb{I}$.

Let us establish an analog of Theorem 2.2. To this end at first we introduce an annihilation operator $a_-(\delta.)$. According to [25] if $g_n \in \mathcal{F}_{n,\mathrm{ext}}$ then g_n can be considered as an element of $\mathcal{F}_{n-1,\mathrm{ext}} \otimes L^2(\mathbb{R}_+)$ and, moreover,

$$|g_n|_{\mathcal{F}_{n-1,\mathrm{ext}}\otimes L^2(\mathbb{R}_+)} \le |g_n|_{\mathcal{F}_{n,\mathrm{ext}}}.$$

Due to this fact the following definition is correct.

Definition 5.2. *An annihilation operator $a_-(\delta.)$ is defined as an unbounded operator*

$$a_-(\delta.)\colon \mathcal{F}_{\mathrm{ext}} \to L^2(\mathbb{R}_+;\mathcal{F}_{\mathrm{ext}}),$$

$$g = (g_n)_{n=0}^{\infty} \mapsto a_-(\delta.)g := ((n+1)g_{n+1}(\cdot))_{n=0}^{\infty}, \tag{5.2}$$

$$(a_-(\delta_t)g)_n(t_1,\dots,t_{n-1}) = ng_n(t_1,\dots,t_{n-1},t)$$

with the dense in $\mathcal{F}_{\mathrm{ext}}$ domain

$$\mathrm{Dom}(a_-(\delta.)) := \left\{ g = (g_n)_{n=0}^{\infty} \in \mathcal{F}_{\mathrm{ext}} \;\middle|\; \sum_{n=0}^{\infty} |g_n(\cdot)|^2_{\mathcal{F}_{n-1,ext}\otimes L^2(\mathbb{R}_+)} n!n < \infty \right\}.$$

From the corresponding statement in [25] we obtain.

Theorem 5.1. *The extended stochastic integral $\mathbb{I}_{\mathrm{ext}}\colon L^2(\mathbb{R}_+;\mathcal{F}_{\mathrm{ext}}) \to \mathcal{F}_{\mathrm{ext}}$ and the annihilation operator $a_-(\delta.) : \mathcal{F}_{\mathrm{ext}} \to L^2(\mathbb{R}_+;\mathcal{F}_{\mathrm{ext}})$ are adjoint one to another. In particular, these operators are closed.*

Remark **5.1.** It is possible to consider $\mathbb{I}$ and $\mathbb{I}_{\mathrm{ext}}$ on intervals $[0,t]$, $t \in \mathbb{R}_+$, using functions $f(\cdot)1\!\!1_{[0,t]}(\cdot)$ instead of $f(\cdot)$ in the corresponding definitions. But in this case it is necessary to keep in mind that the domain of $\mathbb{I}_{\mathrm{ext}}$ depends on t and (even in the case $\eta = 0$) it is possible that f is integrable in the extended sense on $\mathbb{R}_+$ but is not integrable on $[0,t]$. Note that the extended stochastic integral on riggings of $\mathcal{F}_{\mathrm{ext}}$ (see below) has no this lack.

5.2. An extended stochastic integral on the "regular" rigging of $\mathcal{F}_{\mathrm{ext}}$. The space $\mathcal{F}_{\mathrm{ext}}$ has the following "lacks": the extended stochastic integral $\mathbb{I}_{\mathrm{ext}} : L^2(\mathbb{R}_+,\mathcal{F}_{\mathrm{ext}}) \to \to \mathcal{F}_{\mathrm{ext}}$ is an unbounded operator (and, moreover, the domain of $\mathbb{I}_{\mathrm{ext}}(\circ 1\!\!1_{[0,t]})$ depends on t); there is no a multiplication on $\mathcal{F}_{\mathrm{ext}}$ that is naturally connected with $\mathbb{I}_{\mathrm{ext}}$. This constricts an area of possible applications of $\mathbb{I}_{\mathrm{ext}}$. In this subsection we consider a natural in a sense extension of $\mathcal{F}_{\mathrm{ext}}$ that has no the mentioned lacks.

Let $q \in \mathbb{N}$,

$$\mathcal{F}_{\mathrm{ext}}(q) := \bigoplus_{n=0}^{\infty} \mathcal{F}_{n,\mathrm{ext}}\,(n!)^2 2^{qn}$$

be a Hilbert space of sequences $f = (f_n)_{n=0}^{\infty}$, $f_n \in \mathcal{F}_{n,\mathrm{ext}}$, such that

$$\|f\|^2_{\mathcal{F}_{\mathrm{ext}}(q)} = \sum_{n=0}^{\infty} |f_n|^2_{\mathcal{F}_{n,\mathrm{ext}}}(n!)^2 2^{qn} < \infty.$$

We consider the ("regular" in a terminology of [25]) rigging of $\mathcal{F}_{\rm ext}$

$$\mathcal{F}_{\rm ext}^{-}=\operatorname*{ind\,lim}_{q\in\mathbb{N}}\mathcal{F}_{\rm ext}(-q)\supset\mathcal{F}_{\rm ext}(-q)\supset\mathcal{F}_{\rm ext}\supset\mathcal{F}_{\rm ext}(q)\supset\operatorname*{pr\,lim}_{q\in\mathbb{N}}\mathcal{F}_{\rm ext}(q)=\mathcal{F}_{\rm ext}^{+},\tag{5.3}$$

where the space

$$\mathcal{F}_{\rm ext}(-q)=\bigoplus_{n=0}^{\infty}\mathcal{F}_{n,\rm ext}\,2^{-qn},\qquad \|f\|^2_{\mathcal{F}_{\rm ext}(-q)}=\sum_{n=0}^{\infty}|f_n|^2_{\mathcal{F}_{n,\rm ext}}2^{-qn}<\infty$$

is dual of $\mathcal{F}_{\rm ext}(q)$ with respect to $\mathcal{F}_{\rm ext}$.

The extended stochastic integral $\mathbb{I}_{\rm ext}\colon L^2(\mathbb{R}_+;\mathcal{F}_{\rm ext}(-q))\to\mathcal{F}_{\rm ext}(-q)$ and the annihilation operator $a_-(\delta_\cdot)\colon\mathcal{F}_{\rm ext}(q)\to L^2(\mathbb{R}_+;\mathcal{F}_{\rm ext}(q))$ can be defined by formulas (5.1) and (5.2) respectively, one can show by analogy with [25] that now these operators are adjoint one to another and continuous. Moreover, $\mathbb{I}_{\rm ext}$ and $a_-(\delta_\cdot)$ can be continued to adjoint one to another linear continuous operators acting from $\mathcal{F}_{\rm ext}^{-}\otimes L^2(\mathbb{R}_+)$ to $\mathcal{F}_{\rm ext}^{-}$ and from $\mathcal{F}_{\rm ext}^{+}$ to $\mathcal{F}_{\rm ext}^{+}\otimes L^2(\mathbb{R}_+)$ correspondingly. Elements of the Wick calculus on $\mathcal{F}_{\rm ext}^{-}$ can be defined and applied by analogy with the Gaussian analysis.

But rigging (5.3) is not suit in order to define $\mathbb{I}_{\rm ext}$ as a Bochner integral by analogy with (2.17) (because $\delta_t\notin L^2_{\mathbb{C}}(\mathbb{R}_+)$), this can lead to inconvenience in some applications. In the forthcoming subsection we consider the analog of rigging (2.3) that is similar to (5.3) but has no such a lack.

5.3. The "nonregular" rigging of $\mathcal{F}_{\rm ext}$ and elements of the Wick calculus. Excluding from T some indexes (and preserving for this modified set of indexes the notation T) we can formulate the following statement that is a suitable reformulation of Proposition 2.3 in [25].

Proposition 5.1. *For each $\tau\in T$ there exists $q_0=q_0(\tau)\in\mathbb{N}$ such that for all $q\in\mathbb{N}_{q_0}:=\{q_0,q_0+1,\ldots\}$ the dense and continuous embedding $\mathcal{F}(\tau,q)\hookrightarrow\mathcal{F}_{\rm ext}$ takes place.*

In what follows, we accept on default $\tau\in T$ and $q\in\mathbb{N}_{q_0}$. Due to Proposition 5.1 one can construct a rigging of the extended Fock space $\mathcal{F}_{\rm ext}$

$$\mathcal{F}_{{\rm ext},-}\supset\mathcal{F}_{\rm ext}(-\tau,-q)\supset\mathcal{F}_{\rm ext}\supset\mathcal{F}(\tau,q)\supset\mathcal{F}_+,\tag{5.4}$$

where $\mathcal{F}_{\rm ext}(-\tau,-q)$, $\mathcal{F}_{{\rm ext},-}=\operatorname{ind\,lim}_{\tau\in T,q\in\mathbb{N}_{q_0}}\mathcal{F}_{\rm ext}(-\tau,-q)$ are the dual spaces of $\mathcal{F}(\tau,q)$, $\mathcal{F}_+$ with respect to $\mathcal{F}_{\rm ext}$ correspondingly. It is not difficult to show that

$$\mathcal{F}_{\rm ext}(-\tau,-q)=\bigoplus_{n=0}^{\infty}D^{(n)}_{-\tau,\mathbb{C}}2^{-qn},\qquad D^{\widehat{\otimes}0}_{-\tau,\mathbb{C}}:=\mathbb{C},$$

where $D^{(n)}_{-\tau,\mathbb{C}}$, $n\in\mathbb{N}$ are the negative spaces of the chain

$$\operatorname*{ind\,lim}_{\tau\in T}D^{(n)}_{-\tau,\mathbb{C}}=:\mathcal{D}'^{(n)}_{\mathbb{C}}\supset D^{(n)}_{-\tau,\mathbb{C}}\supset\mathcal{F}_{n,\rm ext}\supset D^{\widehat{\otimes}n}_{\tau,\mathbb{C}}\supset\mathcal{D}^{\widehat{\otimes}n}_{\mathbb{C}}:=\operatorname*{pr\,lim}_{\tau\in T}D^{\widehat{\otimes}n}_{\tau,\mathbb{C}}.$$

Hence $\mathcal{F}_{\rm ext}(-\tau,-q)$ consists of sequences $\xi=(\xi_n)_{n=0}^{\infty}$, $\xi_n\in D^{(n)}_{-\tau,\mathbb{C}}$ such that

$$\|\xi\|^2_{\mathcal{F}_{\rm ext}(-\tau,-q)}=\sum_{n=0}^{\infty}|\xi_n|^2_{D^{(n)}_{-\tau,\mathbb{C}}}2^{-qn}<\infty.$$

***Remark* 5.2.** It is easy to see that if $\eta = 0$ then $\mathcal{F}(\tau, q) \subset \mathcal{F}_{\mathrm{ext}}(q)$, therefore $\mathcal{F}_{\mathrm{ext}}(-q) \subset \mathcal{F}_{\mathrm{ext}}(-\tau, -q)$. But in the general case there are no such embeddings, this is connected with the structure of norms in $\mathcal{F}_{n,\mathrm{ext}}$.

Let us denote by $\langle\langle \cdot, \cdot \rangle\rangle_{\mathcal{F}_{\mathrm{ext}}}$ the dual pairing between elements of $\mathcal{F}_{\mathrm{ext}}(-\tau, -q)$ and $\mathcal{F}(\tau, q)$ (just as $\mathcal{F}_{\mathrm{ext},-}$ and $\mathcal{F}_{+}$), this pairing is generated by the scalar product in $\mathcal{F}_{\mathrm{ext}}$. The spaces $\mathcal{F}_{\mathrm{ext}}(-\tau, -q)$ and $\mathcal{F}_{\mathrm{ext},-}$ have a complicated structure as against usual symmetric Fock spaces. However since the positive spaces in riggings (5.4) and (2.3) coincide, there exists a uniquely defined isomorphism

$$U \colon \mathcal{F}_{\mathrm{ext},-} \to \mathcal{F}_{-}$$

such that for all $\xi \in \mathcal{F}_{\mathrm{ext},-}$ and all $\varphi \in \mathcal{F}_{+}$

$$\langle\langle \xi, \varphi \rangle\rangle_{\mathcal{F}_{\mathrm{ext}}} = \langle\langle U\xi, \varphi \rangle\rangle_{\mathcal{F}}.$$

It is clear that $U = \bigoplus_{n=0}^{\infty} U_n$, where each $U_n : \mathcal{D}'^{(n)}_{\mathbb{C}} \to \mathcal{D}'^{\widehat{\otimes} n}_{\mathbb{C}}$ is defined by

$$\langle \xi_n, \varphi_n \rangle_{\mathcal{F}_{n,\mathrm{ext}}} = \langle U_n \xi_n, \varphi_n \rangle, \qquad \xi_n \in \mathcal{D}'^{(n)}_{\mathbb{C}}, \quad \varphi_n \in \mathcal{D}^{\widehat{\otimes} n}_{\mathbb{C}}.$$

One can show [25] that the restrictions of U_n on $D^{(n)}_{-\tau,\mathbb{C}}$ are isometrical isomorphisms between $D^{(n)}_{-\tau,\mathbb{C}}$ and $D^{\widehat{\otimes} n}_{-\tau,\mathbb{C}}$, therefore the restrictions of U on $\mathcal{F}_{\mathrm{ext}}(-\tau, -q)$ are isometrical isomorphisms between $\mathcal{F}_{\mathrm{ext}}(-\tau, -q)$ and $\mathcal{F}(-\tau, -q)$. In what follows, it is convenient for us to understand $\mathcal{F}_{\mathrm{ext},-}$ and $\mathcal{F}_{\mathrm{ext}}(-\tau, -q)$ as the U^{-1}-images of $\mathcal{F}_{-}$ and $\mathcal{F}(-\tau, -q)$ respectively.

Above mentioned realization of the space $\mathcal{F}_{\mathrm{ext},-}$ is convenient for developing of a Wick calculus on it. We do not discuss this in details, but we give a definition of a Wick product on $\mathcal{F}_{\mathrm{ext},-}$. For given $\xi = (\xi_n)_{n=0}^{\infty}, \zeta = (\zeta_n)_{n=0}^{\infty} \in \mathcal{F}_{\mathrm{ext},-}$ a **Wick product** $\xi \Diamond_{\mathrm{ext}} \zeta \in \mathcal{F}_{\mathrm{ext},-}$ is defined by

$$\xi \Diamond_{\mathrm{ext}} \eta := U^{-1}\big(U\xi \Diamond U\zeta\big) = \Big(\sum_{m=0}^{n} \xi_m \diamond \zeta_{n-m}\Big)_{n=0}^{\infty},$$

where for each $\xi_n \in \mathcal{D}'^{(n)}_{\mathbb{C}}$ and each $\zeta_m \in \mathcal{D}'^{(m)}_{\mathbb{C}}$

$$\xi_n \diamond \zeta_m := U_{n+m}^{-1}\big(U_n \xi_n \,\widehat{\otimes}\, U_m \zeta_m\big).$$

The correctness of this definition (and, moreover, the fact that $\Diamond_{\mathrm{ext}}$ is a continuous operation in the topology of $\mathcal{F}_{\mathrm{ext},-}$) follows from results of [25]. We note also that if $\eta = 0$ (the Gaussian and Poissonian cases) then the product $\diamond$ moves to the symmetric tensor product $\widehat{\otimes}$ and $\Diamond_{\mathrm{ext}}$ moves to $\Diamond$.

In order to describe an important property of the product $\diamond$ we adopt the following convention.

Convention 5.1. Elements of the space $\mathcal{F}_{n,\mathrm{ext}} \otimes \mathcal{F}_{m,\mathrm{ext}}$ are equivalence classes, and considering such elements we always choose representatives that vanish on the set

$$\big\{(t_1, \dots, t_n; t_{n+1}, \dots, t_{n+m}) \in \mathbb{R}_{+}^{n+m} \,\big|\, \exists i \in \{1, \dots, n\},$$
$$j \in \{n+1, \dots, n+m\} \colon t_i = t_j\big\}.$$

The following statement follows from [25], Lemma 4.1.

Proposition 5.2. *Let* $f_n \in \mathcal{F}_{n,\mathrm{ext}}$, $g_m \in \mathcal{F}_{m,\mathrm{ext}}$. *Then*

$$f_n \diamond \eta_m = \widehat{f_n g_m} \in \mathcal{F}_{n+m,\mathrm{ext}},$$

where $\widehat{f_n g_m}$ *is the symmetrization of* $f_n \otimes g_m$ *with respect to* $n+m$ *variables.*

Finally, let us consider the creation operator $a_{\mathrm{ext},+}(\delta_t)$ that is defined by

$$a_{\mathrm{ext},+}(\delta_t)\colon \mathcal{F}_{\mathrm{ext}}(-\tau,-q) \to \mathcal{F}_{\mathrm{ext}}(-\tau,-q), \qquad a_{\mathrm{ext},+}(\delta_t) := U^{-1} a_+(\delta_t) U, \tag{5.5}$$

where $a_+(\delta_t)$ is given by (2.7). It is easy to show that the operator $a_{\mathrm{ext},+}(\delta_t)$ is dual of the annihilation operator $a_-(\delta_t)\colon \mathcal{F}(\tau,q) \to \mathcal{F}(\tau,q)$ with respect to the zero space $\mathcal{F}_{\mathrm{ext}}$:

$$\langle\langle a_{\mathrm{ext},+}(\delta_t)\xi, \varphi\rangle\rangle_{\mathcal{F}_{\mathrm{ext}}} = \langle\langle \xi, a_-(\delta_t)\varphi\rangle\rangle_{\mathcal{F}_{\mathrm{ext}}}, \qquad \xi \in \mathcal{F}_{\mathrm{ext}}(-\tau,-q), \quad \varphi \in \mathcal{F}(\tau,q). \tag{5.6}$$

Moreover, a trivial calculation gives

$$a_{\mathrm{ext},+}(\delta_t)\xi = \xi \lozenge_{\mathrm{ext}} (0,\delta_t,0,0,\ldots), \qquad \xi \in \mathcal{F}_{\mathrm{ext}}(-\tau,-q).$$

5.4. An extended stochastic integral on the "nonregular" rigging of $\mathcal{F}_{\mathrm{ext}}$. It follows from Theorem 4.3 and Proposition 5.2 that the Itô integral $\mathbb{I}(f)$ of a simple function

$$f(\cdot) = \sum_{k=0}^{n-1} f_{(k)} 1\!\!1_{(t_k,t_{k+1}]}(\cdot) \in L^2_a(\mathbb{R}_+;\mathcal{F}_{\mathrm{ext}})$$

has the form

$$\mathbb{I}(f) = \sum_{k=0}^{n-1} f_{(k)} \lozenge_{\mathrm{ext}} (0, 1\!\!1_{(t_k,t_{k+1}]}, 0, 0, \ldots) \in \mathcal{F}_{\mathrm{ext}}.$$

Using the same arguments as in Subsection 2.2 it is natural to give the following definition of the extended stochastic integral on the extended Fock spaces.

In what follows, let us fix $\tau \in T$ such that

$$\int_{\mathbb{R}_+} |\delta_t|^2_{D_{-\tau}}\, dt = c(\tau) < \infty \tag{5.7}$$

and $q \in \mathbb{N}_{q_0(\tau)} = \{q_0(\tau), q_0(\tau)+1, \ldots\}$, where $q_0(\tau) \in \mathbb{N}$ is given in Proposition 5.1. The existence of τ with the required property is proved in, e.g., [42], Chapter XIV.

Definition 5.3. *The extended stochastic integral of a function*

$$\xi \in L^2(\mathbb{R}_+;\mathcal{F}_{\mathrm{ext}}(-\tau,-q))$$

is defined by the formula

$$\widehat{\mathbb{I}}_{\mathrm{ext}}(\xi) := \int_{\mathbb{R}_+} a_{\mathrm{ext},+}(\delta_t)\xi(t)\, dt \in \mathcal{F}_{\mathrm{ext}}(-\tau,-q) \tag{5.8}$$

as a Bochner integral of the vector-valued function

$$\mathbb{R}_+ \ni t \mapsto a_{\mathrm{ext},+}(\delta_t)\xi(t) = \xi(t)\lozenge_{\mathrm{ext}}(0,\delta_t,0,0,\ldots) \in \mathcal{F}_{\mathrm{ext}}(-\tau,-q).$$

The correctness of this definition from the following statement follows.

Proposition 5.3. *For all* $\xi \in L^2(\mathbb{R}_+; \mathcal{F}_{\mathrm{ext}}(-\tau, -q))$ *integral* (5.8) *is well-defined as a Bochner one and is continuous as an operator acting from* $L^2(\mathbb{R}_+; \mathcal{F}_{\mathrm{ext}}(-\tau, -q))$ *to* $\mathcal{F}_{\mathrm{ext}}(-\tau, -q)$.

Proof. Let $\xi \in L^2(\mathbb{R}_+; \mathcal{F}_{\mathrm{ext}}(-\tau, -q))$. Using the estimate

$$\|a_{\mathrm{ext},+}(\delta_t)\xi(t)\|_{\mathcal{F}_{\mathrm{ext}}(-\tau,-q)} \le 2^{-\frac{q}{2}} |\delta_t|_{D_{-\tau}} \|\xi(t)\|_{\mathcal{F}_{\mathrm{ext}}(-\tau,-q)}$$

(this inequality follows from (5.5) and results of [22]) we obtain

$$\left\| \int\limits_{\mathbb{R}_+} a_{\mathrm{ext},+}(\delta_t)\xi(t)\, dt \right\|_{\mathcal{F}_{\mathrm{ext}}(-\tau,-q)} \le \int\limits_{\mathbb{R}_+} \|a_{\mathrm{ext},+}(\delta_t)\xi(t)\|_{\mathcal{F}_{\mathrm{ext}}(-\tau,-q)}\, dt \le$$

$$\le 2^{-q/2} \int\limits_{\mathbb{R}_+} |\delta_t|_{D_{-\tau}} \|\xi(t)\|_{\mathcal{F}_{\mathrm{ext}}(-\tau,-q)}\, dt \le$$

$$\le 2^{-q/2} \left(\int\limits_{\mathbb{R}_+} |\delta_t|^2_{D_{-\tau}}\, dt \right)^{1/2} \left(\int\limits_{\mathbb{R}_+} \|\xi(t)\|^2_{\mathcal{F}_{\mathrm{ext}}(-\tau,-q)}\, dt \right)^{1/2} =$$

$$= 2^{-q/2} c(\tau)^{1/2} \|\xi\|_{L^2(\mathbb{R}_+, \mathcal{F}_{\mathrm{ext}}(-\tau,-q))} < \infty,$$

whence the necessary statement follows.

***Remark* 5.3.** It follows from [25] that in the case where (5.7) does not hold, integral (5.8) is well-defined as a Pettis one. Namely, for all $\xi \in L^2(\mathbb{R}_+, \mathcal{F}_{\mathrm{ext}}(-\tau, -q))$, $\tau \in T$, $q \in \mathbb{N}_{q_0(\tau)}$, a function

$$\zeta\colon \mathbb{R}_+ \to \mathcal{F}_{\mathrm{ext}}(-\tau, -q), \qquad t \mapsto \zeta(t) := a_{\mathrm{ext},+}(\delta_t)\xi(t)$$

is Pettis integrable, i.e.,

the function $\langle\langle \zeta(\cdot), \varphi \rangle\rangle_{\mathcal{F}_{\mathrm{ext}}}$ is measurable for any $\varphi \in \mathcal{F}_{\mathrm{ext}}(\tau, q)$;

$\langle\langle \zeta(\cdot), \varphi \rangle\rangle_{\mathcal{F}_{\mathrm{ext}}} \in L^1(\mathbb{R}_+, dt)$ for all $\varphi \in \mathcal{F}_{\mathrm{ext}}(\tau, q)$.

The corresponding Pettis integral of ξ is defined as a unique element of the space $\mathcal{F}_{\mathrm{ext}}(-\tau, -q)$, denoted by $\int_{\mathbb{R}_+} \zeta(t)\, dt$, such that

$$\left\langle\!\!\left\langle \int\limits_{\mathbb{R}_+} \zeta(t)\, dt, \varphi \right\rangle\!\!\right\rangle_{\mathcal{F}_{\mathrm{ext}}} = \int\limits_{\mathbb{R}_+} \langle\langle \zeta(t), \varphi \rangle\rangle_{\mathcal{F}_{\mathrm{ext}}}\, dt.$$

Let us point out a relation between the extended stochastic integral $\widehat{\mathbb{I}}_{\mathrm{ext}}$ and the annihilation operator $a_-(\delta_\cdot)$.

Theorem 5.2. *The integral* $\widehat{\mathbb{I}}_{\mathrm{ext}}\colon L^2(\mathbb{R}_+; \mathcal{F}_{\mathrm{ext}}(-\tau, -q)) \to \mathcal{F}_{\mathrm{ext}}(-\tau, -q)$ *is adjoint of the annihilation operator* $a_-(\delta_\cdot)\colon \mathcal{F}(\tau, q) \to L^2(\mathbb{R}_+, \mathcal{F}(\tau, q))$ *in the sense that*

$$\left\langle\!\!\left\langle \widehat{\mathbb{I}}_{\mathrm{ext}}(\xi), \varphi \right\rangle\!\!\right\rangle_{\mathcal{F}_{\mathrm{ext}}} = \langle\langle \xi, a_-(\delta_\cdot)\varphi \rangle\rangle_{L^2(\mathbb{R}_+; \mathcal{F}_{\mathrm{ext}})} \tag{5.9}$$

for all $\xi \in L^2(\mathbb{R}_+; \mathcal{F}_{\mathrm{ext}}(-\tau, -q))$ *and all* $\varphi \in \mathcal{F}(\tau, q)$.

Proof. Using (5.8) and (5.6) we obtain

$$\langle\langle \mathbb{I}_{\rm ext}(\xi), \varphi\rangle\rangle_{\mathcal{F}_{\rm ext}} = \left\langle\!\!\left\langle \int\limits_{\mathbb{R}_+} a_{{\rm ext},+}(\delta_t)\xi(t)\,dt, \varphi \right\rangle\!\!\right\rangle_{\mathcal{F}_{\rm ext}} = \int\limits_{\mathbb{R}_+} \langle\langle a_{{\rm ext},+}(\delta_t)\xi(t), \varphi\rangle\rangle_{\mathcal{F}_{\rm ext}} dt =$$

$$= \int\limits_{\mathbb{R}_+} \langle\langle \xi(t), a_-(\delta_t)\varphi\rangle\rangle_{\mathcal{F}_{\rm ext}} dt = \langle\langle \xi, a_-(\delta_\cdot)\varphi\rangle\rangle_{L^2(\mathbb{R}_+;\mathcal{F}_{\rm ext})}$$

for all $\xi \in L^2(\mathbb{R}_+; \mathcal{F}_{\rm ext}(-\tau, -q))$ and all $\varphi \in \mathcal{F}(\tau, q)$.

From this statement and Theorem 5.1 we obtain the following corollary.

***Corollary* 5.1.** *Let* $f \in L^2(\mathbb{R}_+; \mathcal{F}_{\rm ext}) \subset L^2(\mathbb{R}_+, \mathcal{F}_{\rm ext}(-\tau, -q))$ *be integrable in the sense of Definition* 5.1. *Then the extended stochastic integral* $\widehat{\mathbb{I}}_{\rm ext}(f)$ *that is defined by* (5.8) *coincides with the extended stochastic integral* $\mathbb{I}_{\rm ext}(f)$ *from Definition* 5.1, *i.e.,*

$$\widehat{\mathbb{I}}_{\rm ext}(f) = \mathbb{I}_{\rm ext}(f), \qquad f \in {\rm Dom}\,(\mathbb{I}_{\rm ext})$$

(*this explains why we use the same name for these integrals*).

In order to rewrite integral (5.8) by analogy with (5.1) we define

$$\widehat{\xi}_n := U_{n+1}^{-1}\big({\rm Pr}((U_n \otimes 1)\xi_n)\big) \in D_{-\tau,\mathbb{C}}^{(n+1)}$$

for all $\xi_n \in D_{-\tau,\mathbb{C}}^{(n)} \otimes L^2(\mathbb{R}_+)$, where Pr denotes the symmetrization operator. The next statement follows from [25], Theorem 4.4.

Theorem 5.3. *Let* $\xi = (\xi_n)_{n=0}^{\infty} \in L^2(\mathbb{R}_+, \mathcal{F}_{\rm ext}(-\tau, -q))$ (*now* $\xi_n \in D_{-\tau,\mathbb{C}}^{(n)} \otimes$ $\otimes\, L^2(\mathbb{R}_+)$). *Then*

$$\widehat{\mathbb{I}}_{\rm ext}(\xi) = (0, \widehat{\xi}_0, \widehat{\xi}_1, \ldots) \in \mathcal{F}_{\rm ext}(-\tau, -q).$$

***Remark* 5.4.** All results of this subsection can be rewritten with obvious modifications for the space $\mathcal{F}_{{\rm ext},-}$ instead of $\mathcal{F}_{\rm ext}(-\tau, -q)$.

***Remark* 5.5.** Using the generalized Wiener–Itô–Segal isomorphism $I\colon \mathcal{F}_{\rm ext} \to$ $\to (L^2)$, defined by (3.4), one can reformulate all the above definitions and statements in terms of test and generalized functions on $\mathcal{D}'$ whose dual paring is generated by the scalar product in the space (L^2). In such a way one obtains a natural generalization of the Itô integral with respect to Meixner processes, see [25] for more details.

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