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ON FUZZY SEMI $\delta - V$ CONTINUITY IN FUZZY $\delta - V$ TOPOLOGICAL SPACE

ПРО НЕЧІТКУ НАПІВ $\delta - V$ НЕПЕРЕРВНІСТЬ В НЕЧІТКОМУ $\delta - V$ ТОПОЛОГІЧНОМУ ПРОСТОРІ

New concepts of fuzzy semi $\delta - V$ and fuzzy semi $\delta - \Lambda$ sets were introduced in the work „On fuzzy semi $\delta - \Lambda$ sets and fuzzy semi $\delta - V$ sets $V - 6$" by the authors (*J. Trip. Math. Soc.,* **6**, 81 – 88 (2004)). It was shown that the family of all fuzzy semi $\delta - V$ sets forms a fuzzy supra topological space on X denoted by $(X, FS^{\delta V})$. The aim of this paper is to introduce the concept of fuzzy semi $\delta - V$ continuity in a fuzzy $\delta - V$ topological space. Finally, some properties, preservation theorems, etc., are studied.

Нові поняття нечітких напів $\delta - V$ та нечітких напів $\delta - \Lambda$ множин введено у роботі авторів „On fuzzy semi $\delta - \Lambda$ sets and fuzzy semi $\delta - V$ sets $V - 6$" (*J. Trip. Math. Soc.* – 2004. – **6**. – С. 81 – 88). Було показано, що сім'я усіх нечітких напів $\delta - V$ множин формує нечіткий супра-топологічний простір в X, що позначається як $(X, FS^{\delta V})$. Метою даної статті є введення поняття нестійкої напів $\delta - V$ неперервності у нестійкому $\delta - V$ топологічному просторі. Також досліджено деякі її властивості, наведено теорему про збереження та інші питання.

1. Introduction. The notion of fuzzy semi $\delta - V$ and fuzzy semi $\delta - \Lambda$ set has been introduced by the authors in [1]. It was shown that the set of all fuzzy semi $\delta - V$ sets forms a fuzzy supra topological space denoted by $(X, FS^{\delta V})$.

In Section 2, we study some properties on fuzzy semi $\delta - V$ continuity. Some theorems are also studied. Finally, in Section 3, we study some relationships between the above two continuities. Here we denote the complement of λ, i.e., $1 - \lambda$, by λ^c, and the fuzzy topological space is denoted by (X, F).

Below, we present several definitions and results for reader's convenience.

1.1 [2]. Let S be a fuzzy subset of a fuzzy space (X, F). We define $\Lambda_\delta(S)$ and $V_\delta(S)$ as follows

$$\Lambda_\delta(S) = \inf\{G : S \le G,\ G \in F_\delta O(X, F)\}$$

and

$$V_\delta(S) = \sup\{D : D \le S,\ D \in F_\delta C(X, F)\}.$$

1.2 [2]. Fuzzy subsets S, Q, and $\{S_j, j \in J\}$ of a fuzzy space (X, F) possess the following properties:

(1) $S \le \Lambda_\delta(S)$,

(2) $Q \le S$ implies that $\Lambda_\delta(Q) \le \Lambda_\delta(S)$,

(3) $\Lambda_\delta \Lambda_\delta(S) = \Lambda_\delta(S)$,

(4) if $S \in F_\delta O(X, F)$, then $S = \Lambda_\delta(S)$,

(5) $\Lambda_\delta\{\bigcup (S_j, j \in J)\} = \bigcup\{\Lambda_\delta(S_j), j \in J\}$,

(6) $\Lambda_\delta\{\bigcap (S_j, j \in J)\} \le \bigcap\{\Lambda_\delta(S_j), j \in J\}$,

(7) $\Lambda_\delta(1 - S) = 1 - V_\delta(S)$,

(8) $\Lambda_\delta(0) = 0$ and $\Lambda_\delta(1) = 1$,

(9) VqA iff $Vq\Lambda_\delta(A)$, where V is a δ-closed subset of X.

1.3 [2]. Fuzzy subsets S, Q, and $\{S_j, j \in J\}$ of a space (X, F) possess the following properties:

(1) $V_\delta(0) = 0$ and $V_\delta(1) = 1$,

(2) $V_\delta(S) \le S$,

(3) $Q \le S$ implies that $V_\delta(Q) \le V_\delta(S)$,

(4) $V_\delta V_\delta(S) = V_\delta(S)$,

(5) if $S \in F_\delta C(X, F)$, then $S = V_\delta(S)$,

(6) $V_\delta\{\bigcap (S_j, j \in J)\} = \bigcap \{V_\delta(S_j), j \in J\}$,

(7) $\bigcup \{V_\delta(S_j), j \in J\} \le V_\delta\{\bigcup (S_j, j \in J)\}$,

(8) MqA iff $V_\delta(A)qM$, where M is a fuzzy δ-open subset of X.

1.4 [2]. A fuzzy subset S of a fuzzy space (X, F) is called a fuzzy $\delta - \Lambda$ set (resp., a fuzzy $\delta - V$ set) if $S = \Lambda_\delta(S)$ (resp., $V_\delta(S) = S$).

1.5 [2]. Let S be a fuzzy subset of a fuzzy space (X, F). We define $\eta_\delta(S)$ and $\gamma_\delta(S)$ as follows:

$$\eta_\delta(S) = V_\delta \Lambda_\delta(S)$$

and

$$\gamma_\delta(S) = \Lambda_\delta V_\delta(S).$$

1.6 [2]. Fuzzy subsets S, Q, and $\{S_j, j \in J\}$ of a space (X, F) possess the following properties:

(1) $Q \le S$ implies that $\eta_\delta(Q) \le \eta_\delta(S)$,

(2) $V_\delta(S) \le \eta_\delta(S) \le \Lambda_\delta(S)$,

(3) $\eta_\delta(0) = 0$, $\eta_\delta(1) = 1$,

(4) if $S \in F_\delta O(X, F)$, then $\eta_\delta(S) = V_\delta(S)$,

(5) $\eta_\delta(1 - S) = 1 - \gamma_\delta(S)$,

(6) $\eta_\delta(S)$ is bounded,

(7) $\eta_\delta\{\bigcup (S_j, j \in J)\} \ge \bigcup \{\eta_\delta(S_j), j \in J\}$,

(8) $\bigcap \{\eta_\delta(S_j), j \in J\} \ge \eta_\delta\{\bigcap (S_j, j \in J)\}$,

(9) VqA iff $\eta_\delta(A)q\Lambda_\delta(V)$, where V is a δ-closed subset of X.

1.7 [2]. Fuzzy subsets S, Q, and $\{S_j, j \in J\}$ of a fuzzy space (X, F) possess the following properties:

(1) $Q \le S$ implies that $\gamma_\delta(Q) \le \gamma_\delta(S)$,

(2) $V_\delta(S) \le \gamma_\delta(S) \le \Lambda_\delta(S)$,

(3) $\gamma_\delta(0) = 0$, $\gamma_\delta(1) = 1$,

(4) if $S \in F_\delta C(X, F)$ then $\gamma_\delta(S) = \Lambda_\delta(S)$,

(5) $\gamma_\delta(S)$ is bounded,

(6) $\gamma_\delta\{\bigcup (S_j, j \in J)\} \ge \bigcup \{\gamma_\delta(S_j), j \in J\}$,

(7) $\gamma_\delta\{\bigcap (S_j, j \in J)\} \le \bigcap \gamma_\delta\{(S_j), j \in J\}$,

(8) VqA iff $\gamma_\delta(A)qV_\delta(V)$, where V is a δ-open subset of X.

1.8 [2]. A fuzzy subset S of a fuzzy space (X, F) is called a fuzzy $\delta - \eta$ set (resp., a fuzzy $\delta - \gamma$ set) iff $S = \eta_\delta(S)$ (resp., $S = \gamma_\delta(S)$).

1.9 [2]. A fuzzy set S of a fuzzy space X is a $\delta - \eta$ set iff S^c is a fuzzy $\delta - \gamma$ set.

1.10 [2]. The families of all fuzzy $\delta - \Lambda$ sets and all fuzzy $\delta - V$ sets form fuzzy Alexandroff spaces. We denote them by (X, F^{Λ_δ}) and (X, F^{V_δ}) respectively, and call them the fuzzy $\delta - \Lambda$ topological space and the fuzzy $\delta - V$ topological space, respectively.

1.11 [1]. A fuzzy subset S of a fuzzy space (X, F) is called a fuzzy semi $\delta - V$ set if there exists a fuzzy $\delta - V$ set H such that

$$H \leq S \leq \Lambda_\delta(H) = \gamma_S(H),$$

and a fuzzy subset G is said to be a fuzzy semi $\delta - \Lambda$ set if there exists a fuzzy $\delta - \Lambda$ set K such that $V_\delta(K) \leq G \leq K$.

1.12 [1]. If S is a fuzzy $\delta - V$ set, then it is obviously a fuzzy semi $\delta - V$ set, but the converse statement is not necessarily true. Similarly, if G is a fuzzy $\delta - \Lambda$ set, then it is obviously a fuzzy semi $\delta - \Lambda$ set.

1.13 [1]. For any fuzzy subset λ of a fuzzy space X, the following statements are equivalent:

(a) λ is a fuzzy semi $\delta - V$ set,

(b) λ^c is a fuzzy semi $\delta - \Lambda$ set,

(c) $\eta_\delta(\lambda^c) \leq \lambda^c$,

(d) $\gamma_\delta(\lambda) \geq \lambda$.

1.14 [1]. Let S be a fuzzy subset of a fuzzy space (X, F). We define $\omega_\delta(S)$ and $m_\delta(S)$ as follows:

$$\omega_\delta(S) = \bigcap \{ G \colon G \geq S, \ \ G \text{ is a fuzzy semi } \delta - \Lambda \text{ set} \}$$

and

$$m_\delta(S) = \bigcup \{ G \colon G \leq S, \ G \text{ is a fuzzy semi } \delta - V \text{ set} \}.$$

1.15 [1]. A fuzzy subset S of a fuzzy space (X, F) is a fuzzy semi $\delta - V$ set (resp., a fuzzy semi $\delta - \Lambda$ set) iff $m_\delta(A) = A$ [resp., $\omega_\delta(A) = A$].

1.16 [1]. Fuzzy subsets S, Q, and $\{S_j, j \in J\}$ of a fuzzy space (X, F) possess the following properties:

(1) $S \geq m_\delta(S)$,

(2) $Q \leq S$ implies that $m_\delta(Q) \leq m_\delta(S)$,

(3) $m_\delta m_\delta(S) = m_\delta(S)$,

(4) $m_\delta\{\bigcup (S_j, j \in J)\} \geq \bigcup \{m_\delta(S_j), j \in J\}$,

(5) $m_\delta\{\bigcap (S_j, j \in J)\} \leq \bigcap \{m_\delta(S_j), j \in J\}$,

(6) $m_\delta(1 - S) = 1 - \omega_\delta(S)$,

(7) $m_\delta(0) = 0$ and $m_\delta(1) = 1$,

(8) $m_\delta(S)$ is a fuzzy semi $\delta - V$ set,

(9) $m_\delta(S) \leq \Lambda_\delta(S)$.

1.17 [1]. Fuzzy subsets S, Q, and $\{S_j, j \in J\}$ of a fuzzy space (X, F) possess the following properties:

(1) $S \leq \omega_\delta(S)$,

(2) $Q \leq S$ implies that $\omega_\delta(Q) \leq \omega_\delta(S)$,

(3) $\omega_\delta \omega_\delta(S) = \omega_\delta(S)$,

(4) $\omega_\delta\{\bigcup (S_j, j \in J)\} \geq \bigcup \{\omega_\delta(S_j), j \in J\}$,

(5) $\omega_\delta\{\bigcap (S_j, j \in J)\} \le \bigcap \{\omega_\delta(S_j), j \in J\}$,

(6) $\omega_\delta(0) = 0$ and $\omega_\delta(1) = 1$,

(7) $\omega_\delta(S)$ is a fuzzy semi $\delta - \Lambda$ set,

(9) $V_\delta(S) \le \omega_\delta(S)$.

1.18 [1]. The family of all fuzzy semi $\delta - V$ sets forms a fuzzy supra topological space and is denoted by $FS^{\delta V}$; the supra topological space may be denoted by $(X, FS^{\delta V})$ and can be written as a fuzzy supra semi $\delta - V$ topological space.

1.19 [3]. A function $f\colon (X, FS_1^{\delta V}) \to (Y, FS_2^{\delta V})$ is said to be fuzzy supra semi $\delta -$ $- V$ continuous if the inverse image of every fuzzy semi $\delta - V$ set in Y is a fuzzy semi $\delta - V$ set in X.

1.20 [2]. A function $f\colon (X, F_1^{\Lambda_\delta}) \to (Y, F_2^{\Lambda_\delta})$ is called fuzzy Λ_δ-continuous iff the inverse image of a fuzzy Λ_δ-open set in $(Y, F_2^{\Lambda_\delta})$ is a fuzzy Λ_δ-open set in $(X, F_1^{\Lambda_\delta})$.

1.21 [4]. A function $f\colon (X, F_1) \to (Y, F_2)$ is called fuzzy $\delta - \Lambda$ continuous iff the inverse image of a $\delta - \Lambda$ set in (Y, F_2) is a fuzzy δ-open set in (X, F_1).

2. Fuzzy semi $\delta - V$ continuity in a fuzzy $\delta - V$ topological space. In this section, we introduce fuzzy semi $\delta - V$ continuity in the fuzzy $\delta - V$ topological space. Some related properties are to be studied. Some important theorems are also introduced.

Definition 2.1. *A function $f\colon (X, F_1^{V_\delta}) \to (Y, F_2^{V_\delta})$ from a fuzzy topological space X into a fuzzy topological space Y is called fuzzy semi $\delta - V$ continuous if $f^{-1}(B)$ is a fuzzy semi $\delta - V$ set of X for each $B \in F_2^{V_\delta}$.*

***Example* 2.1.** Let $X = \{a, b, c\}$ and let A and B be fuzzy sets of X defined as follows:

$$A(a) = 0.3, \quad A(b) = 0.2, \quad A(c) = 0.4,$$
$$B(a) = 0.15, \quad B(b) = 0.1, \quad B(c) = 0.2.$$

We set $F_1 = \{0, A, 1\}$ and $F_2 = \{0, B, 1\}$ and define $f\colon (X, F_1^{V_\delta}) \to (Y, F_2^{V_\delta})$ as $f(x) = x$. It is clear that $F_1^{V_\delta} = \{0, A^c, 1\}$ and $F_2^{V_\delta} = \{0, B^c, 1\}$. Here, $f^{-1}(0) = 0$, $f^{-1}(1) = 1$, and $f^{-1}(B^c) = B^c \notin F_1^{V_\delta}$. Then $V_\delta(A) = 0$. Hence, $0 = V_\delta(A) \le B \le A$. If A is a fuzzy $\delta - \Lambda$ set in X, then B is a fuzzy semi $\delta - \Lambda$ set in X. Hence, B^c is a fuzzy semi $\delta - V$ set in X. Therefore, the inverse image of a fuzzy $\delta - V$ set in Y is a fuzzy semi $\delta - V$ set in X. Hence, f is fuzzy semi $\delta - V$ continuous but not fuzzy V_δ-continuous. Again, f is also fuzzy supra semi $\delta - V$ continuous. Taking into account that B^c is a fuzzy $\delta - V$ set and using 1.12, we conclude that B^c is a fuzzy semi $\delta - V$ set.

Theorem 2.1. *Let $f\colon (X, F_1^{V_\delta}) \to (Y, F_2^{V_\delta})$ be a mapping from a fuzzy topological space $(X, F_1^{V_\delta})$ into a fuzzy topological space $(X, F_2^{V_\delta})$. Then the following statements are equivalent*:

(i) *f is fuzzy semi $\delta - V$ continuous*;

(ii) *$f^{-1}(B)$ is a fuzzy semi $\delta - \Lambda$ set in X for every fuzzy $\delta - \Lambda$ set B in Y*;

(iii) *$f[\omega_\delta(A)] \le \Lambda_\delta[f(A)]$ for every fuzzy set A of X*;

(iv) *$\omega_\delta[f^{-1}(B)] \le f^{-1}[\Lambda_\delta(B)]$ for every fuzzy set B of Y*;

(v) $f^{-1}[V_\delta(B^c)] \le m_\delta[f^{-1}(B^c)]$, *where* B^c *is the complement of* B.

Proof. (i) $\Rightarrow$ (ii). f is fuzzy semi $\delta - V$ continuous iff $f^{-1}(B^c)$ is a fuzzy semi $\delta - V$ set in X for every $B^c \in F_2^{V_\delta}$ iff $1 - f^{-1}(B^c)$ is fuzzy semi $\delta - \Lambda$ in X for every $1 - B^c$ being a fuzzy $\delta - \Lambda$ set in Y iff $f^{-1}(B)$ is a fuzzy semi $\delta - \Lambda$ set in X for every B being a fuzzy $\delta - \Lambda$ set in Y [from 1.13, 1.2 (7), and 1.4]. The required implication is proved.

(ii) $\Rightarrow$ (iii). Let A be a fuzzy set of X and let $f(A)$ be a fuzzy set of Y. Then $\Lambda_\delta[f(A)]$ [from 1.2 (3) and 1.4] is a fuzzy $\delta - \Lambda$ set in Y. Hence, it follows from (ii) that $f^{-1}\Lambda_\delta[f(A)]$ is a fuzzy semi $\delta - \Lambda$ set in X and

$$\omega_\delta(A) \le \omega_\delta[f^{-1}f(A)] \le \omega_\delta f^{-1}\Lambda_\delta[f(A)] \text{ [from 1.2 (1)] } =$$
$$= f^{-1}\Lambda_\delta[f(A)] \text{ [from 1.15]}.$$

Therefore, $f[\omega_\delta(A)] \le ff^{-1}\Lambda_\delta[f(A)] \le \Lambda_\delta[f(A)]$.

(iii) $\Rightarrow$ (iv). Let $A = f^{-1}(B)$. Then $f(A) = ff^{-1}(B) \le B$, and from 1.2 (2) we have $\Lambda_\delta[f(A)] \le \Lambda_\delta(B)$. Hence, it follows from (iii) that $f[\omega_\delta[f^{-1}(B)]] \le \Lambda_\delta(B)$, i.e., $f^{-1}f[\omega_\delta[f^{-1}(B)]] \le f^{-1}\Lambda_\delta(B)$, i.e., $\omega_\delta[f^{-1}(B)] \le f^{-1}f[\omega_\delta[f^{-1}(B)]] \le f^{-1}\Lambda_\delta(B)$.

(iv) $\Rightarrow$ (v). $1 - \omega_\delta[f^{-1}(B)] \ge 1 - f^{-1}\Lambda_\delta(B)$, i.e., $f^{-1}[V_\delta(B^c)] \le m_\delta[f^{-1}(B^c)]$ [from 1.2 (7) and 1.16 (6)].

(v) $\Rightarrow$ (i). Let B^c be a fuzzy $\delta - V$ set. Then it follows from 1.4 that $B^c = V_\delta(B^c)$, i.e., $f^{-1}(B^c) = f^{-1}[V_\delta(B^c)] \le m_\delta[f^{-1}(B^c)] \le f^{-1}(B^c)$ [from 1.16 (1)]. Hence, $m_\delta[f^{-1}(B^c)] = f^{-1}(B^c)$.

Thus, f is fuzzy semi $\delta - V$ continuous.

Theorem 2.2. *Let* $f\colon (X, F_1^{V_\delta}) \to (Y, F_2^{V_\delta})$ *be a bijective mapping from a fuzzy topological space* $(X, F_1^{V_\delta})$ *into a fuzzy topological space* $(X, F_2^{V_\delta})$. *Then* f *is fuzzy semi* $\delta - V$ *continuous iff* $f[m_\delta(A^c)] \ge V_\delta[f(A^c)]$.

Proof. $f[m_\delta(A^c)] \ge V_\delta[f(A^c)]$ iff $1 - f[m_\delta(A^c)] \le 1 - V_\delta[f(A^c)]$ iff $f[\omega_\delta(A)] \le$ $\le \Lambda_\delta[f(A)]$ [from 1.2 (7) and 1.16 (6)] iff f is fuzzy semi $\delta - V$ continuous [from Theorem 2.1].

Theorem 2.3. *Let* $f\colon (X, F_1^{V_\delta}) \to (Y, F_2^{V_\delta})$ *be a mapping from a fuzzy topological space* $(X, F_1^{V_\delta})$ *into a fuzzy topological space* $(X, F_2^{V_\delta})$. *Then the following statements are equivalent*:

(i) f *is fuzzy semi* $\delta - V$ *continuous*;

(ii) $\eta_\delta[f^{-1}(B)] \le f^{-1}[\Lambda_\delta(B)]$ *for any fuzzy set* B *of* Y;

(iii) $\gamma_\delta[f^{-1}(B^c)] \ge f^{-1}[V_\delta(B^c)]$, *where* B^c *is the complement of* B;

(iv) $f[\eta_\delta(A)] \le \Lambda_\delta f(A)$ *for any fuzzy set* A^c *of* X.

Proof. (i) $\Rightarrow$ (iii). It follows from 1.3 (4) and 1.4 that $V_\delta(B^c)$ is a fuzzy $\delta - V$ set in X. If f is fuzzy semi $\delta - V$ continuous, then $f^{-1}[V_\delta(B^c)]$ is a fuzzy semi $\delta - V$ set in X. Hence, $\gamma_\delta[f^{-1}(V_\delta(B^c))] \ge f^{-1}[V_\delta(B^c)]$ [from 1.13], i.e., $f^{-1}[V_\delta(B^c)] \le \gamma_\delta[f^{-1}(V_\delta(B^c))] \le \gamma_\delta[f^{-1}(B^c)]$ [from 1.3 (2)].

(iii) $\Leftrightarrow$ (ii). $1 - f^{-1}[V_\delta(B^c)] \ge 1 - \gamma_\delta[f^{-1}(B^c)]$ iff $\eta_\delta[f^{-1}(B)] \le f^{-1}[\Lambda_\delta(B)]$

[from 1.6 (5) and 1.2 (7)] for any fuzzy set B of Y.

(ii) $\Rightarrow$ (iv). Let $B = f(A)$. Then $f^{-1}(B) = f^{-1}f(A) \geq A$. Hence, it follows from (ii) that $\eta_\delta(A) \leq \eta_\delta[f^{-1}(B)]$ [from 1.6 (1)] $\leq f^{-1}[\Lambda_\delta(B)] \leq f^{-1}[\Lambda_\delta f(A)]$, i.e., $ff^{-1}[\Lambda_\delta f(A)] \geq f[\eta_\delta(A)]$, i.e., $[\Lambda_\delta f(A)] \geq ff^{-1}[\Lambda_\delta f(A)] \geq f[\eta_\delta(A)]$.

(iv) $\Rightarrow$ (ii). Let $A = f^{-1}(B)$. Then $f(A) = ff^{-1}(B) \leq B$, i.e., $\Lambda_\delta(B) \geq [\Lambda_\delta f(A)]$ [from 1.2 (2)] $\geq f[\eta_\delta[f^{-1}(B)]]$. Consequently, $f^{-1}[\Lambda_\delta(B)] \geq f^{-1}f[\eta_\delta[f^{-1}(B)]] \geq \geq \eta_\delta[f^{-1}(B)]$.

(iii) $\Rightarrow$ (i). Let B^c be a fuzzy $\delta - V$ set in Y. Then $V_\delta(B^c) = B^c$ [from 1.4] and $f^{-1}(B^c) = f^{-1}[V_\delta(B^c)] \leq \gamma_\delta[f^{-1}(B^c)]$. Hence, it follows from 1.13 that $f^{-1}(B^c)$ is a fuzzy semi $\delta - V$ set in X. Therefore, f is fuzzy semi $\delta - V$ continuous. Thus, we have shown that (i) $\Rightarrow$ (iii) $\Leftrightarrow$ (ii) $\Leftrightarrow$ (iv) and (iii) $\Rightarrow$ (i).

Theorem 2.4. *Let $f\colon (X, F_1^{V_\delta}) \to (Y, F_2^{V_\delta})$ be a bijective mapping from a fuzzy topological space $(X, F_1^{V_\delta})$ into a fuzzy topological space $(X, F_2^{V_\delta})$. Then f is fuzzy semi $\delta - V$ continuous iff $f[\gamma_\delta(A^c)] \geq V_\delta[f(A^c)]$.*

Proof. $f[\gamma_\delta(A^c)] \geq V_\delta[f(A^c)]$ iff $1 - V_\delta[f(A^c)] \geq 1 - f[\gamma_\delta(A^c)]$ iff $[\Lambda_\delta f(A)] \geq f[\eta_\delta(A)]$ [from 1.2 (7) and 1.6 (5)] iff f is fuzzy $\delta - V$ continuous [from Theorem 2.3].

3. Relationship between fuzzy supra semi $\delta - V$ continuity and fuzzy semi $\delta -$ $- V$ continuity. In this section, we introduce the relationship between fuzzy semi $\delta - V$ continuity, fuzzy supra semi $\delta - V$ continuity, and fuzzy Λ_δ continuity.

Theorem 3.1. *If a function $f\colon X \to Y$ is fuzzy supra semi $\delta - V$ continuous, then f is fuzzy semi $\delta - V$ continuous.*

Proof. Let A be a fuzzy $\delta - V$ set in Y. Then it follows from 1.12 that A is a fuzzy semi $\delta - V$ set. Since f is fuzzy supra semi $\delta - V$ continuous, $f^{-1}(A)$ is a fuzzy semi $\delta - V$ set in X, i.e., the inverse image of a fuzzy $\delta - V$ set in Y is a fuzzy semi $\delta - V$ set in X. Hence, f is fuzzy semi $\delta - V$ continuous.

***Remark* 3.1.** The converse statement is not necessarily true because a fuzzy semi $\delta - V$ set is not necessarily a fuzzy $\delta - V$ set.

Theorem 3.2. *If a function $f\colon X \to Y$ is fuzzy Λ_δ continuous, then it is also fuzzy semi $\delta - V$ continuous.*

Proof. Let A be a fuzzy $\delta - \Lambda$ set in Y. Since f is fuzzy Λ_δ continuous, we conclude that $f^{-1}(A)$ is a fuzzy $\delta - \Lambda$ set in X, i.e., $f^{-1}(A)$ is a fuzzy semi $\delta - \Lambda$ set in X [from 1.12]. Hence, by virtue of Theorem 2.1, f is fuzzy semi $\delta - V$ continuous.

***Remark* 3.2.** The converse statement is not necessarily true because a fuzzy semi $\delta - \Lambda$ set is not necessarily a fuzzy $\delta - \Lambda$ set. This follows from Example 2.1.

1. *Mukherjee A., Halder S.* On fuzzy semi $\delta - \Lambda$ sets and fuzzy semi $\delta - V$ sets *V-6* // J. Trip. Math. Soc. – 2004. – **6**. – P. 81 – 88.
2. *Mukherjee A., Halder S.* Some properties of fuzzy $\delta - \Lambda$ sets and fuzzy $\delta - V$ sets *V-6* // Ibid. – P. 23 – 34.
3. *Mukherjee A., Halder S.* On fuzzy supra semi $\delta - V$ continuity in fuzzy supra semi $\delta - V$ topological space // Acta math. hung. (to appear).
4. *Mukherjee A., Halder S.* On fuzzy Λ_δ continuous functions // Indian J. Pure and Appl. Math. (to appear).

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