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FRECHET-VALUED HOLOMORPHIC FUNCTIONS ON COMPACT SETS IN (DFN)-SPACES

ФРЕШЕ-ЗНАЧНІ ГОЛОМОРФНІ ФУНКЦІЇ НА КОМПАКТНИХ МНОЖИНАХ У (DFN)-ПРОСТОРАХ

The aim of this paper is to give the equivalence between the weak holomorphicity and the holomorphicity of Frechet-valued functions on compact polydiscs in (DFN)-spaces. Moreover, the relations between separately holomorphic functions and holomorphic functions on compact polydiscs in (DFN)-spaces are also given.

Мета цієї статті — встановити еквівалентність між слабкою голоморфністю та голоморфністю Фреше-значних функцій на компактних полідисках у (DFN)-просторах. Також наведено співвідношення між нарізно голоморфними функціями та голоморфними функціями на компактних полідисках у (DFN)-просторах.

Introduction. Let E be a Frechet space (i.e., a complete metrizable locally convex space) with a fundamental system of semi-norms $\{\|\cdot\|_k\}$. For each subset B of E, we define $\|\cdot\|_B^*\colon E' \to [0,+\infty]$ by

$$\|u\|_B^* = \sup\big\{|u(x)|\colon x \in B\big\},$$

where $u \in E'$, E' is the topological dual space of E.

Instead of $\|\cdot\|_{U_k}^*$ we write $\|\cdot\|_k^*$, where $U_k = \big\{x \in E\colon \|x\|_k \leqslant 1\big\}$. Using this notation, we say that E has the property

$$\left.\begin{array}{ll} (DN) & \exists p\ \forall q, d>0\ \exists k, C>0 \\ & \\ (\overline{DN}) & \exists p\ \forall q\ \exists k\ \forall d>0\ \exists C>0 \end{array}\right\} \|x\|_q^{1+d} \leqslant C\|x\|_k\|x\|_p^d \quad \forall x \in E.$$

$$\left.\begin{array}{ll} (\Omega) & \forall p\ \exists q\ \forall k\ \exists d, C>0 \\ & \\ (\tilde{\Omega}) & \forall p\ \exists q, d>0\ \forall k\ \exists C>0 \end{array}\right\} \|u\|_q^{*1+d} \leqslant C\|u\|_k^*\|u\|_p^{*d} \quad \forall u \in E'.$$

Throughout this paper, if the Frechet space E has the property (DN) (respectively, $(\overline{DN})$, (Ω), $(\tilde{\Omega})$), then we write $E \in (DN)$ (respectively, $E \in (\overline{DN})$, $E \in (\Omega)$, $E \in (\tilde{\Omega})$). The above properties have been introduced and investigated by Vogt [1] – [3].

In this paper, for all notions concerning the theory of holomorphic functions on locally convex spaces and the theory of nuclear locally convex spaces, we refer readers to the books of S. Dineen [4] and A. Pietsch [5]. However, for convenience of readers, we recall some important notions which we use frequently here.

Let $(E_\alpha)_{\alpha\in\Gamma}$ be a collection of locally convex spaces. The locally convex space E is the locally convex inductive limit of $(E_\alpha)_{\alpha\in\Gamma}$ and we write $E = \lim\limits_{\alpha}\operatorname{ind} E_\alpha$ if for each α in Γ, there exists a linear mapping $i_\alpha\colon E_\alpha \to E$ such that E has the finest locally convex topology for which each i_α is continuous. A locally convex inductive limit of normed spaces is called a bornological space.

Let X be a compact set in the Frechet space E. By $\mathcal{H}(X)$ we denote the space of germs of holomorphic function on X. This space is equipped with the inductive topology

$$\mathcal{H}(X) = \lim_{U\downarrow X}\operatorname{ind}\mathcal{H}^\infty(U);$$

here, for each neighborhood U of X, we denote by $\mathcal{H}^\infty(U)$ the Banach space of bounded holomorphic functions on U with the sup-norm

$$\|f\|_U = \sup\big\{|f(z)|\colon z \in U\big\}.$$

A locally convex space E is called to be quasi-Montel if every closed bounded subset of E is compact. If p is a semi-norm on the vector space E, we set $E_p = \widehat{(E/_{p^{-1}(0)}, p)}$ (i.e., E_p is the Banach space obtained by factoring out the kernel of p and completing the normed linear space $(E/_{p^{-1}(0)}, p)$. A locally convex space E is called to be Schwartz space if for each continuous semi-norm p on E there exists a continuous semi-norm q on E, $q \geqslant p$, such that the canonical mapping (i.e., the mapping induced by the indentity on E) from E_q to E_p is compact. A linear mapping T between the Banach spaces E and F is nuclear if there exists a sequence $(\lambda_n)_{n=1}^\infty$ in l_1, a bounded sequence $(x_n)_{n=1}^\infty$ in F, and a bounded sequence $(\psi_n)_{n=1}^\infty$ in E' such that $Tx = \sum_{n=1}^\infty \lambda_n \psi_n(x) x_n$ for every x in E. A locally convex space E is called to be nuclear if for each continuous semi-norm p on E there exists a continuous semi-norm q on E, $q \geqslant p$, such that the canonical mapping from E_q to E_p is nuclear.

A sequence of vectors $(e_n)_{n=1}^\infty$ in a locally convex space E is called a basis if for each x in E there exists a unique sequence of scalars x_n such that

$$x = \lim_{m\to\infty} \sum_{n=1}^m x_n e_n = \sum_{n=1}^\infty x_n e_n.$$

If the mapping $P_m\colon E \to E$,

$$P_m\left(\sum_{n=1}^\infty x_n e_n\right) = \sum_{n=1}^m x_n e_n$$

are continuous for all m, the basis is called a Schauder basis. The Schauder basis $(e_n)_{n=1}^\infty$ is said to be absolute if for any semi-norm p on E there exists semi-norm q on E such that

$$\sum_{n=1}^\infty |x_n| p(e_n) \leqslant q\left(\sum_{n=1}^\infty x_n e_n\right)$$

for any $\sum_{n=1}^\infty x_n e_n \in E$.

A compact polydisc X in E', the topological dual space of E, is called to be a compact determining polydisc if every holomorphic function g on X such that $g|_X = 0$ then $g = 0$ on a neighbourhood of X in E'.

Let E and F be locally convex spaces and let X be an open set in E. A function $f\colon X \to F$ is called to be holomorphic on X if f is continuous and for every finite-dimensional subspace G of E, $f|_{G\cap X}$ is holomorphic function of several complex variables. If the above-mentioned request holds for all $u \circ f$, $u \in F'$, the topological dual space of F, then the function f is said to be weakly holomorphic on X.

By $\mathcal{H}(X, F)$ (respectively, $\mathcal{H}_w(X, F)$) we denote the vector space of all holomorphic (respectively, weakly holomorphic) functions on X with values in F. The aim of the present paper is to find some conditions for which

(A) $\mathcal{H}(X, F) = \mathcal{H}_w(X, F)$.

This problem has been interested by some authors [6] – [10]. In [7], L. M. Hai has shown that $F \in (DN)$ if and only if (A) holds for every $\widetilde{L}$-regular compact set X in a Frechet space E, where a compact subset X in a Frechet space E is called $\widetilde{L}$-regular if $[\mathcal{H}(X)]' \in (\tilde{\Omega})$. Moreover, he also has shown that for a compact polydisc X in a dual space of a nuclear Frechet space E with a basis, (A) holds for every Banach space F if and only if $E \in (DN)$.

In this paper, we shall prove the following theorems:

Theorem A. *Let E be a Frechet nuclear space with a basis having a continuous norm. Then the following statements are equivalent*:

(i) E has the property (*DN*);

(ii) $\mathcal{H}(X, F) = \mathcal{H}_w(X, F)$ and $\mathcal{H}(X)$ is quasi-Montel for every compact determining polydisc X in E' and for every Frechet space $F \in (\overline{DN})$.

To state the second theorem, we give the following notion. Let X be a compact set in a locally convex space E and let $f\colon X \to \mathcal{H}(F)$ be a continuous function with values in $\mathcal{H}(F)$. The function f is called to be separately holomorphic if $\delta_x \circ f \in \mathcal{H}(X)$ for every $x \in F$, where $\delta_x\colon \mathcal{H}(F) \to \mathbb{C}$ is given by

$$\delta_x(\varphi) = \varphi(x) \quad \text{for each} \quad \varphi \in \mathcal{H}(F).$$

By $\mathcal{H}_\delta(X, \mathcal{H}(F))$ we denote the vector space of separately holomorphic functions on X with values in $\mathcal{H}(F)$.

Let E' denote the strong dual space of a Frechet space E. A holomorphic function on E' is said to be of bounded type if it is bounded on every bounded set in E'. By $\mathcal{H}_b(E')$ we denote the metric locally convex space of entire functions of bounded type on E' equipped with the topology of the convergence on bounded sets in E'.

Theorem B. *Let E be a Frechet nuclear space with a basis and have a continuous norm. Then the following statements are equivalent*:

(i) E has the property (DN);

(ii) $\mathcal{H}(X, \mathcal{H}_b(F')) = \mathcal{H}_\delta(X, \mathcal{H}_b(F'))$ holds for every compact determining polydisc X in E' and either every Frechet – Schwartz space $F \in (\overline{DN})$ having an absolute basis or every Banach space F.

Theorem C. *Let E be a nuclear Frechet space with a basis and have a continuous norm. Then*

$$\mathcal{H}(X, F') = \mathcal{H}_w(X, F')$$

holds for every compact determining polydisc X in E' and for every Frechet space F if and only if $E \in (DN)$.

1. Proof of Theorem A. For the proof of Theorem A, we need the following two lemmas:

Lemma 1 [6]. *Let B be a Banach space and let $\mathcal{H}(O_B)$ denote the space of germs of holomorphic functions at O in B. Then $\left[\mathcal{H}(O_B)\right]'_\beta \in (\Omega)$.*

Lemma 2 [11]. *Every continuous linear map from a Frechet space $E \in (\Omega)$ into a Frechet space $F \in (\overline{DN})$ can be factorized through a Banach space. This means that there exists a continuous semi-norm ρ on E and a continuous linear map $g : E_\rho \to F$, where E_ρ is the Banach space associated to the continuous semi-norm such that $f = g \circ \Phi_\rho$, $\Phi_\rho\colon E \to E_\rho$ is the canonical quotient map.*

We now prove Theorem A.

Sufficiency. Assume that $E \in (DN)$, $F \in (\overline{DN})$, and X is a determining compact polydisc in E' of the form

$$X = \left\{ \omega = (\omega_n) \in E' \colon \sup_{n \geqslant 1} |\omega_n \alpha_n| \leqslant 1 \right\},$$

where $\{\alpha_n\}_{n \geqslant 1}$ is a sequence of positive numbers. Since $\mathcal{H}(X)$ is regular [4], $\mathcal{H}(X)$ is quasi-Montel. It suffices to show that $\mathcal{H}_w(X, F) \subset \mathcal{H}(X, F)$. Let $f \in \mathcal{H}_w(X, F)$. Since X is compact, $\alpha_n \neq 0$ for every $n \geqslant 1$. Note that $\mathcal{H}(X)$ is regular because $E \in (DN)$ [4] and since X is determinating then we can consider the linear map $\widehat{f}\colon F' \to \mathcal{H}(X)$ given $\widehat{f}(u) = \widehat{uf}$, a holomorphic extension of uf to a neighbourhood of X.

By [4], on a neighbourhood (depending on u) of X, we have

$$\widehat{f}(u)(\omega) = \sum_{m \in N^{(N)}} b_m(u) \omega^m,$$

where

$$b_m(u) = \frac{1}{(2\pi i)^n} \int\limits_{|\lambda_1| = \frac{1}{|\alpha_1|}} \ldots \int\limits_{|\lambda_n| = \frac{1}{|\alpha_n|}} \frac{\widehat{f}(u)(\lambda_1 e_1^* + \ldots + \lambda_n e_n^*)}{\lambda_1^{m_1+1} \ldots \lambda_n^{m_n+1}} d\lambda$$

and

$$N^{(N)} = \Big\{ m = (m_n)_{n=1}^{\infty};\ \ m_n \ \text{ is a nonnegative integer for all } \ n \ \ \text{and } \ m_n = 0 \ \text{ for all } \ n \ \text{ sufficiently large} \Big\},$$

$\{e_j\}_{j \geqslant 1}$ and $\{e_j^*\}_{j \geqslant 1}$ are bases of E and E', respectively.

We check that $b_m(u)$ are continuous on F'. Fix $m \in N^{(N)}$ and put

$$X_m = X \cap \operatorname{span}\{e_1^*, \ldots, e_n^*\} = \left\{ (\omega_1, \ldots, \omega_n) \colon |\omega_i| \leqslant \frac{1}{|\alpha_i|}, \ \ i = \overline{1, n} \right\}.$$

Consider $f_m = f|_{X_m}$. By the hypothesis, $f_m \in \mathcal{H}_w(X_m, F)$. By [7] and by the $\widetilde{L}$-regularity of X_m in span $\{e_1^*, \ldots, e_n^*\}$, we have $f_m \in \mathcal{H}(X_m, F)$. Thus, there exists a neighbourhood V_m of X_m in span $\{e_1^*, \ldots, e_n^*\}$ for which f_m is extended to a holomorphic function $\widehat{f}_m \colon V_m \to F$. Hence,

$$b_m(u) = \frac{1}{(2\pi i)^n} \int\limits_{|\lambda_1| = \frac{1}{|\alpha_1|}} \ldots \int\limits_{|\lambda_n| = \frac{1}{|\alpha_n|}} \frac{\widehat{f}(u)(\lambda_1 e_1^* + \ldots + \lambda_n e_n^*)}{\lambda^{m+1}} d\lambda =$$

$$= \frac{1}{(2\pi i)^n} \int\limits_{|\lambda_1| = \frac{1}{|\alpha_1|}} \ldots \int\limits_{|\lambda_n| = \frac{1}{|\alpha_n|}} \frac{\widehat{f}_m(u)(\lambda_1 e_1^* + \ldots + \lambda_n e_n^*)}{\lambda^{m+1}} d\lambda, \tag{1}$$

where $\lambda^{m+1} = \lambda_1^{m_1+1} \ldots \lambda_n^{m_n+1}$ is continuous on F'.

We now prove that $\widehat{f}\colon F'_{\text{bor}} \to \mathcal{H}(X)$ is continuous, where F'_{bor} means that the space F' is equipped with the bornological topology. Take arbitrary $\mu \in [\mathcal{H}(X)]' \cong \mathcal{H}(U)$ [12], where

$$U = \left\{ z = (z_n) \in E \colon \sup \left| \frac{z_n}{\alpha_n} \right| < 1 \right\}$$

is an open polydisc in E.

By [4], we can write

$$\mu(z) = \sum_{N^{(N)}} a_m(\mu) z^m, \quad z \in U,$$

and

$$\langle \widehat{f}(u), u \rangle = \sum_{N^{(N)}} b_m(u) a_m(\mu). \tag{2}$$

From

$$\sup \left\{ \left| \sum_J b_m(u) a_m(\mu) \right| : J \subset N^{(N)}, \ \ J \ \text{ is finite} \right\} < \infty,$$

for $u \in F'$, we infer that

$$\sup \left\{ \left| \sum_J b_m(u) a_m(\mu) \right| : J \subset N^{(N)}, \ \ J \ \text{ is finite}, \ \ u \in B' \right\} < \infty$$

for all bounded sets $B' \subset F'$.

This implies the continuity of $\widehat{f}\colon F'_{\text{bor}} \to \mathcal{H}(X)$. By the regularity of $\mathcal{H}(X)$, there exists an increasing sequence of bounded sets $\{B_k\}$ in E such that $\widehat{f}$ maps continuously F'_{bor} into $\lim\limits_{k}\text{ind}\mathcal{H}^\infty(B_k^0)$. Choose $\{\varepsilon_k\} \downarrow 0$ such that $B = \bigcup\limits_{k=1}^{\infty} \varepsilon_k B_k$ is bounded in E. Let P and Q denote the space $E'/_{\ker\| \, \|_B^*}$ equipped with the topology generated by $\{\| \, \|_{B_k}^*\}$ and $\| \, \|_B^*$ respectively. Note that

$$\text{id}\colon \quad \big(E'/_{\ker\| \, \|_B^*}, Q\big) \to \big(E'/_{\ker\| \, \|_B^*}, P\big)$$

is continuous. It follows that

$$\lim_{k}\text{ind}\mathcal{H}^\infty(B_k^0) = \mathcal{H}(O_P)$$

which is continuously embedded into $\mathcal{H}(O_Q)$. Lemmas 1 and 2 imply that there exists a neighbourhood W of O in F'_{bor} and a neighbourhood U of X such that $\widehat{f}(W) \subset \mathcal{H}^\infty(U)$. Define a holomorphic function

$$g\colon U \to F$$

given by

$$g(x)(u) = \widehat{f}(u)(x)$$

for $x \in U$, $x \in F'$. We have

$$g(x)(u) = \widehat{f}(u)(x) = f(u)(x)$$

for $x \in X$, $u \in F'$. Thus, $g|_X = f$ and f is extended holomorphically to U.

Necessity. Let $\{e_i\}$ be a basis of E with the dual basis $\{e_j^*\} \subset E$. Since E has a continuous norm, there exists an open polydisc in E of the form

$$U = \left\{ z = \sum_{j=1}^{\infty} z_j e_j \in E\colon \sup_{j \geqslant 1} |z_j| p_j < 1 \right\},$$

where $p_j > 0$ for all $j \geqslant 1$.

Hence,

$$X = U^M = \left\{ \omega = \sum_{j=1}^{\infty} \omega_j e_j^* : |\omega_j| \leqslant p_j \ \text{ for } \ j \geqslant 1 \right\} = p_1 \overline{\Delta} \times Y$$

is a compact determining polydisc in E'; here $\Delta = \big\{\omega_1 \in \mathbb{C} \colon |\omega_1| \leqslant 1\big\}$ and $Y = \big\{(\omega_j)_{j \geqslant 2} \colon |\omega_j| \leqslant p_j \text{ for } j \geqslant 2\big\}$.

Indeed, given $f \in \mathcal{H}(X)$ such that $f|_X = 0$. Take a convex neighbourhood W of X in E' such that $f \in \mathcal{H}(W)$. For each $m \geqslant 1$, put $L = \operatorname{span}\{e_1^*, e_2^*, \ldots, e_m^*\}$ and consider

$$V = \left\{ \sum_{1 \leqslant j \leqslant m} \lambda_j p_j e_j^* \colon \sum_{j=1}^{m} |\lambda_j| \leqslant 1 \right\}.$$

Note that V is a neighbourhood of $0 \in L$ which is contained in X. By the hypothesis, $f|_V = 0$. Thus, $f|_{W \cap L} = 0$. Hence,

$$f|_{W \cap \operatorname{span}\{e_j^* \colon j \geqslant 1\}} = 0.$$

The density of $W \cap \operatorname{span}\{e_j^* \colon j \geqslant 1\}$ in W and the continuity of f imply that $f|_W = 0$.

First, we show that $\mathcal{H}(X)$ is regular. Since $\mathcal{H}(X)$ is quasi-Montel, $\mathcal{H}(X)$ is quasi-reflexive.

Now given a balanced convex bounded set A in $\mathcal{H}(X)$. Consider the normed space $E_1 = \mathcal{H}(X)(A)$ spanned by A and the function $f \colon X \to E_1'$ given by $f(x)(\sigma) = \sigma(x)$ for $x \in X$, $\sigma \in E_1$.

Since $\mathcal{H}(X) = \mathcal{H}(X)''$, we infer that f is weakly holomorphic. By the hypothesis, f can be extended to a bounded holomorphic function $\widehat{f}$ on a neighbourhood V_1 of X in E'. From the relation

$$\sigma(x) = f(x)(\sigma) = \widehat{f}(x)(\sigma)$$

for every $x \in X$ and $\sigma \in A$ and from the uniqueness of X it follows that A is contained and bounded in $\mathcal{H}^{\infty}(V_1)$. Thus, $\mathcal{H}(X)$ is regular and hence, by [4], $\mathcal{H}(U)$ is bornological. Since $L_b(F, \mathcal{H}(p_1\Delta)) \cong F' \widehat{\otimes}_{\Pi} \mathcal{H}(p_1\Delta)$ contained in

$$\mathcal{H}(W_1) \widehat{\otimes}_{\Pi} \mathcal{H}(p_1\Delta) \cong \mathcal{H}(W_1) \widehat{\otimes}_{\varepsilon} \mathcal{H}(p_1\Delta) \cong \mathcal{H}(p_1\Delta \times W_1) = \mathcal{H}(U)$$

is a complemented subspace of one, where $E = \mathbb{C}e_1 \otimes F$, $U = p_1\Delta \times W_1$, it follows that $L_b\big(F, \mathcal{H}(p_1\Delta)\big)$ is bornological. By [3] (Theorem 4.9), F and hence, $E \in (DN)$.

2. Proof of Theorem B. We need the following lemma:

Lemma 3 [13]. *Let E, F be Frechet spaces with $E \in (\Omega)$, $F \in (\overline{DN})$. Assume that F is a Schwartz space with an absolute basis. Then every E'-valued holomorphic function on F' is factorized through a Banach space.*

We now give the proof of Theorem B.

Sufficiency. Given $f \in \mathcal{H}_\delta(X, \mathcal{H}_b(F'))$.

(i) First assume that F is a Frechet – Schwartz space with an absolute basis and $F \in (\overline{DN})$. Consider the map $\widehat{f} \colon F' \to \mathcal{H}(X)$ given by $\widehat{f}(u) = \delta(u) \circ f$ for $u \in F'$. We use again the notations of Theorem A. By (1) and (2), for each $u \in [\mathcal{H}(X)]'$, the set

$$\left\{ \sum_{j} b_m(\cdot) a_m(\mu) \colon J \subset M, J \ \text{ is finite} \right\}$$

is bounded in $\mathcal{H}_b(F')$ and hence, it is relatively compact in $\mathcal{H}_b(F')$. This yields that the function $\widehat{f}\colon F' \to \mathcal{H}(X)$ is weakly holomorphic. Since F' is a DFS-space, it implies that $\widehat{f}$ is holomorphic. Now, as in the proof of Theorem A, by Lemma 3, $\widehat{f}$ is holomorphically factorized through a Banach space. By the regularity of $\mathcal{H}(X)$, as in Theorem A, we can extend f to an element of $\mathcal{H}\big(X, \mathcal{H}_b(F')\big)$.

(ii) Let F be a Banach space. As in (i), $\widehat{f}\colon F' \to \mathcal{H}(X)$ is a holomorphic function of bounded type. Again, by the regularity of $\mathcal{H}(X)$, it implies that f can be holomorphically extended to an element of $\mathcal{H}(X, \mathcal{H}_b(F'))$.

Necessity. As in the proof of Theorem A, it suffices to show that $\mathcal{H}(X)$ is regular. Given A a balanced convex closed bounded set in $\mathcal{H}(X)$. As in Theorem A, consider the function $f: X \to \mathcal{H}(X)(A)$. Since $\mathcal{H}(X)(A) \subset \mathcal{H}_b([H(X)(A)]')$, $f \in \mathcal{H}_\delta\big(X, \mathcal{H}_b\big([\mathcal{H}(X)(A)]'\big)\big)$. By the hypothesis, f is extended to a bounded holomorphic function $\widehat{f}\colon V \to \mathcal{H}_\delta([\mathcal{H}(X)(A)]')$. Since X is an uniqueness set, we may assume that $\widehat{f}\colon V \to \mathcal{H}(X)(A)$ is holomorphic. This yields that A is contained and bounded in $\mathcal{H}^\infty(V)$. The theorem is proved.

3. Proof of Theorem C. Given $f \in \mathcal{H}_w(X, F')$. Since $f(X)$ is bounded, we can find a neighbourhood V of $0 \in F$ such that $f(X)$ is contained and bounded in $F'(V^0)$. Since $F'(V^0) \cong (\widetilde{F}(V))'$, where $\widetilde{F}(V)$ is the Banach space associated to V, it implies that $f\colon X \to F'(V^0) \subset \mathcal{H}_b(\widetilde{F}(V))$ and $f \in \mathcal{H}_\delta(X, \mathcal{H}_b(\widetilde{F}(V)))$. Theorem B yields that $f \in \mathcal{H}(X, \mathcal{H}_b(\widetilde{F}(V)))$ and as in the proof of Theorem B we infer that $f \in \mathcal{H}(X, F')$.

The necessity follows from the proof of Theorem A.

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