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M. Svanadze (Ilia Chavchavadze State Univ., Tbilisi, Georgia),
P. Giordano, V. Zampoli (Univ. Salerno, Italy)

ON THE REPRESENTATIONS OF GENERAL SOLUTION IN THE THEORY OF MICROPOLAR THERMOELASTICITY WITHOUT ENERGY DISSIPATION

ЗОБРАЖЕННЯ ЗАГАЛЬНОГО РОЗВ'ЯЗКУ В ТЕОРІЇ МІКРОПОЛЯРНОЇ ТЕРМОЕЛАСТИЧНОСТІ БЕЗ РОЗСІЮВАННЯ ЕНЕРГІЇ

In the present paper, the linear theory of micropolar thermoelasticity without energy dissipation is considered. This work is articulated as follows. Section 2 regards the basic equations for micropolar thermoelastic materials, supposed isotropic and homogeneous, and the assumptions on the constitutive constants. In Section 3 some theorems connected with the representations of general solution are studied.

Розглянуто лінійну теорію мікрополярної термоеластичності без розсіювання енергії. Статтю побудовано таким чином. Другий пункт присвячено базовим рівнянням для мікрополярних термоеластичних матеріалів, які вважаються ізотропними та однорідними, та припущенням щодо основних констант. У третьому пункті доведено деякі теореми про зображення загального розв'язку.

1. Introduction. In [1], Eringen established the theory of micropolar thermoelasticity. In recent years there has been very much written on the subject of this theory. The basis results and extensive review of works on the theory of micropolar thermoelasticity can be found in the books of Eringen [2] and Nowacki [3].

In [4], Boschi and Iesan extended a generalized theory of micropolar thermoelasticity that permits the transmission of heat as thermal waves at finite speed. Recently, Green and Naghdi [5] introduced a theory of thermoelasticity without energy dissipation. In [6], Ciarletta presented a linear theory of micropolar thermoelasticity without energy dissipation. This theory permits the transmission of heat as thermal waves at finite speed, and the heat flow does not involve energy dissipation.

Contemporaly treatment of the various boundary-value problems on the elasticity theory usually begins with the representation of a general solution of field equations in terms of elementary (harmonic, biharmonic, metaharmonic and etc.) functions. In the classical theory of elasticity the Boussinesq – Somiliana – Galerkin, Boussinesq – Papkovitch – Neuber, Green – Lamé and Cauchy – Kovalevski – Somiliana solutions are well known (see Gurtin [7], Kupradze and al. [8], Nowacki [9]). An excellent review of the history of these solutions is given in Gurtin [7].

The representations of Galerkin-type solutions in the theory of micropolar thermoelasticity without energy dissipation, in the theory of thermoelastic materials with voids, and in the dynamical theory of binary mixture consisting of a gas and an elastic solid are established by Ciarletta [6, 10, 11]. In the theories of binary mixtures of elastic solids and fluid-saturated porous media the representations of general solutions are presented by Basheleishvili [12], Svanadze [13], and Svanadze and de Boer [14].

In this article the linear theory of isotropic and homogeneous micropolar thermoelastic materials without energy dissipation [6] is considered. The representations of general solution of the system of steady oscillations in terms of metaharmonic functions are obtained.

2. Basic equations. Let $\mathbf{x} = (x_1, x_2, x_3)$ be the point of the Euclidean three-dimensional space E^3, and let t denote the time variable. We consider a linear micropolar thermoelastic material which occupies the region Ω of E^3. The system of linearized equations of motion in the theory of micropolar thermoelasticity without energy dissipation for isotropic elastic solids can be written as [6]

$$
\begin{gathered}
(\mu+\kappa)\Delta\widetilde{\mathbf{u}} + (\lambda+\mu)\,\mathrm{grad}\,\mathrm{div}\,\widetilde{\mathbf{u}} + \kappa\,\mathrm{curl}\,\widetilde{\boldsymbol{\varphi}} - m\,\mathrm{grad}\,\widetilde{\theta} = \rho(\ddot{\widetilde{\mathbf{u}}} - \mathbf{G}'),\\
\gamma\Delta\widetilde{\boldsymbol{\varphi}} + (\alpha+\beta)\,\mathrm{grad}\,\mathrm{div}\,\widetilde{\boldsymbol{\varphi}} + \kappa\,\mathrm{curl}\,\widetilde{\mathbf{u}} - 2\kappa\widetilde{\boldsymbol{\varphi}} = \rho_1\ddot{\widetilde{\boldsymbol{\varphi}}} - \rho\mathbf{G}'', \qquad (1)\\
k_0\Delta\widetilde{\theta} - aT_0\ddot{\widetilde{\theta}} - mT_0\,\mathrm{div}\,\ddot{\widetilde{\mathbf{u}}} = -\rho\dot{S},
\end{gathered}
$$

where $\widetilde{\mathbf{u}} = (\widetilde{u}_1, \widetilde{u}_2, \widetilde{u}_3)$ is the displacement vector, $\widetilde{\boldsymbol{\varphi}} = (\widetilde{\varphi}_1, \widetilde{\varphi}_2, \widetilde{\varphi}_3)$ is the microrotation vector, $\widetilde{\theta}$ is the temperature measured from the constant absolute temperature T_0 ($T_0 > 0$); λ, μ, κ, m, α, β, γ, a, k_0 are constitutive coefficients, ρ ($\rho > 0$) is the reference mass density, ρ_1 ($\rho_1 > 0$) is a coefficient of inertia, $\mathbf{G}'$ is the body force density, $\mathbf{G}''$ is the body couple density, and S is the heat source density [6]; Δ is the Laplacian, and dot denotes differentiation with respect to t: $\dot{\widetilde{\mathbf{u}}} = \dfrac{\partial\widetilde{\mathbf{u}}}{\partial t}$, $\ddot{\widetilde{\mathbf{u}}} = \dfrac{\partial^2\widetilde{\mathbf{u}}}{\partial t^2}$.

If the body forces $\mathbf{G}'$, $\mathbf{G}''$ and the heat source density S are assumed to be absent, and the displacement vector $\widetilde{\mathbf{u}}$, the microrotation vector $\widetilde{\varphi}$ and the temperature $\widetilde{\theta}$ are postulated to have a harmonic time variation, that is

$$
\widetilde{\mathbf{u}}(\mathbf{x},t) = \mathrm{Re}\left[\mathbf{u}(\mathbf{x})e^{-i\omega t}\right], \quad \widetilde{\boldsymbol{\varphi}}(\mathbf{x},t) = \mathrm{Re}\left[\boldsymbol{\varphi}(\mathbf{x})e^{-i\omega t}\right], \quad \widetilde{\theta}(\mathbf{x},t) = \mathrm{Re}\left[\theta(\mathbf{x})e^{-i\omega t}\right],
$$

then from the system of equations of motion (1) we obtain the following system of equations of steady oscillations (steady vibrations):

$$
\begin{gathered}
(\mu+\kappa)\Delta\mathbf{u} + (\lambda+\mu)\,\mathrm{grad}\,\mathrm{div}\,\mathbf{u} + \kappa\,\mathrm{curl}\,\boldsymbol{\varphi} - m\,\mathrm{grad}\,\theta + \rho\omega^2\mathbf{u} = \mathbf{0},\\
\gamma\Delta\boldsymbol{\varphi} + (\alpha+\beta)\,\mathrm{grad}\,\mathrm{div}\,\boldsymbol{\varphi} + \kappa\,\mathrm{curl}\,\mathbf{u} + \mu_1\boldsymbol{\varphi} = \mathbf{0}, \qquad (2)\\
k_0\Delta\theta + a_0\theta + m_0\,\mathrm{div}\,\mathbf{u} = 0,
\end{gathered}
$$

where $\mu_1 = \rho_1\omega^2 - 2\kappa$, $a_0 = aT_0\omega^2$, $m_0 = mT_0\omega^2$, and ω is the oscillation frequency ($\omega > 0$).

Throughout this article, it is assumed that all functions are continuous and differentiable up to the required order on Ω. We assume that the constitutive coefficients satisfy the conditions [6]

$$
\begin{gathered}
3\lambda + 2\mu + \kappa > 0, \qquad 2\mu + \kappa > 0, \qquad \kappa > 0, \qquad k_0 > 0,\\
3\alpha + \beta + \gamma > 0, \qquad \gamma \pm \beta > 0, \qquad a > 0.
\end{gathered}
$$

In this article the representations of general solution of system (2) in terms of metaharmonic functions are obtained.

3. Representations of general solution. We consider separately two possible cases: $\omega \neq \omega_0$ and $\omega = \omega_0$, where $\omega_0 = \sqrt{\dfrac{2\kappa}{\rho_1}}$.

1. Let $\omega \neq \omega_0$. In the sequel we use the following lemmas.

Lemma 1. *If* $(\mathbf{u}, \boldsymbol{\varphi}, \theta)$ *is a solution of system* (2), *then*

$$
\begin{gathered}
\mathbf{u} = -\frac{1}{\rho\omega^2}\Big[\mathrm{grad}(\mu_0\,\mathrm{div}\,\mathbf{u} - m\theta) - (\mu+\kappa)\,\mathrm{curl}\,\mathrm{curl}\,\mathbf{u} + \kappa\,\mathrm{curl}\,\boldsymbol{\varphi}\Big],\\
\boldsymbol{\varphi} = -\frac{\gamma_0}{\mu_1}\,\mathrm{grad}\,\mathrm{div}\,\boldsymbol{\varphi} + \frac{1}{\mu_1}(\gamma\,\mathrm{curl}\,\mathrm{curl}\,\boldsymbol{\varphi} - \kappa\,\mathrm{curl}\,\mathbf{u}).
\end{gathered} \qquad (3)
$$

From system (2) directly follows (3).

Lemma 2. *If* $(\mathbf{u}, \boldsymbol{\varphi}, \theta)$ *is a solution of system* (2), *then*

$$(\Delta + k_1^2)(\Delta + k_2^2)\operatorname{div}\mathbf{u} = 0, \qquad (\Delta + k_3^2)(\Delta + k_4^2)\operatorname{curl}\mathbf{u} = \mathbf{0}, \tag{4}$$

$$(\Delta + k_5^2)\operatorname{div}\boldsymbol{\varphi} = 0, \qquad (\Delta + k_3^2)(\Delta + k_4^2)\operatorname{curl}\boldsymbol{\varphi} = \mathbf{0}, \tag{5}$$

$$(\Delta + k_1^2)(\Delta + k_2^2)\theta = 0, \tag{6}$$

where k_1^2, k_2^2 *and* k_3^2, k_4^2 *are the roots of the equations* (*with respect to* ξ)

$$\mu_0 k_0 \xi^2 - (\rho k_0 + aT_0\mu_0 + m^2 T_0)\omega^2\xi + aT_0\rho\omega^4 = 0$$

and

$$\gamma(\mu + \kappa)\xi^2 - [\rho\omega^2\gamma + \mu_1(\mu + \kappa) + \kappa^2]\xi + \rho\omega^2\mu_1 = 0,$$

respectively, $\mu_0 = \lambda + 2\mu + \kappa$, $k_5^2 = \dfrac{\mu_1}{\gamma_0}$, $\gamma_0 = \alpha + \beta + \gamma$.

Proof. Applying the operator div to Eq. $(2)_2$ we obtain Eq. $(5)_1$. Applying the operator div to Eq. $(2)_1$ and taking into account Eq. $(2)_3$ we get

$$\begin{gathered}(\mu_0\Delta + \rho\omega^2)\operatorname{div}\mathbf{u} - m\Delta\theta = 0,\\ m_0 \operatorname{div}\mathbf{u} + (k_0\Delta + a_0)\theta = 0.\end{gathered} \tag{7}$$

From system (7) we have

$$\begin{gathered}[\mu_0\kappa_0\Delta^2 + (\rho\kappa_0 + aT_0\mu_0 + m^2T_0)\omega^2\Delta + aT_0\rho\omega^4]\operatorname{div}\mathbf{u} = 0,\\ [\mu_0\kappa_0\Delta^2 + (\rho\kappa_0 + aT_0\mu_0 + m^2T_0)\omega^2\Delta + aT_0\rho\omega^4]\theta = 0.\end{gathered} \tag{8}$$

On the basis of (8) we obtain Eqs. $(4)_1$ and (6).

Applying the operators $(\gamma\Delta + \mu_1)\operatorname{curl}$ and curl to Eqs. $(2)_1$ and $(2)_2$, respectively, we get

$$\begin{gathered}(\gamma\Delta + \mu_1)[(\mu + \kappa)\Delta + \rho\omega^2]\operatorname{curl}\mathbf{u} + \kappa(\gamma\Delta + \mu_1)\operatorname{curl}\operatorname{curl}\boldsymbol{\varphi} = \mathbf{0},\\ (\gamma\Delta + \mu_1)\operatorname{curl}\boldsymbol{\varphi} + \kappa\operatorname{curl}\operatorname{curl}\mathbf{u} = \mathbf{0}.\end{gathered} \tag{9}$$

Taking into account Eq. $(9)_2$ and equality $\operatorname{curl}\operatorname{curl}\mathbf{u} = \operatorname{grad}\operatorname{div}\mathbf{u} - \Delta\mathbf{u}$ from $(9)_1$ we have

$$\big\{(\gamma\Delta + \mu_1)[(\mu + \kappa)\Delta + \rho\omega^2] + \kappa^2\Delta\big\}\operatorname{curl}\mathbf{u} = \mathbf{0}. \tag{10}$$

Obviously, from Eq. (10) we obtain Eq. $(4)_2$. In the same way from Eqs. $(2)_1$ and $(2)_2$ we get Eq. $(5)_2$.

***Remark* 1.** It is easily seen that

i) $k_1^2 > 0$, $k_2^2 > 0$, $k_1^2 \neq k_2^2$;

ii) $k_3^2 > 0$, $k_4^2 > 0$, $k_3^2 \neq k_4^2$, $k_5^2 > 0$ for $\omega > \omega_0$, $k_3^2 > 0$, $k_4^2 < 0$, $k_5^2 < 0$ for $\omega < \omega_0$;

iii) $\mu_0 k_2^2 - \rho\omega^2 \neq 0$, $(\mu + \kappa)k_4^2 - \rho\omega^2 \neq 0$.

In the following we use the notations

$$\alpha_1 = k_0 k_1^2 - a_0, \qquad \alpha_2 = -m, \qquad \alpha_3 = -m_0 k_1^2, \qquad \alpha_4 = \mu_0 k_2^2 - \rho\omega^2,$$

$$\beta_1 = \gamma k_3^2 - \mu_1, \qquad \beta_2 = \kappa k_4^2, \qquad \beta_3 = \kappa, \qquad \beta_4 = (\mu+\kappa) k_4^2 - \rho\omega^2, \tag{11}$$

$$\lambda_1^{-1} = \alpha_2 \alpha_3 k_2^2 - \alpha_1 \alpha_4 k_1^2 = k_0 k_1^2 (k_2^2 - k_1^2)\alpha_4.$$

It is obvious that

$$\begin{gathered} \mu_0 \alpha_j k_{j+2}^2 + m\alpha_{j+2} = \rho\omega^2 \alpha_j, \\ [(\mu+\kappa)\beta_j - \kappa\beta_{j+2}] k_{j+2}^2 = \rho\omega^2 \beta_j, \\ \gamma\beta_{j+2} k_{j+2}^2 - \kappa\beta_j = \mu_1 \beta_{j+2}, \\ (k_0 k_j^2 - a_0)\alpha_{j+2} + m\alpha_j k_j^2 = 0, \quad j = 1, 2. \end{gathered} \tag{12}$$

Let

$$\begin{gathered} \psi_1 = \lambda_1 (\Delta + k_2^2)(\mu_0 \operatorname{div} \mathbf{u} - m\theta), \\ \psi_2 = -\lambda_0 k_0 k_1^2 (\Delta + k_1^2)\theta, \\ \psi_3 = -\frac{\gamma_0}{\mu_1} \operatorname{div} \boldsymbol{\varphi}. \end{gathered} \tag{13}$$

On the basis of Eqs. $(4)_1$, $(5)_1$ and (6) we have

$$(\Delta + k_j^2)\psi_j = 0, \quad j = 1, 2, 3. \tag{14}$$

On the other hand, by virtue of (7) and (11), from (13) follows that

$$\psi_1 = \lambda_1 (\alpha_4 \operatorname{div} \mathbf{u} + \alpha_2 k_2^2 \theta), \qquad \psi_2 = -\lambda_1 (\alpha_3 \operatorname{div} \mathbf{u} + \alpha_1 k_1^2 \theta). \tag{15}$$

From Eqs. $(13)_3$ and (15) we get

$$\begin{gathered} \operatorname{div} \mathbf{u} = -(\alpha_1 k_1^2 \psi_1 + \alpha_2 k_2^2 \psi_2), \qquad \operatorname{div} \boldsymbol{\varphi} = -\frac{\mu_1}{\gamma_0}\psi_3, \\ \theta = \alpha_3 \psi_1 + \alpha_4 \psi_2. \end{gathered} \tag{16}$$

We introduce the notation

$$\begin{gathered} \mathbf{w}_1 = (w_{11}, w_{12}, w_{13}) = \frac{1}{\beta_3 k_3^2 (k_4^2 - k_3^2)} (\Delta + k_4^2) \operatorname{curl} \boldsymbol{\varphi}, \\ \mathbf{w}_2 = (w_{21}, w_{22}, w_{23}) = \frac{1}{\beta_4 k_4^2 (k_3^2 - k_4^2)} (\Delta + k_3^2) \operatorname{curl} \boldsymbol{\varphi}. \end{gathered} \tag{17}$$

Taking into account Eqs. $(4)_2$ and $(5)_3$, from (17) we have

$$(\Delta + k_{j+2})\mathbf{w}_j = 0, \qquad \operatorname{div} \mathbf{w}_j = 0, \quad j = 1, 2, \tag{18}$$

and

$$\operatorname{curl}\boldsymbol{\varphi} = \beta_3 k_3^2 \mathbf{w}_1 + \beta_4 k_4^2 \mathbf{w}_2. \tag{19}$$

By virtue of Eq. $(12)_3$ and (19) from $(9)_2$ we get

$$\operatorname{curl}\operatorname{curl}\mathbf{u} = -\frac{1}{\beta_3}(\gamma\Delta + \mu_1)\operatorname{curl}\boldsymbol{\varphi} = \frac{1}{\beta_3}\left[\beta_1\beta_3 k_3^2 \mathbf{w}_1 + \beta_4 k_4^2(\gamma k_4^2 - \mu_1)\mathbf{w}_2\right] =$$

$$= \beta_1 k_3^2 \mathbf{w}_1 + \beta_2 k_4^2 \mathbf{w}_2. \tag{20}$$

From Eq. $(9)_1$ it follows that

$$\operatorname{curl}\mathbf{u} = -\frac{1}{\rho\omega^2}[(\mu+\kappa)\Delta\operatorname{curl}\mathbf{u} + \kappa\operatorname{curl}\operatorname{curl}\boldsymbol{\varphi}] =$$

$$= -\frac{1}{\rho\omega^2}[-(\mu+\kappa)\operatorname{curl}\operatorname{curl}\operatorname{curl}\mathbf{u} + \kappa\operatorname{curl}\operatorname{curl}\boldsymbol{\varphi}]. \tag{21}$$

On the basis of Eqs. (19) and (20) from (21) we obtain

$$\operatorname{curl}\mathbf{u} =$$

$$= -\frac{1}{\rho\omega^2}\Big[-(\mu+\kappa)\operatorname{curl}(\beta_1 k_3^2\mathbf{w}_1 + \beta_2 k_4^2 \mathbf{w}_2) - \kappa\operatorname{curl}(\beta_3 k_3^2 \mathbf{w}_1 + \beta_4 k_4^2 \mathbf{w}_2)\Big] =$$

$$= \frac{1}{\rho\omega^2}\sum_{j=1}^{2}\left[(\mu+\kappa)\beta_j - \kappa\beta_{j+2}\right]k_{j+2}^2\operatorname{curl}\mathbf{w}_j. \tag{22}$$

In view of $(12)_2$ from Eq. (22) we have

$$\operatorname{curl}\mathbf{u} = \operatorname{curl}(\beta_1\mathbf{w}_1 + \beta_2\mathbf{w}_2). \tag{23}$$

Theorem 1. *If $\omega \neq \omega_0$ and $(\mathbf{u}, \boldsymbol{\varphi}, \theta)$ is a solution of system* (2), *then*

$$\begin{gathered}\mathbf{u} = \operatorname{grad}(\alpha_1\psi_1 + \alpha_2\psi_2) + \beta_1\mathbf{w}_1 + \beta_2\mathbf{w}_2,\\ \boldsymbol{\varphi} = \operatorname{grad}\psi_3 + \operatorname{curl}(\beta_3\mathbf{w}_1 + \beta_4\mathbf{w}_2),\\ \theta = \alpha_3\psi_1 + \alpha_4\psi_2,\end{gathered} \tag{24}$$

where ψ_1, ψ_2, ψ_3 are metaharmonic functions and $\mathbf{w}_1$, $\mathbf{w}_2$ are metaharmonic vectors satisfy Eqs. (14) *and* (18), *respectively*; *α_j and β_j, $j = 1, 2, 3, 4$, are defined by Eqs.* (11).

Proof. Let $(\mathbf{u}, \boldsymbol{\varphi}, \theta)$ be a solution of system (2). Taking into account Eqs. (16), (19) and (20), from Eq. $(3)_1$ we have

$$\mathbf{u} = -\frac{1}{\rho\omega^2}\Big\{\operatorname{grad}\big[-\mu_0(\alpha_1 k_1^2\psi_1 + \alpha_2 k_2^2\psi_2) - m(\alpha_3\psi_1 + \alpha_4\psi_2)\big] -$$

$$-(\mu+\kappa)(\beta_1 k_3^2\mathbf{w}_1 + \beta_2 k_4^2\mathbf{w}_2) + \kappa(\beta_3 k_3^2\mathbf{w}_1 + \beta_4 k_4^2\mathbf{w}_2)\Big\} =$$

$$= \frac{1}{\rho\omega^2}\sum_{j=1}^{2}\Big\{(\mu_0\alpha_j k_j^2 + m\alpha_{j+2})\operatorname{grad}\psi_j + \big[(\mu+\kappa)\beta_j - \kappa\beta_{j+2}\big]k_{j+2}^2\mathbf{w}_j\Big\}. \tag{25}$$

In view of Eqs. (12) from (25) we obtain Eq. $(24)_1$.

On the basis of Eqs. $(16)_2$, (19) and (23) from $(3)_2$ we get

$$\boldsymbol{\varphi} = \operatorname{grad}\psi_3 + \frac{1}{\mu_1}\operatorname{curl}\sum_{j=1}^{2}(\gamma\beta_{j+2}k_{j+2}^2 - \kappa\beta_j)\mathbf{w}_j. \tag{26}$$

By virtue of $(12)_3$ from Eq. (26) we have Eq. $(24)_2$.

Theorem 2. *If* $\omega \neq \omega_0$ *and* $\mathbf{u}$, $\boldsymbol{\varphi}$ *and* θ *given by Eqs.* (24), *where* ψ_1, ψ_2, ψ_3 *and* $\mathbf{w}_1$, $\mathbf{w}_2$ *satisfy Eqs.* (14) *and* (18), *respectively, then* $(\mathbf{u}, \boldsymbol{\varphi}, \theta)$ *is the solution of system* (2) *in* Ω.

Proof. From Eqs. (24) we get

$$\begin{gathered}\Delta\mathbf{u} = -\sum_{j=1}^{2}\left[\alpha_j k_j^2 \operatorname{grad}\psi_j + \beta_j k_{j+2}^2 \mathbf{w}_j\right],\\ \operatorname{grad}\operatorname{div}\mathbf{u} = -\sum_{j=1}^{2}\alpha_j k_j^2 \operatorname{grad}\psi_j, \\ \operatorname{curl}\boldsymbol{\varphi} = \sum_{j=1}^{2}\beta_{j+2}k_{j+2}^2\mathbf{w}_j.\end{gathered} \tag{27}$$

Taking into account Eqs. (12), $(24)_3$ and (27) we have

$$(\mu+\kappa)\Delta\mathbf{u} + (\lambda+\mu)\operatorname{grad}\operatorname{div}\mathbf{u} + \kappa\operatorname{curl}\boldsymbol{\varphi} - m\operatorname{grad}\theta + \rho\omega^2\mathbf{u} =$$

$$= -\sum_{j=1}^{2}\left(\mu_0\alpha_j k_j^2 + m\alpha_{j+2} - \rho\omega^2\alpha_j\right)\operatorname{grad}\psi_j +$$

$$+\sum_{j=1}^{2}\left\{\left[(\mu+\kappa)k_{j+2}^2 - \rho\omega^2\right]\beta_j - \kappa\beta_{j+2}k_{j+2}^2\right\}\mathbf{w}_j = 0.$$

On the other hand from Eqs. (24) follows that

$$\begin{gathered}\Delta\boldsymbol{\varphi} = -k_5^2\operatorname{grad}\psi_3 - \sum_{j=1}^{2}\beta_{j+2}k_{j+2}^2\operatorname{curl}\psi_j,\\ \operatorname{grad}\operatorname{div}\boldsymbol{\varphi} = -k_5^2\operatorname{grad}\psi_3,\\ \operatorname{curl}\mathbf{u} = \operatorname{curl}(\beta_1\mathbf{w}_1 + \beta_2\mathbf{w}_2).\end{gathered} \tag{28}$$

By virtue of Eqs. (12), (28) we get

$$\gamma\Delta\boldsymbol{\varphi} + (\alpha+\beta)\operatorname{grad}\operatorname{div}\boldsymbol{\varphi} + \kappa\operatorname{curl}\mathbf{u} + \mu_1\boldsymbol{\varphi} +$$

$$= (-\gamma_0 k_5^2 + \mu_1)\operatorname{grad}\psi_3 + \sum_{j=1}^{2}(\gamma\beta_{j+2}k_{j+2}^2 - \kappa\beta_j - \mu_1\beta_{j+2})\operatorname{curl}\mathbf{w}_j = 0.$$

Similarly, in view of (12) and

$$\begin{gathered}\Delta\theta = -\alpha_3 k_1^2\psi_1 - \alpha_4 k_2^2\psi_2,\\ \operatorname{div}\mathbf{u} = -\alpha_1 k_1^2\psi_1 - \alpha_2 k_2^2\psi_2\end{gathered}$$

we obtain

$$k_0\Delta\theta + a_0\theta + m_0\operatorname{div}\mathbf{u} = \sum_{j=1}^{2}\left[(k_0 k_j^2 - a_0)\alpha_{j+2} + m_0\alpha_j k_j^2\right]\psi_j = 0.$$

Hence, we have Eq. $(2)_3$.

Thus, the general solution $(\mathbf{u}, \boldsymbol{\varphi}, \theta)$ (vector with seven components) of system of homogeneous equations (4) in terms of nine metaharmonic functions ψ_j, w_{1j} and w_{2j}, $j = 1, 2, 3$, is obtained.

2. Let $\omega = \omega_0$. Obviously, $\mu_1 = k_4 = k_5 = 0$, $k_3^2 = \dfrac{\rho\omega_0^2\gamma + \kappa^2}{\gamma(\mu+\kappa)}$. From system (2) we obtain Eqs. $(3)_1$, $(4)_1$, (6), and

$$\Delta \operatorname{div} \boldsymbol{\varphi} = 0, \qquad \Delta(\Delta + k_3^2) \operatorname{curl} \mathbf{u} = \mathbf{0}, quad \Delta(\Delta + k_3^2) \operatorname{curl} \boldsymbol{\varphi} = \mathbf{0}. \tag{29}$$

We introduce the notations

$$\beta_1^* = \gamma k_3^2, \qquad \beta_2^* = -\frac{\kappa}{\rho\omega_0^2}, \qquad \lambda^* = \alpha + \beta - \gamma + 2\kappa\beta_2^*, \quad \mu^* = \gamma - \kappa\beta_2^*. \tag{30}$$

Theorem 3. *If $\omega = \omega_0$ and $(\mathbf{u}, \boldsymbol{\varphi}, \theta)$ is a solution of system* (2), *then*

$$\begin{gathered}\mathbf{u} = \operatorname{grad}(\alpha_1\psi_1 + \alpha_2\psi_2) + \beta_1^*\mathbf{v} + \beta_2^* \operatorname{curl} \mathbf{u}_0,\\ \boldsymbol{\varphi} = \mathbf{u}_0 + \kappa \operatorname{curl} \mathbf{v},\\ \theta = \alpha_3\psi_1 + \alpha_4\psi_2,\end{gathered} \tag{31}$$

where ψ_1, ψ_2 satisfy Eq. (14), *the vectors $\mathbf{v} = (v_1, v_2, v_3)$ and $\mathbf{u}_0$ are solutions of following equations*:

$$(\Delta + k_3^2)\mathbf{v} = \mathbf{0}, \quad \operatorname{div} \mathbf{v} = 0, \tag{32}$$

and

$$\mu^*\Delta\mathbf{u}_0 + (\lambda^* + \mu^*) \operatorname{grad} \operatorname{div} \mathbf{u}_0^* = \mathbf{0}; \tag{33}$$

respectively, α_1, α_2, α_3, α_4 are defined by (12).

Proof. Let

$$\mathbf{v} = -\frac{1}{\kappa k_3^4}\Delta \operatorname{curl} \boldsymbol{\varphi}, \qquad \mathbf{u}_0 = \boldsymbol{\varphi} - \kappa \operatorname{curl} \mathbf{v}. \tag{34}$$

Taking into account Eq. $(29)_3$ from $(34)_1$ we have (32).

On the basis of Eqs. (11), (13) and (31) from $(3)_1$ we obtain Eq. $(31)_1$. By virtue of Eqs. $(31)_1$ and (34) we get

$$\begin{gathered}\mu^*\Delta\mathbf{u}_0 + (\lambda^* + \mu^*) \operatorname{grad} \operatorname{div} \mathbf{u}_0 =\\ = (\gamma - \kappa\beta_2^*)\Delta\mathbf{u}_0 + (\alpha + \beta + \kappa\beta_2^*) \operatorname{grad} \operatorname{div} \mathbf{u}_0 + (\beta_1^* - \gamma k_3^2)\kappa \operatorname{curl} \mathbf{v} =\\ = \gamma(\Delta\mathbf{u}_0 - \kappa k_3^2 \operatorname{curl} \mathbf{v}) + (\alpha + \beta) \operatorname{grad} \operatorname{div} \mathbf{u}_0 +\\ +\kappa(\beta_1^* \operatorname{curl} \mathbf{v} + \beta_2^* \operatorname{curl} \operatorname{curl} \mathbf{u}_0).\end{gathered} \tag{35}$$

In view of Eq. $(2)_2$ from (35) we have Eq. (33).

Obviously, from $(34)_2$ it follows Eq. $(31)_2$.

Thus, if $\omega = \omega_0$, then the general solution of system (2) is presented by 5 metaharmonic functions $\psi_1, \psi_2, v_1, v_2, v_3$ and by vector function $\mathbf{u}_0$, that is solution of the homogeneous equilibrium equation of the classical theory of elasticity with Lamé constants λ^* and μ^*.

***Remark* 2.** As in classical theory of elasticity [7, 8], by virtue of Theorems 1 to 3 it is possible to construct the solutions of boundary-value problems in the linear theory of micropolar thermoelasticity without energy dissipation.

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