К О Р О Т К І П О В І Д О М Л Е Н Н Я

UDC 517.9

Vu Tuan, Vu Thi Thu Houng (Hanoi Univ. Education, Vietnam)

SOME RESULTS ON THE ASYMPTOTIC STABILITY OF ORDER α

ДЕЯКІ РЕЗУЛЬТАТИ ПРО АСИМПТОТИЧНУ СТІЙКІСТЬ ПОРЯДКУ α

The quasi-equiasymptotic stability of order α $\left(\alpha \in \mathbb{R}_+^*\right)$ with respect to a part of variables is considered. Some sufficient conditions, a converse theorem and a theorem for multistability are proved.

Розглянуто проблему квазірівномірно асимптотичної стійкості порядку α $\left(\alpha \in \mathbb{R}_+^*\right)$ відносно частини змінних. Доведено деякі достатні умови, обернену теорему та теорему про мультистійкість.

1. Introduction. Consider the differential system of the form

$$\frac{dx}{dt} = X(t, x), \tag{1}$$

$$X(t, 0) \equiv 0 \quad \text{for all} \quad t \in I = [a, \infty) \subset \mathbb{R}_+.$$

Denoting

$$x = (x_1, \ldots, x_n)^T = \left(y_1, \ldots, y_m, z_1, \ldots, z_p\right)^T = \begin{bmatrix} y \\ z \end{bmatrix},$$

$$X = (X_1, \ldots, X_n)^T = \left(Y_1, \ldots, Y_m, Z_1, \ldots, Z_p\right)^T = \begin{bmatrix} Y \\ Z \end{bmatrix},$$

$$n = m + p, \quad p \geq 0, \quad m \geq 0,$$

$$D = \{(t, x) \colon t \in I, \ \|y\| < H, \ \|z\| < +\infty\}, \quad H > 0,$$

we assume that

$$X \colon D \to \mathbb{R}^n,$$

$$(t, x) \mapsto X(t, x)$$

is continuous and satisfies some uniqueness condition of solutions in D (see [1]). In the paper, we introduce the notion of quasi-equiasymptotic stability of order α $\left(\alpha \in \mathbb{R}_+^*\right)$, establish some sufficient conditions for this stability, prove a converse theorem and a theorem for the x-uniform stability and (at the same time) y-quasi-equiasymptotic stability of order α (one case of multistability) [2]. First, we give some definitions.

Definition 1. *The trivial solution* $x = 0$ *of* (1) *is said to be:*

i) x- (*or* y-, *respectively*) *quasi-equiasymptotically stable of order* α $\left(\alpha \in \mathbb{R}_+^*\right)$, *if given any* $\varepsilon > 0$ *and any* $t_0 \in I$, *there exist* $\delta = \delta(t_0, \varepsilon)$ *and* $T = T(t_0, \varepsilon)$ *such that if* $\| x_0 \| < \delta$, *then*

$$\| x(t; t_0, x_0) \| < \varepsilon (t - t_0)^{-\alpha}$$

$$\left(or \quad \| y(t; t_0, x_0) \| < \varepsilon (t - t_0)^{-\alpha}, \text{ respectively} \right)$$

for all $t \geq t_0 + T$;

ii) x- (*or* y-, *respectively*) *equiasymptotically stable of order* α $\left(\alpha \in \mathbb{R}_+^*\right)$, *if it is stable in the sense of Liapunov and* (*at the same time*) x- (*or* y-, *respectively*) *quasi-equiasymptotically stable of order* α;

iii) *stable* (*uniformly stable, respectively*) *in the sense of Liapunov and* (*at the same time*) *y-quasi-equiasymptotically stable of order* α *if given any* $\varepsilon > 0$ *and any* $t_0 \in I$, *there exist* $\delta = \delta(t_0, \varepsilon) > 0$ $(\delta = \delta(\varepsilon) > 0$, *respectively*) *and* $T = T(t_0, \varepsilon)$ *such that if* $\| x_0 \| < \delta$, *then* $\| x(t; t_0, x_0) \| < \varepsilon$ *for all* $t \geq t_0$ *and* $\| y(t; t_0, x_0) \| < \varepsilon (t - t_0)^{-\alpha}$ *for all* $t \geq t_0 + T$.

2. Asymptotic stability of order α. ***2.1. Sufficient conditions.***

Theorem 1. *Suppose that there exists a Liapunov function* $V(t, x)$ *defined on* D *such that*

i) $V(t, 0) \equiv 0$;

ii) $\| x \| \leq V(t, x)$;

iii) $\dot{V}_{(1)}(t, x) \leq -\dfrac{\alpha}{t} V(t, x)$ *for* $\alpha > 0$, $t \in I$.

Then the trivial solution $x = 0$ *of* (1) *is equiasymptotically stable of order* β $(0 < \beta < \alpha)$.

The proof of this theorem is similar to the proof of Theorem 1 in the paper [2].

Theorem 2. *Suppose that there exists a Liapunov function* $V(t, x)$ *defined on* D *such that*

i) $V(t, 0) \equiv 0$;

ii) $\| y \| \leq V(t, x)$;

iii) $\dot{V}_{(1)}(t, x) \leq -\dfrac{\alpha}{t} V(t, x)$ *for* $\alpha > 0$, $t \in I$.

Then the trivial solution $x = 0$ *of* (1) *is y-equiasymptotically stable of order* β $(0 < \beta < \alpha)$.

Proof. Given any $\varepsilon > 0$ $(\varepsilon < H)$ and any $t \in I = [a, +\infty)$ for any x satisfying the condition $\| y \| = \varepsilon$, inequality ii) implies $V(t, x) \geq \varepsilon$. Because of the continuity of $V(t, x)$ and $V(t_0, 0) \equiv 0$, given $t_0 \in I$, there exists $\delta = \delta(t_0, \varepsilon) > 0$ such that if $\| x_0 \| < \delta$, then $V(t_0, x_0) < \varepsilon$. Assume that there exists a solution $x(t; t_0, x_0)$ of (1) such that $\| x_0 \| < \delta$ and $\| y(t_1; t_0, x_0) \| = \varepsilon$ at $t_1 \in I$. From iii) it follows that $V\left(t_1, x(t_1; t_0, x_0)\right) \leq V(t_0, x_0)$ and then

$$\varepsilon \leq V\left(t_1, x(t_1; t_0, x_0)\right) \leq V(t_0, x_0) < \varepsilon.$$

This contradiction shows that if $\| x_0 \| < \delta$ then $\| y(t; t_0, x_0) \| < \varepsilon$ for all $t \geq t_0$, i.e., the trivial solution $x = 0$ is y-stable in the sense of Liapunov. Given $\gamma > 0$, we denote by $x(t; t_0, x_0)$ the solution of (1) satisfying the condition $\| x_0 \| < \gamma$. By virtue of iii), we have

$$V\big(t, x(t; t_0, x_0)\big) \;\le\; V(t_0, x_0)\left(\frac{t}{t_0}\right)^{-\alpha} \;\le\; t_0^{\alpha} V(t_0, x_0)(t - t_0)^{-\alpha} \tag{2}$$

for all t sufficiently large. Let $M(t_0, \gamma) = \max_{\| x_0 \| = \gamma} V(t_0, x_0)$, $T = T(t_0, \varepsilon, \gamma)$ such that

$$0 \;\le\; \frac{M(t_0, \gamma)}{(t - t_0)^{\alpha - \beta}} \;<\; \frac{\varepsilon}{t_0^{\alpha}}$$

for $0 < \beta < \alpha$ and for all $t \ge t_0 + T$. From (2) it follows that for all $t \ge t_0 + T$ we have

$$y(t; t_0, x_0) \;\le\; V\big(t, x(t; t_0, x_0)\big) \;\le\; t_0^{\alpha} V(t_0, x_0)(t - t_0)^{-\alpha} \;\le$$

$$\le\; t_0^{\alpha} \frac{M(t_0, \gamma)}{(t - t_0)^{\alpha - \beta}} (t - t_0)^{-\beta} \;<\; t_0^{\alpha} \frac{\varepsilon}{t_0^{\alpha}} (t - t_0)^{-\beta} \;=\; \varepsilon (t - t_0)^{-\beta}$$

which proves that the trivial solution $x = 0$ of (1) is y-quasiasymptotically stable of order β.

The theorem is proved.

2.2. ***Converse theorems for linear system.*** We shall study now converse theorems on the quasi-equiasymptotic stability. We consider the linear system

$$\frac{dx}{dt} \;=\; A(t)x, \tag{3}$$

where $A(t)$ is a continuous $n \times n$ matrix on I (see [3, 4]).

Theorem 3. *Suppose that there exist* $M > 1$ *and* $\alpha > 0$ *such that*

$$\| x(t; t_0, x_0) \| \;\le\; M \| x_0 \| \left(\frac{t_0}{t}\right)^{\alpha} \quad \text{for all} \quad t \ge t_0, \tag{4}$$

where $x(t; t_0, x_0)$ *is a solution of* (3). *Then there exists a Liapunov function* $V(t, x)$, *which satisfies the following conditions*:

i) $\| x \| \le V(t, x) \le M \| x \|$;

ii) $\big| V(t, x) - V(t, x') \big| \le M \big\| x - x' \big\|$;

iii) $\dot{V}_{(3)}(t, x) \le -\dfrac{\alpha}{t} V(t, x)$.

Proof. Let $V(t, x)$ be defined by

$$V(t, x) \;=\; \sup_{\tau \ge 0} \| x(t + \tau; t, x) \| \left(\frac{t + \tau}{t}\right)^{\alpha}.$$

It is clear that $\| x \| \le V(t, x)$. On the other hand, from (4) we have

$$\| x(t + \tau; t, x) \| \;\le\; M \| x \| \left(\frac{t}{t + \tau}\right)^{\alpha},$$

which implies

$$V(t, x) \;=\; \sup_{\tau \ge 0} \| x(t + \tau; t, x) \| \left(\frac{t + \tau}{t}\right)^{\alpha} \;\le\; \sup_{\tau \ge 0} M \| x \| \left(\frac{t + \tau}{t}\right)^{\alpha} \left(\frac{t}{t + \tau}\right)^{\alpha} \;=\; M \| x \|.$$

Thus, we obtain

$$\| x \| \;\le\; V(t, x) M \| x \|.$$

Since the system (3) is linear, we have

$$x(\tau; t, x) - x(\tau; t, x') = x(\tau; t, x - x'). \tag{5}$$

Hence,

$$\left|V(t, x) - V(t, x')\right| = \left|\sup_{\tau \geq 0} \|x(t + \tau; t, x)\| \left(\frac{t+\tau}{t}\right)^{\alpha} - \sup_{\tau \geq 0} \left\|x(t + \tau; t, x')\right\| \left(\frac{t+\tau}{t}\right)^{\alpha}\right| \leq$$

$$\leq \sup_{\tau \geq 0} \left\{ \left| \|x(t + \tau; t, x)\| - \left\|x(t + \tau; t, x')\right\| \right| \right\} \left(\frac{t+\tau}{t}\right)^{\alpha} \leq$$

$$\leq \sup_{\tau \geq 0} \left\|x(t + \tau; t, x) - x(t + \tau; t, x')\right\| \left(\frac{t+\tau}{t}\right)^{\alpha} = \sup_{\tau \geq 0} \left\|x(t + \tau; t, x - x')\right\| \left(\frac{t+\tau}{t}\right)^{\alpha} \leq$$

$$\leq \sup_{\tau \geq 0} M \left\|x - x'\right\| \left(\frac{t}{t+\tau}\right)^{\alpha} \left(\frac{t+\tau}{t}\right)^{\alpha} = M \left\|x - x'\right\|.$$

Thus, the condition ii) is established. Now we shall prove the continuity of $V(t, x)$.

From i) and ii) it follows that $V(t, x)$ is continuous at 0. It remains to prove the continuity of $V(t, x)$ at $x \neq 0$. For $\delta \geq 0$ we have

$$\left|V(t + \delta, x') - V(t, x)\right| \leq \left|V(t + \delta, x') - V(t + \delta, x)\right| +$$

$$+ |V(t + \delta, x) - V(t + \delta, x(t + \delta; t, x))| + |V(t + \delta, x(t + \delta; t, x)) - V(t, x)|. \tag{6}$$

Since $V(t, x)$ is Lipschitzian in x and $x(t + \delta; t, x)$ is continuous, the first two terms are small when $\left\|x - x'\right\|$ and δ are small. Let us consider the third term. Since

$$x(t + \delta + \tau; t + \delta, x(t + \delta; t, x)) = x(t + \delta + \tau; t, x),$$

we have

$$|V(t + \delta, x(t + \delta; t, x)) - V(t, x)| =$$

$$= \left|\sup_{\tau \geq 0} \|x(t + \delta + \tau; t + \delta, x(t + \delta; t, x))\| \left(\frac{t+\delta+\tau}{t+\delta}\right)^{\alpha} - \sup_{\tau \geq 0} \|x(t + \tau; t, x)\| \left(\frac{t+\tau}{t}\right)^{\alpha}\right| =$$

$$= \left|\sup_{\tau \geq 0} \|x(t + \delta + \tau; t, x)\| \left(\frac{t+\delta+\tau}{t+\delta}\right)^{\alpha} - \sup_{\tau \geq 0} \|x(t + \tau; t, x)\| \left(\frac{t+\tau}{t}\right)^{\alpha}\right| =$$

$$= \left|\sup_{\tau \geq \delta} \left\{ \|x(t + \tau; t, x)\| \left(\frac{t+\tau}{t}\right)^{\alpha} \right\} \left(\frac{t}{t+\delta}\right)^{\alpha} - \sup_{\tau \geq 0} \|x(t + \tau; t, x)\| \left(\frac{t+\tau}{t}\right)^{\alpha}\right|.$$

Put $a(\delta) = \max_{\tau \geq \delta} \|x(t + \tau; t, x)\| \left(\frac{t+\tau}{t}\right)^{\alpha}$. Then $a(\delta)$ is continuous and bounded because

$$a(\delta) \leq a(0) = V(t, x) \leq M\|x\| < \infty$$

and $a(\delta) \to a(0)$ as $\delta \to 0$. Thus,

$$|V(t + \delta, x(t + \delta; t, x)) - V(t, x)| = \left|a(\delta)\left(\frac{t}{t+\delta}\right)^{\alpha} - a(0)\right| \to 0$$

as $\delta \to 0$. Therefore, $V(t, x)$ is continuous. Finally, we shall establish condition iii). We have

$$V(t+h, x(t+h; t, x)) =$$

$$= \sup_{\tau \ge 0} \| x(t+h+\tau; t+h, x(t+h; t, x)) \| \left(\frac{t+h+\tau}{t+h} \right)^{\alpha} =$$

$$= \sup_{\tau \ge 0} \| x(t+h+\tau; t+h, x(t+h; t, x)) \| \left(\frac{t+h+\tau}{t} \right)^{\alpha} \left(\frac{t}{t+h} \right)^{\alpha} =$$

$$= \left\{ \sup_{\tau \ge h} \| x(t+\tau; t, x) \| \left(\frac{t+\tau}{t} \right)^{\alpha} \right\} \left(\frac{t}{t+h} \right)^{\alpha} \le$$

$$\le \left\{ \sup_{\tau \ge 0} \| x(t+\tau; t, x) \| \left(\frac{t+\tau}{t} \right)^{\alpha} \right\} \left(\frac{t}{t+h} \right)^{\alpha} = V(t, x) \left(\frac{t}{t+h} \right)^{\alpha},$$

which implies

$$\frac{V(t+h, x(t+h; t, x)) - V(t, x)}{h} \le V(t, x) \frac{\left(\frac{t}{t+h} \right)^{\alpha} - 1}{h}.$$

Since

$$\lim_{h \to 0} \frac{\left(\frac{t}{t+h} \right)^{\alpha} - 1}{h} = \lim_{h \to 0} \alpha \left(\frac{t}{t+h} \right)^{\alpha - 1} \frac{-t}{(t+h)^2} = -\frac{\alpha}{t},$$

we obtain

$$\dot{V}_3(t, x) \le -\frac{\alpha}{t} V(t, x).$$

This completes the proof.

Theorem 4. *Suppose that there exist* $M > 1$ *and* $\alpha > 0$ *such that*

$$\| y(t; t_0, x_0) \| \le M \| x_0 \| \left(\frac{t_0}{t} \right)^{\alpha}$$

for all $t \ge t_0$, *where* $x(t; t_0, x_0) = \left(y^T, z^T \right)^T$ *is a solution of* (3). *Then there exists a Liapunov function* $V(t, x)$, *which satisfies the following*:

i) $\| y \| \le V(t, x) \le M \| x \|$;

ii) $\left| V(t, x) - V(t, x') \right| \le M \left\| x - x' \right\|$;

iii) $\dot{V}_{(3)}(t, x) \le -\frac{\alpha}{t} V(t, x)$.

This theorem can be proved by the same argument used in the proof of Theorem 3.

3. Liapunov stability and *y*-quasi-equiasymptotic stability of order α. Consider the differential system of the form

$$\begin{aligned} \dot{y} &= A(t) y + B(t) z + Y(t, y, z), \\ \dot{z} &= C(t) y + D(t) z + Y(t, y, z) \end{aligned} \tag{7}$$

and the linear system relatively

$$\begin{aligned} \dot{y} &= A(t) y + B(t) z, \\ \dot{z} &= C(t) y + D(t) z. \end{aligned} \tag{8}$$

Assume that the following conditions are valid:

$$\begin{gathered} Y(t, 0, 0) \equiv Y(t, 0, z) \equiv 0, \qquad Z(t, 0, 0) \equiv Z(t, 0, z) \equiv 0, \\ \frac{t(\| Y(t, y, z) \| + \| Z(t, y, z) \|)}{\| y \|} \to 0 \end{gathered} \tag{9}$$

as $\|y\| + \|z\| \to 0$. We shall prove a theorem, generalized Theorem 1 in [5], on case of multistability [6 – 8].

Theorem 5. *Suppose that the trivial solution* $x = 0$ *of the linear system* (8) *is uniformly stable and there exist* $M > 1$, $\alpha > 0$ *such that*

$$\|y(t; t_0, x_0)\| \le M\|x_0\|\left(\frac{t_0}{t}\right)^{\alpha} \quad \text{for all} \quad t \ge t_0,$$

where $x(t; t_0, x_0) = \left(y^T, z^T\right)^T$ *is a solution of* (8). *Then the trivial solution* $y = z = 0$ *of* (7), *for which condition* (9) *holds, is* x*-stable in the sense of Liapunov and* (*at the same time*) *is* y*-quasi-equiasymptotically stable of order* α_1 $(0 < \alpha_1 < \alpha)$.

Proof. Since $\|y(t; t_0, x_0)\| \le M\|x_0\|\left(\frac{t_0}{t}\right)^{\alpha}$ or all $t \ge t_0$, by virtue of Theorem 4, there exists a Liapunov function $V(t, x)$ satisfying the following conditions:

$$\|y\| \le V(t, x) \le M\|x\|, \tag{10}$$

$$\left|V(t, x) - V(t, x')\right| \le M\left\|x - x'\right\|,$$

$$\dot{V}_{(8)}(t, x) \le -\frac{\alpha}{t} V(t, x). \tag{11}$$

Differentiating the function V along the system (7), we have

$$\dot{V}_{(7)}(t, x) = \dot{V}_{(8)}(t, x) + R(t, x),$$

where

$$R(t, x) = \left\langle \frac{\partial V}{\partial x}, X^*(t, x) \right\rangle,$$

$$X^* = \left(Y^T, Z^T\right)^T,$$

$\langle \cdot \rangle$ is inner product. By virtue of (9) – (11), the following conditions hold in the domain $\{t \ge 0,\ \|x\| \le h\}$:

$$R(t, x) \le \frac{M\varepsilon}{t}\|y\| \le \frac{\varepsilon M}{t} V(t, x),$$

where $\varepsilon \to 0$ as $\|x\| \to 0$. Consequently, there exists β $(0 < \beta < h)$ such that, in the domain $\{t \ge t_0,\ \|x\| \le \beta\}$, we have

$$\dot{V}_{(7)}(t, x) \le -\frac{\alpha}{t} V(t, x) + \frac{\varepsilon M}{t} V(t, x) \le -\frac{\alpha_1}{t} V(t, x), \tag{12}$$

where $\alpha_1 = \alpha - \varepsilon M$, $0 < \alpha_1 < \alpha$, for sufficiently small $\varepsilon > 0$. We consider a solution $x(t; t_0, x_0)$ of (7), where $t_0 \ge a$, $\|x_0\| \le \delta$ $(0 < \delta < \beta)$, for which the inequality $\|x(t; t_0, x_0)\| \le \beta$ holds at least in an interval $T = \left(t_0, t^*\right)$. Therefore, by virtue of (12) and (10), we have

$$\|y(t; t_0, x_0)\| \le V\big(t, x(t; t_0, x_0)\big) \le M\|x_0\|\left(\frac{t_0}{t}\right)^{\alpha_1} \Rightarrow$$

$$\Rightarrow \|y(t; t_0, x_0)\| \le M\|x_0\| t_0^{\alpha_1}(t - t_0)^{-\alpha_1} \tag{13}$$

for $t \in T_*$. It follows from (9) and (13) that

$$\| X(t, x(t; t_0, x_0)) \| \le \varepsilon_1 M \| x_0 \| t_0^{\alpha_1} t^{-(1+\alpha_1)}$$

($t \in T$; $\alpha_2 = \varepsilon_1 M \to 0$ as $\| x \| \to 0$). It is clear that the solution of the nonlinear system (7) is of the form

$$x(t; t_0, x_0) = K(t; t_0) x_0 + \int_{t_0}^{t} K(t, \tau) X(\tau, x(\tau; t_0, x_0)) d\tau, \tag{14}$$

where $U(t)$ is a fundamental matrix of the linear system (8) and $K(t, t_0) = U(t) U^{-1}(t_0)$. Because of the uniform stability in the sense of Liapunov of the trivial solution of (8), there exists $N = \text{const} \ge 1$ such that $|K(t, t_0)| \le N$ for $t \ge t_0 \ge a$ (see [9]). Then (14) implies

$$\| x(t; t_0, x_0) \| \le N \| x_0 \| + \int_{t_0}^{t} N \alpha_2 \| x_0 \| t_0^{\alpha_1} \tau^{-(\alpha_1 + 1)} d\tau \le$$

$$\le N \| x_0 \| \left(1 + \alpha_2 \alpha_1^{-1} - \alpha_2 \alpha_1^{-1} t_0^{\alpha_1} t^{-\alpha_1}\right) \le N \| x_0 \| \left(1 + \alpha_2 \alpha_1^{-1}\right). \tag{15}$$

Given $\varepsilon > 0$ ($\varepsilon < \beta$) we choose $\delta = \delta(\varepsilon, t_0) > 0$ and $\| x_0 \| < \delta$ such that

$$\delta < \min\left\{ M^{-1} t_0^{-\alpha_1}; \left[N\left(1 + \alpha_2 \alpha_1^{-1}\right) \right]^{-1} \right\} \varepsilon .$$

Then $\| x(t; t_0, x_0) \| < \varepsilon$, $t \in T$. By virtue of (13), we have

$$\| y(t; t_0, x_0) \| < \varepsilon (t - t_0)^{-\alpha_1}, \quad t \in T .$$

Thus, for all t satisfying the condition

$$\| x(t; t_0, x_0) \| \le \beta,$$

the inequality

$$\| x(t; t_0, x_0) \| < \varepsilon$$

is valid. Hence, $\varepsilon < \beta$, the inequality

$$\| x(t; t_0, x_0) \| < \varepsilon$$

is satisfied for all $t \ge t_0$, and

$$\| y(t; t_0, x_0) \| < \varepsilon (t - t_0)^{-\alpha_1} \quad \text{for all} \quad t \ge t_0,$$

that is the trivial solution $x = 0$ of (7) is Liapunov stable and (at the same time) is y-quasi-equiasymptotically stable of order α.

The theorem is proved.

Example.

$$\begin{aligned} \frac{dx}{dt} &= -\frac{2x}{t} + \frac{y^2 \cos t}{t}, \\ \frac{dy}{dt} &= -x t^2 \sin t + y \sin t + \frac{x^2 \cos(t^2 + 1)}{t}, \end{aligned} \qquad t \ge 1. \tag{16}$$

First, it is easy to see that the general solution of the linear system

$$\frac{dx}{dt} = -\frac{2x}{t},$$

$$\frac{dy}{dt} = -xt^2 \sin t + y \sin t$$

is

$$x = \frac{C_1}{t^2}, \qquad y = C_1 + C_2 e^{-\cos t}, \qquad t \geq 1.$$

Hence, it is clear that the trivial solution $x = y = 0$ of (7) is uniformly stable in the sense of Liapunov. On the other hand, the zero solution is x-quasi-equiasymptotically stable of order 2. Since the nonlinear part of (16) satisfies condition (9), by virtue of Theorem 5 the zero solution of (16) is uniformly stable and, at the same time, is x-quasi-equiasymptotically stable of order 2.

1. *Румянцев В. В., Озиранер А. С.* Устойчивость и стабилизация движения по отношению к части переменных. – М.: Наука, 1987. – 253 с.
2. *Vu Tuan, Dao Thi Lien.* Asymptotic stability of order k of differential systems // Int. Conf. „Abstract and Applied Analysis" (August 13 – 17, Hanoi). – 2002.
3. *Yoshizawa.* Stability theory by Liapunov's second method // Math. Soc. Jap. – 1966.
4. *Pham Van Viet.* On the asymptotic behavior of solutions of differential systems // Vietnam Conf. Math. (6, Hue, 9-2002). – 2002.
5. *Воротников В. И.* К задачам устойчивости по части переменных // Прикл. математика и механика. – 1999. – **33**(5). – С. 536 – 545.
6. *Martynyuk A. A.* On multistability of motion with respect to some of the variables // Rus. Acad. Sci. Dokl. Math. – 1992. – **45** (**3**). – P. 533 – 536.
7. *Martynyuk A. A.* On the exponential multistability of separating motions // Ibid. – 1994. – **49** (**3**). – P. 528 – 531.
8. *Мартынюк А. А.* Полуустойчивость — новое направление в анализе нелинейных систем (обзор) // Прикл. механика. – 1994. – **30** (5). – С. 3 – 17.
9. *Taniguchi T.* Stability theories of perturbed linear ordinary differential equations // J. Math. Anal. and Appl. – 1990. – **149**. – P. 583 – 598.

Received 12.12.2003