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STOCHASTIC SEMIGROUPS AND COAGULATION EQUATIONS
СТОХАСТИЧНІ НАПІВГРУПИ І РІВНЯННЯ КОАГУЛЯЦІЇ

A general class of bilinear systems of discrete or continuous coagulation equations is considered. It is shown that their solutions can be approximated by the solutions of appropriate stochastic systems describing the coagulation process in terms of stochastic semigroups.

Розглянуто загальний клас білінійних систем дискретних або неперервних рівнянь коагуляції. Показано, що їх розв'язки можуть бути наближені розв'язками стохастичних систем, які описують процес коагуляції через стохастичні напівгрупи.

1. Coagulation equations — mesoscopic description. Coagulation processes are important in various physical situations. Entities (clusters, particles, droplets, ...) merge by coalescence to form larger ones. Such a phenomena takes place in polymer science, atmosphere physics, colloidal chemistry, biology, and immunology — see references in [1–4]. A system of infinite number of equations (discrete model) was introduced by Smoluchowski [5] to describe the coagulation of colloids moving according to a Brownian motion.

In the discrete model the size of entities (clusters) is characterized by an integer $r \in \mathbb{N} = \{1, 2, \dots\}$ identified with the number of identical elementary entities. In the continuous model the size of entities is characterized by a real nonnegative number $r \in [0, \infty[$. The discrete Smoluchowski coagulation equation is an infinite system of bilinear ODEs whereas the continuous Smoluchowski coagulation equation is a bilinear integro-differential equation. We unify them in the following notation:

$$\partial_t f = Q_1[f], \qquad t > 0, \quad r \in \mathcal{J}, \tag{1.1}$$

where $\mathcal{J}$ is either $\mathbb{N}$ or $[0, \infty[$, $f = f(t, r)$ is the density of clusters of size r at time $t \geq 0$,

$$Q_1[f](r) = \frac{1}{2} \int\limits_{\mathcal{J}_r} \alpha(r - r_1, r_1) f(r - r_1) f(r_1)\, d\lambda(r_1) - f(r) \int\limits_{\mathcal{J}} \alpha(r, r_1) f(r_1)\, d\lambda(r_1),$$

$$\mathcal{J}_r = \Big\{ r_1 \in \mathcal{J} : \quad r_1 < r \Big\}, \quad r \in \mathcal{J},$$

λ is the counting measure in the case $\mathcal{J} = \mathbb{N}$ or the Lebesgue measure in the case $\mathcal{J} = [0, \infty[$, $\alpha(r, r_1)$ are the coagulation rate.

The Smoluchowski coagulation equation has been the subject of several modifications and studies. Another coagulation model was proposed by Oort and van de Hulst (and then by Safronov) to describe the process of aggregation of protoplanetary bodies in astrophysics (see references in [3]). The Oort–Hulst–Safronov coagulation equation reads

$$\begin{gathered} \partial_t f = Q_0[f], \qquad t > 0, \quad r \in [0, \infty[, \\ Q_0[f](r) = -\partial_r \Big(f(r) \int\limits_0^r r_1 \alpha(r, r_1) f(r_1)\, dr_1 \Big) - f(r) \int\limits_r^\infty \alpha(r, r_1) f(r_1)\, dr_1. \end{gathered} \tag{1.2}$$

In [3] the following class of generalized coagulation equations was introduced

$$\partial_t f = Q_{GC}[f], \qquad t > 0, \quad r \in [0, \infty[, \tag{1.3}$$

where

$$Q_{GC}[f](r) = \frac{1}{2} \int_0^\infty \int_0^\infty A(r; r_1, r_2) a(r_1, r_2) f(r_1) f(r_2)\, dr_1\, dr_2 - \\ - f(r) \int_0^\infty \alpha(r, r_1) f(r_1)\, dr_1. \tag{1.4}$$

A is the weighted probability that the interaction of a cluster of size r_1 and another cluster of size r_2 generates a cluster of size r and is a nonnegative function satisfying

$$A(r; r_1, r_2) = A(r; r_2, r_1), \qquad r, \ \ r_1, \ \ r_2 \in [0, \infty[, \tag{1.5a}$$

$$\int_0^\infty r\, A(r; r_1, r_2)\, dr = r_1 + r_2, \qquad r_1, \ \ r_2 \in [0, \infty[. \tag{1.5b}$$

The structure of Eq. (1.3) can be related to the large class of bilinear Generalized Kinetic Models. A general class of bilinear systems of Boltzmann-like integro-differential equations describing the dynamics of individuals undergoing kinetic (stochastic) interactions was proposed and analyzed in [6]. These equations can model interactions between pairs of individuals of various populations at the mesoscopic level. The class of equations in [6] can be regarded as a generalization of the Jäger and Segel kinetic model [7], as well as those of Arlotti and Bellomo [8, 9], Arlotti, Bellomo and Lachowicz [10], Lachowicz and Wrzosek [4], Geigant, Ladizhansky and Mogilner [11]. In the literature these kind of models are referred to as the GKM — Generalized Kinetic Models. Paper [6] was a first step in the description of the mathematical properties of a large class of General Kinetic Models. It provides some existence and uniqueness theorems for the GKM, discusses its equilibrium solutions, and studies its diffusive limit. In [6] the existence of unstable equilibrium solutions which are inhomogeneous was proved. The case when only homogeneous equilibrium solutions exist was specified. Under suitable scaling it was proved that the one-dimensional version of the GKM is asymptotically equivalent to the nonlinear porous medium equation also used in mathematical biology as the model for density dependent population dispersal. In [12] research perspectives for finding possible transitions from the mesoscopic to the macroscopic level were presented. In [13] the results on the relationships between the microscopic and the mesoscopic levels and then in [14] between the microscopic and the macroscopic levels are proved.

Condition (1.5b) ensures that the total volume is preserved during the coagulation reaction. In fact we have

$$\int_0^\infty Q_{GC}[f] \phi\, dr = \\ = \int_0^\infty \int_0^{r_1} \left(\left(\int_0^\infty A(r; r_1, r_2) \phi(r)\, dr \right) - \phi(r_1) - \phi(r_2) \right) \alpha(r_1, r_2) f(r_1) f(r_2)\, dr_2\, dr_1 \tag{1.6}$$

for any test function ϕ.

In [3] a family of generalized coagulation equations connecting the Smoluchowski and the OHS equations was introduced. For $\varepsilon \in]0, 1]$ and $r_1, r_2 \in [0, \infty[$ it was defined

$$A_\varepsilon(r;r_1,r_2) = \delta\Big(r - r_1 \vee r_2 - \varepsilon r_1 \wedge r_2\Big) + (1-\varepsilon)\delta\Big(r - r_1 \wedge r_2\Big) \tag{1.7}$$

where

$$r_1 \vee r_2 = \max\big\{r_1, r_2\big\}, \qquad r_1 \wedge r_2 = \min\big\{r_1, r_2\big\},$$

δ is the Dirac distribution,

$$\alpha_\varepsilon(r_1,r_2) = \frac{\alpha(r_1,r_2)}{\varepsilon}. \tag{1.8}$$

Putting A_ε instead of A and a_ε instead of a in (1.6) we set $Q_{GC} = Q_\varepsilon$. Thus we have

$$\partial_t f_\varepsilon = Q_\varepsilon[f], \qquad t > 0, \quad r \in [0,\infty[\,, \tag{1.9}$$

where

$$Q_\varepsilon[f](r) = \frac{1}{\varepsilon}\int\limits_0^{\frac{r}{1+\varepsilon}} \alpha(r-\varepsilon r_1, r_1) f(r-\varepsilon r_1) f(r_1)\, dr_1 +$$

$$+\frac{1-\varepsilon}{\varepsilon} f(r) \int\limits_r^\infty \alpha(r,r_1) f(r_1)\, dr_1 - \frac{1}{\varepsilon} f(r) \int\limits_0^\infty \alpha(r,r_1) f(r_1)\, dr_1\,. \tag{1.10}$$

We have

$$\int\limits_0^\infty Q_\varepsilon[f]\phi\, dr =$$

$$= \int\limits_0^\infty \int\limits_0^{r_1} \Big(\frac{\phi(r_1+\varepsilon r_2) - \phi(r_1)}{\varepsilon} - \phi(r_2)\Big)\alpha(r_1,r_2) f(r_1) f(r_2)\, dr_1\, dr_2, \tag{1.11}$$

for any test function ϕ.

It is straightforward to see that the choice $\varepsilon = 1$ yields (1.1) in the case $\mathcal{J} = [0,\infty[$. On the other hand in [3] the convergence of the weak solution f_ε to Eq. (1.9) with the initial datum $f^{(0)}$ towards a weak solution to Eq. (1.2) with the same initial datum was proved.

The models presented in this section can be related to the level of statistical description of test-particle (a “*mesoscopic*” description). Using the general approach of [13] we are going to show that the solutions of the mesoscopic models above can be approximated by the solutions of appropriate systems describing the coagulation process of particles undergoing stochastic interactions (a “microscopic” description) in terms of stochastic semigroups.

The mathematical relationships between the particle systems and various Smoluchowski coagulation equations were studied in a number of papers — see [2, 15 – 17], and references therein. Our approach however is simpler and makes use of a general theory developed in [13, 18, 19].

2. Particle systems — microscopic description. Here we follow the idea of [20] and [13]. We show that the solutions of Eq. (1.9) can be approximated by solutions of (linear) equations describing the dynamics of a suitable system of interacting particles.

For simplicity of notation we consider here only the continuous case $\mathcal{J} = [0,\infty[$ but the discrete case $\mathcal{J} = \mathbb{N}$ can be treated in the same way.

Consider a system composed of N interacting particles. Every particle $n \in \in \{1, 2, ..., N\}$ is characterized by $\mathbf{u}_n = (r_n, u_n)$, where $r_n \in \mathcal{J}$ characterizes the size of the n-particle and $u_n \in \mathcal{U}$ — its inner state. Here $\mathcal{J} = [0, \infty[$ and $\mathcal{U} = [0, 1]$. Actually u_n plays an auxiliary rôle but it may be related to the measure of "*coagulation intensity*" of the particle. The n-particle interacts with the m-particle and the interaction take place at random times. After the interaction both particles may merge or/and change their inner state.

Consider the Markov process of N-particles with infinitesimal generator given by

$$\Lambda_N \phi(r_1, u_1, \ldots, r_N, u_N) =$$
$$= \frac{1}{N\varepsilon} \sum_{\substack{1 \le n,m \le N \\ n \ne m}} \alpha(r_n, r_m)\, u_m \Bigg(\int\limits_{\mathcal{U}} \Big(\chi(r_m < r_n) B_1(v, u_n) \times$$
$$\times \phi\big(r_1, u_1, \ldots, r_{n-1}, u_{n-1}, r_n + \varepsilon r_m, v, r_{n+1}, u_{n+1}, \ldots, r_N, u_N\big) +$$
$$+ \chi(r_n < r_m) B_2(v, u_n) \times$$
$$\times \phi\big(r_1, u_1, \ldots, r_{n-1}, u_{n-1}, r_n, v, r_{n+1}, u_{n+1}, \ldots, r_N, u_N\big)\Big)\, dv -$$
$$- \phi\big(r_1, u_1, \ldots, r_N, u_N\big)\Bigg), \qquad r_j \in \mathcal{J}, \quad u_j \in \mathcal{U}, \quad j = 1, \ldots, N, \tag{2.1}$$

where ϕ is an appropriate test function, $\varepsilon \in]0, 1[$, B_1 and B_2 are measurable functions such that

$$\int\limits_{\mathcal{U}} B_i(u, u_1)\, du = 1, \qquad \int\limits_{\mathcal{U}} u B_i(u, u_1)\, du = u_1 \kappa_i,$$
$$\text{for} \quad \text{a.a. } u_1 \in \mathcal{U}, \quad i = 1, 2, \qquad \kappa_1 = 1, \quad \kappa_2 = 1 - \varepsilon, \tag{2.2}$$
$$\chi(\text{true}) = 1, \qquad \chi(\text{false}) = 0\,.$$

In the present paper we assume rather restrictive case

$$0 \le \alpha(\mathbf{u}, \mathbf{u}_1) \le c_a, \qquad \text{for a.a. } \mathbf{u}, \mathbf{u}_1 \in \mathcal{J} \times \mathcal{U}, \tag{2.3}$$

where c_a is a positive constant. However the general case of unbounded α can be treated by the usual approximation methods (cf. [3]).

Assume that the system is initially distributed according to the probability density $F^N \in L_1^{(N)}$, where $L_1^{(N)}$ is the space equipped with the norm

$$\|f\|_{L_1^{(N)}} = \int\limits_{\mathcal{J}\times\mathcal{U}} \ldots \int\limits_{\mathcal{J}\times\mathcal{U}} |f(\mathbf{u}_1, \ldots, \mathbf{u}_N)|\, d\mathbf{u}_1 \ldots d\mathbf{u}_N\,.$$

The time evolution is described by the probability density

$$f^N(t) = \exp\Big(t\Lambda_N^*\Big) F^N\,. \tag{2.4}$$

It satisfies $\Big(\text{in } L_1^{(N)}\Big)$

$$\partial_t f^N = \Lambda_N^* f^N\,, \qquad f^N\Big|_{t=0} = F^N, \tag{2.5}$$

where

$$\Lambda_N^* f(r_1, u_1, \ldots, r_N, u_N) =$$

$$= \frac{1}{N\,\varepsilon} \sum_{\substack{1\le n,m\le N\\ n\ne m}} u_m \Bigg(\int\limits_{\mathcal{U}} \Big(\chi\big((1+\varepsilon)r_m < r_n\big) \times$$
$$\times \alpha(r_n - \varepsilon r_m, r_m) B_1(u_n, v) \times$$
$$\times f\big(r_1, u_1, \ldots, r_{n-1}, u_{n-1}, r_n - \varepsilon r_m, v, r_{n+1}, u_{n+1}, \ldots, r_N, u_N\big) +$$
$$+ \chi(r_n < r_m)\alpha(r_n, r_m) B_2(u_n, v) \times$$
$$\times\, f\big(r_1, u_1, \ldots, r_{n-1}, u_{n-1}, r_n, v, r_{n+1}, u_{n+1}, \ldots, r_N, u_N\big)\Big)\, dv -$$
$$-\alpha(r_n, r_m) f\big(r_1, u_1, \ldots, r_N, u_N\big)\Bigg), \qquad r_j \in \mathcal{J}, \quad u_j \in \mathcal{U}, \quad j = 1, \ldots, N, \quad (2.6)$$

Under the assumptions (2.3) the operator Λ_N^* is a bounded linear operator in the space $L_1^{(N)}$. Therefore the Cauchy problem (2.5) has a unique solution (2.4) in $L_1^{(N)}$ for all $t \ge 0$. Moreover, by standard argument we see that the solution is nonnegative for nonnegative initial data and the $L_1^{(N)}$-norm is conserved

$$\|f^N(t)\|_{L_1^{(N)}} = \|F^N\|_{L_1^{(N)}} = 1 \qquad \text{for} \quad t > 0\,. \qquad (2.7)$$

We assume that all functions are symmetric

$$f^N(\mathbf{u}_1, \ldots, \mathbf{u}_N) = f^N(\mathbf{u}_{p_1}, \ldots, \mathbf{u}_{p_N}), \qquad (2.8)$$

for a.a. $\mathbf{u}_1$, $\ldots$, $\mathbf{u}_N$ in $\mathcal{J} \times \mathcal{U}$ and for any permutation $\{p_1, \ldots, p_N\}$ of the set $\{1, \ldots, N\}$. Note that if f is invariant with respect to permutations of variables then $\Lambda_N^* f$ is invariant too.

We introduce the s-individual marginal density ($1 \le s < N$)

$$f^{N,s}(\mathbf{u}_1, \ldots, \mathbf{u}_s) = \int\limits_{(\mathcal{J}\times\mathcal{U})^{N-s}} f^N(\mathbf{u}_1, \ldots, \mathbf{u}_N)\, d\mathbf{u}_{s+1} \ldots d\mathbf{u}_N,$$

and $f^{N,N} = f^N$.

The function f^N satisfies Eq. (2.5) iff $f^{N,s}$ satisfy the following finite hierarchy of equations

$$\partial_t f^{N,s} = \frac{s}{N} \Lambda_s^* f^{N,s} + \frac{N-s}{N} \Theta_{s+1} f^{N,s+1}, \qquad s = 1, \ldots, N, \qquad (2.9)$$

where

$$(\Theta_{s+1} f)(r_1, u_1, \ldots, r_s, u_s) =$$
$$= \frac{1}{\varepsilon} \sum_{n=1}^{s} \int\limits_{\mathcal{J}\times\mathcal{U}} u_{s+1} \Bigg(\int\limits_{\mathcal{U}} \Big(\chi\big((1+\varepsilon)r_{s+1} < r_n\big) \times$$
$$\times \alpha(r_n - \varepsilon r_{s+1}, r_{s+1}) B_1(u_n, v) \times$$
$$\times f\big(r_1, u_1, \ldots, r_{n-1}, u_{n-1}, r_n - \varepsilon r_{s+1}, v, r_{n+1}, u_{n+1}, \ldots, r_{s+1}, u_{s+1}\big) +$$
$$+ \chi(r_n < r_{s+1}) \alpha(r_n, r_{s+1}) B_2(u_n, v) \times$$
$$\times f\big(r_1, u_1, \ldots, r_{n-1}, u_{n-1}, r_n, v, r_{n+1}, u_{n+1}, \ldots, r_{s+1}, u_{s+1}\big)\Big)\, dv -$$
$$-\alpha(r_n, r_{s+1}) f\big(r_1, u_1, \ldots, r_{s+1}, u_{s+1}\big)\Bigg)\, dr_{s+1}\, du_{s+1}. \qquad (2.10)$$

Taking N sufficiently large we may expect that the solution of the finite hierarchy (2.9) approximates solution of the following infinite hierarchy of equations

$$\partial_t f^s = \Theta_{s+1} f^{s+1}, \qquad s = 1, 2, \ldots . \tag{2.11}$$

The integral versions of hierarchies (2.9) and (2.11) read

$$f^{N,s}(t) = F^{N,s} + \frac{s}{N} \int_0^t \Lambda_s f^{N,s}(t_1)\, dt_1 + \frac{N-s}{N} \int_0^t \Theta_{s+1} f^{N,s+1}(t_1)\, dt_1, \tag{2.12}$$
$$s = 1, \ldots, N,$$

and

$$f^s(t) = F^s + \int_0^t \Theta_{s+1} f^{s+1}(t_1)\, dt_1, \qquad s = 1, 2, \ldots, \tag{2.13}$$

respectively.

Definition 2.1. *An admissible hierarchy* $\{f^s\}_{s=1,2,3,\ldots}$ *is a sequence of functions* f^s *satisfying* (*for* $s = 1, 2, \ldots$):

(i) f^s *is a probability density on* $\big(\mathcal{J} \times \mathcal{U}\big)^s$;

(ii) $f^s(\mathbf{u}_1, \ldots, \mathbf{u}_s) = f^s(\mathbf{u}_{r_1}, \ldots, \mathbf{u}_{r_s})$ *for a.a.* $\mathbf{u}_1, \ \ldots, \ \mathbf{u}_s$ *in* $\mathcal{J} \times \mathcal{U}$ *and for any permutation* $\{r_1, \ldots, r_s\}$ *of the set* $\{1, \ldots, s\}$;

(iii) $f^s(\mathbf{u}_1, \ldots, \mathbf{u}_s) = \int_{\mathcal{J} \times \mathcal{U}} f^{s+1}(\mathbf{u}_1, \ldots, \mathbf{u}_{s+1})\, d\mathbf{u}_{s+1}$ *for a.a.* $\mathbf{u}_1, \ \ldots, \ \mathbf{u}_s$ *in* $\mathcal{J} \times \mathcal{U}$.

By (2.3) we have

$$\|\Theta_{s+1} f\|_{L_1^{(s)}} \leq \frac{c_1 s}{\varepsilon} \|f\|_{L_1^{(s+1)}}, \tag{2.14}$$

and

$$\int_{(\mathcal{J} \times \mathcal{U})^s} (\Theta_{s+1} f)(\mathbf{u}_1, \ldots, \mathbf{u}_s)\, d\mathbf{u}_1 \ldots d\mathbf{u}_s = 0, \tag{2.15}$$

for all $f \in L_1^{(s+1)}$ and $s = 1, 2, \ldots$, where c_1 is a constant. Moreover,

$$\int_{\mathcal{J} \times \mathcal{U}} (\Theta_{s+1} f)(\mathbf{u}_1, \ldots, \mathbf{u}_s)\, d\mathbf{u}_s = (\Theta_s \hat{f})(\mathbf{u}_1, \ldots, \mathbf{u}_{s-1}), \tag{2.16}$$

for all $f \in L_1^{(s+1)}$ and $s = 1, 2, \ldots$, where

$$\hat{f}(\mathbf{u}_1, \ldots, \mathbf{u}_s) = \int_{\mathcal{J} \times \mathcal{U}} f(\mathbf{u}_1, \ldots, \mathbf{u}_{s-1}, \mathbf{u}_{s+1}, \mathbf{u}_s)\, d\mathbf{u}_{s+1}.$$

We have the following theorem.

Theorem 2.1. *Let* $\{F^s\}_{s=1,2,\ldots}$ *be an admissible hierarchy. Then, for all* $t > 0$, *there exists a unique hierarchy* $\{f^s(t)\}_{s=1,2,\ldots}$, *such that* $f^s(t) \in L_1^{(s)}$, $s = 1, 2, \ldots$, *that is a solution of Eq.* (2.13) *with initial data* $f^s(0) = F^s$, $s = 1, 2, \ldots$. *Moreover* $\{f^s(t)\}_{s=1,2,\ldots}$, *for all* $t > 0$, *is an admissible hierarchy.*

The proof is the same as the proof of Theorem 3.1 in [13].

3. The main result — asymptotic relationship. We assume now that the Markov process starts with chaotic (i.e. factorized) probability density and we consider the hierarchy (2.13) with initial data

$$F^s = F \otimes \ldots \otimes F = (F)^{s\otimes}, \qquad s = 1, 2, \ldots, \tag{3.1}$$

i.e. s-fold outer product of a probability density F defined on $\mathcal{J} \times \mathcal{U}$. We may see that the propagation of chaos is held and the solution $f^s(t)$ to Eq. (2.13) is the s-product of solution $f(t)$ of the following bilinear equation:

$$\partial_t f(r,u) = \Gamma_\varepsilon[f](r,u), \qquad (r,u) \in \mathcal{J} \times \mathcal{U}, \tag{3.2}$$

$$\Gamma_\varepsilon[f](r,u) = \frac{1}{\varepsilon} \int\limits_0^{\frac{r}{1+\varepsilon}} \int\limits_0^1 \alpha(r - \varepsilon r_1, r_1) B_1(u,v) f(r - \varepsilon r_1, v)\, \bar{f}(r_1)\, dv\, dr_1 +$$

$$+\frac{1}{\varepsilon} \int\limits_r^{\infty} \int\limits_0^1 \alpha(r, r_1) B_2(u,v) f(r,v) \bar{f}(r_1)\, dv\, dr_1 - \frac{1}{\varepsilon} \int\limits_0^{\infty} \alpha(r, r_1) f(r,u) \bar{f}(r_1)\, dr_1, \tag{3.3}$$

where

$$\bar{f}(r) = \int\limits_0^1 u f(r,u)\, du\,. \tag{3.4}$$

Equation (3.2) can be related to the class of Generalized Kinetic Models. The existence theory for Eq. (3.2) in the $L_1^{(1)}$ setting is standard. From Theorem 2.1 we immediately have the following corollary.

***Corollary* 3.1.** *Let F be a probability density on $\mathcal{J} \times \mathcal{U}$. Then, for each $t_0 > 0$, there exists an admissible hierarchy $\{f^s\}_{s=1,2,\ldots}$ such that*

(i) *it is a unique solution of Eq.* (2.13) *with chaotic initial data* (3.1),

(ii) *$f^s(t)$ is chaotic*

$$f^s(t) = \Big(f(t)\Big)^{s\,\otimes},$$

for all $0 < t \le t_0$ and $s = 1, 2, \ldots$, where $f(t)$ is the unique solution in $L_1^{(1)}$ of Eq. (3.2) *with the initial datum F. Moreover, $\bar{f}(t)$, given by* (3.4)*, is a unique solution of Eq.* (1.9) *with the initial datum $\bar{F}$.*

We may now formulate the main result, namely the theorem stating that the solution of Eq. (1.9) is approximated by the solutions of Eq. (2.5) as $N \to \infty$ (the proof follows the line of [20], see [13]).

Theorem 3.1. *Let F be a probability density on $\mathcal{J} \times \mathcal{U}$. Then, for each $t_0 > 0$, there exists N_0 such that for $N \ge N_0$*

$$\sup_{[0,t_0]} \|f^{N,1} - f\|_{L_1^{(1)}} \le \frac{c_2}{N^\eta},$$

where the nonnegative functions $f^{N,s} \in L_1^{(s)}$, $s = 1, \ldots, N$, form the unique solution of Eq. (2.12) *corresponding to the initial datum*

$$f^{N,s}(0) = (F)^{s\,\otimes}, \qquad s = 1, \ldots, N\,;$$

$f \in L_1^{(1)}$ is the unique, nonnegative solution of Eq. (3.2) *corresponding to the initial datum F; η and c_2 are positive constants that depend on t_0.*

***Corollary* 3.2.** *Under the assumptions of Theorem* 3.1

$$\sup_{t\in[0,t_0]} \int\limits_{\mathcal{J}} \Big|\bar{f}^{N,1}(t,r) - \bar{f}(t,r)\Big|\, dr \le \frac{c_2}{N^\eta},$$

where $\bar{f}(t)$, given by (3.4)*, is a unique solution of Eq.* (1.9) *corresponding to the initial datum $\bar{F}$.*

Corollary 3.2 shows that the solution of the coagulation bilinear integro-differential equations can be approximated by the solutions of linear equations describing the stochastic system of individuals — provided that the parameters of the stochastic system are suitably chosen.

The estimates are not optimized. One can hope that some of them can be improved to make them uniform with respect to t_0.

There are some possible generalizations of the above result. With slight modifications one can consider k interacting clusters (another approach can be find in [21] and [22]).

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