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EXACT SOLUTIONS OF A MATHEMATICAL MODEL FOR FLUID TRANSPORT IN PERITONEAL DIALYSIS*

ТОЧНІ РОЗВ'ЯЗКИ ОДНІЄЇ МАТЕМАТИЧНОЇ МОДЕЛІ ПЕРЕНОСУ РІДИНИ ПРИ ОЧЕРЕВИННОМУ ДІАЛІЗІ

A mathematical model for fluid transport in peritoneal dialysis is constructed. The model is based on a nonlinear system of two-dimensional partial differential equations (PDE) with the relevant boundary and initial conditions. Using the classical Lie scheme, we have established that the based PDE system (under some restrictions on coefficients) is invariant under the infinite-dimensional Lie algebra, therefore families of exact solutions were found. Moreover, exact solutions with a more general structure were found using another (non-Lie) technique. Finally, it was shown that some of the solutions obtained describe the hydrostatic pressure and the glucose concentration in peritoneal dialysis.

Побудовано математичну модель переносу рідини при очеревинному діалізі, яка базується на нелінійній системі двовимірних диференціальних рівнянь з частинними похідними (ДРЧП) з відповідними крайовими та початковими умовами. Шляхом застосування класичного методу Лі встановлено, що базова система ДРЧП (при певних обмеженнях на коефіцієнти) інваріантна відносно нескінченновимірної алгебри Лі, що дозволило побудувати сім'ї точних розв'язків. Крім того, точні розв'язки більш загальної структури знайдено за допомогою іншого неліївського методу. Також встановлено, що деякі з отриманих розв'язків описують гідростатичний тиск та концентрацію глюкози при очеревинному діалізі.

1. Introduction. Peritoneal dialysis is a life saving treatment for chronic patients with end stage renal disease [1]. Dialysis fluid is infused into the peritoneal cavity, and, during its dwell there, small metabolites (urea, creatinine) and other uremic toxins diffuse from blood to the fluid, and after some time (usually a few hours) are removed together with the drained fluid. The treatment is repeated continuously. The peritoneal transport occurs between dialysis fluid in the peritoneal cavity and blood passing down capillaries in tissue surrounding the peritoneal cavity. The capillaries are distributed within the tissue at different distance from the tissue surface that is in contact with dialysis fluid. The solutes, which are transported between blood and dialysis fluid, have to cross two transport barriers: the capillary wall and a tissue layer. Typically, many solutes are transported from blood to dialysate, but some solutes that are present in high concentration in dialysis fluid are transported to blood. This kind of transport system happens also in other medical treatments, as local delivery of anticancer medications, and some experimental or natural physiological phenomena. Mathematical description of these systems was obtained using partial differential equations based on the simplification that capillaries are homogeneously distributed within the tissue [2, 3]. Experimental evidence confirmed the good applicability of such models [2, 3].

Another objective of peritoneal dialysis is to remove excess water from the patient [1]. This is gained by inducing high osmotic pressure in dialysis fluid by adding a solute in high concentration. The most often used solute is glucose. This medical application of high osmotic pressure is rather unique for peritoneal dialysis. Mathematical description of osmotically induced fluid transport from blood to dialysis fluid has not been formulated

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fully yet, in spite of the well known basic physical law for such transport. A previous attempt did not result in a satisfactory description, and was falsified later on [2].

The paper is organized in the following way. In Section 2, a mathematical model of fluid transport in peritoneal dialysis is constructed. In Section 3, the classical Lie method (see, e.g., [4, 5]) is applied for finding exact solutions of a simplification of the model constructed. Moreover, exact solutions were found using a non-Lie ansatz. Finally, in Section 4, the exact solutions are compared with numerical solutions, which were found using a numerical technique [6] based on the finite elements method and Galerkin's method. It was shown that some of these solutions describe the hydrostatic pressure and the glucose concentration in peritoneal dialysis.

2. Mathematical model. The mathematical description of transport processes within the tissue consists in local balance of fluid volume and solute mass. For incompressible fluid, the change of volume may occur due to elasticity of the tissue. The fractional void volume, i.e., the volume occupied by the fluid in the interstitium (the rest of the tissue being cells and macromolecules forming interstitium) expressed per one unit volume of the whole tissue is denoted ν, and its time evolution is described by the following equation:

$$\frac{\partial \nu}{\partial t} = -\frac{\partial j_V}{\partial x} + q_V \tag{1}$$

where j_V is the volumetric fluid flux across the tissue, q_V is the density of volumetric fluid flux from blood to the tissue, t is time, and x is the distance from the tissue surface in contact with dialysis fluid (flat geometry of the tissue is here assumed). The solute (glucose) is distributed only within the interstitial fluid, and its concentration in this fluid is denoted by C_G. The equation that describes the local changes of solute amount, νC_G, is as follows:

$$\frac{\partial(\nu C_G)}{\partial t} = -\frac{\partial j_G}{\partial x} + q_G, \tag{2}$$

where j_G is glucose flux through the tissue, and q_G is the density of glucose flux from blood. The flows of fluid and solute are described according to linear nonequilibrium thermodynamics. Osmotic pressure of glucose is described by van't Hoff law, i.e., it is proportional to glucose concentration. The volumetric flux across the tissue is generated by hydrostatic and osmotic pressure gradients:

$$j_V = -\nu K \frac{\partial P}{\partial x} + \sigma_{TG} \nu K R T \frac{\partial C_G}{\partial x}, \tag{3}$$

whereas for the density of fluid flux from blood to tissue we assume that it is generated by osmotic pressure difference between blood and tissue:

$$q_V = -L_p a \sigma_{CG} R T (C_{GB} - C_G). \tag{4}$$

The solute flux across the tissue is composed of diffusive component (proportional to glucose concentration gradient) and convective component (proportional to glucose concentration and volumetric flux):

$$j_G = -\nu D_G \frac{\partial C_G}{\partial x} + S_{TG} C_G j_V. \tag{5}$$

Similarly, the density of glucose flux from blood to tissue has diffusive component (proportional to the difference of glucose concentration in blood, C_{GB}, and glucose concentration in tissue, C_G) and convective component (proportional to the density of volumetric flow from blood to tissue, q_V):

$$q_G = p_G a(C_{GB} - C_G) + S_{CG} q_V \left((1 - F_G)C_{GB} + F_G C_G\right). \tag{6}$$

The coefficients in the above equations are: K — hydraulic permeability of tissue, σ_{TG} — the Staverman reflection coefficient for glucose in tissue, R — gas constant, T — temperature, L_p — hydraulic permeability of the capillary wall, a — density of capillary surface area, σ_{CG} — the Staverman reflection coefficient for glucose in the capillary wall, D_G — diffusivity of glucose in tissue, $S_{TG} = 1 - \sigma_{TG}$ — sieving coefficient of glucose in tissue, p_G — diffusive permeability of the capillary wall, $S_{CG} = 1 - \sigma_{CG}$ — sieving coefficient for glucose in the capillary wall, and F_G — weighing factor.

Equations (1), (2) together with equations (3)–(6) for flows form a system of two nonlinear partial differential equations with three variables: ν, P, and C_G. Therefore, an additional, constitutive, equation is necessary, and this is the equation describing how fractional fluid volume, ν, depends on interstitial pressure, P:

$$\nu(P) = \nu_{\min} + \frac{\nu_{\max} - \nu_{\min}}{1 + \left(\dfrac{\nu_{\max} - \nu_{\min}}{\nu_0 - \nu_{\min}} - 1\right)\exp(-b(P - P_0))}, \tag{7}$$

where $\nu_{\min}$, $\nu_{\max}$, ν_0, and b are empirically measured constants. Boundary conditions for a tissue layer of width L impermeable at $x = L$ and in contact with dialysis fluid at $x = 0$ are as follows:

$$\begin{aligned} x = 0: &\quad P = P_D, \qquad C_G = C_{GD}, \\ x = L: &\quad \frac{\partial P}{\partial x} = 0, \qquad \frac{\partial C_G}{\partial x} = 0. \end{aligned} \tag{8}$$

Initial conditions describe equilibrium within the tissue without any contact with dialysis fluid at $x = 0$:

$$t = 0: \quad P = P_0, \qquad C_G = C_{GB}. \tag{9}$$

It is easily seen that equations (1)–(7) can be united into two equations for finding the glucose concentration $C_G \equiv U(t, x)$ and the hydrostatic pressure $P(t, x)$, namely:

$$\begin{aligned} \frac{\partial \nu}{\partial t} &= K\frac{\partial}{\partial x}\left(\nu\frac{\partial P}{\partial x}\right) - \sigma_1\frac{\partial}{\partial x}\left(\nu\frac{\partial U}{\partial x}\right) + h_1 U - h_0, \\ \frac{\partial(\nu U)}{\partial t} &= D\frac{\partial}{\partial x}\left(\nu\frac{\partial U}{\partial x}\right) + SK\frac{\partial}{\partial x}\left(\nu U\frac{\partial P}{\partial x}\right) - \\ &\quad - S\sigma_1\frac{\partial}{\partial x}\left(\nu U\frac{\partial U}{\partial x}\right) + b_2 U^2 - b_1 U + b_0, \end{aligned} \tag{10}$$

where

$$\begin{aligned} \sigma_1 = \sigma_{TG}KRT, &\quad D = D_G, \quad S = S_{TG}, \\ h_0 = C_{GB}h_1, &\quad h_1 = L_p aRT\sigma_{CG}, \\ b_0 = P_G a C_{GB} - S_{CG}(1 - F_G)h_0 C_{GB}, & \\ b_1 = P_G a - (1 - 2F_G)S_G h_0, &\quad b_2 = S_{CG}F_G h_1. \end{aligned} \tag{11}$$

Thus, we obtain the boundary-value problem (7)–(10) to find the functions $\nu(t, x)$, $P(t, x)$ and $U(t, x)$.

Note that possible values of the parameters arising in this problem can be established from experimental data published in [2].

3. Exact solutions of system (10). In this section we restrict ourselves to the consideration of the nonlinear balance equations (10) together with an approximation of the switch type relation (7) in the form of the linear piecewise continuous function

$$\nu(P) = \begin{cases} \nu_{\min}, & P < P_{\min}, \\ \nu_{\min} + b(P - P_{\min}), & P_{\min} \le P \le P_{\max}, \\ \nu_{\max}, & P > P_{\max}. \end{cases} \tag{12}$$

Substituting (12) into the balance equations (10), we arrive at the following equations:

$$\begin{gathered} 0 = KP_{xx} - \sigma_1 U_{xx} + h_1^* U - h_0^*, \\ U_t = DU_{xx} + KS(UP_x)_x - S\sigma_1(UU_x)_x + b_2^* U^2 - b_1^* U + b_0^* \end{gathered} \tag{13}$$

in the cases $\nu = \nu_{\min}$ or $\nu = \nu_{\max}$ (here $h_i^* = h_i/\nu_{\min}$ or $h_i^* = h_i/\nu_{\max}$, $i = 0, 1$; $b_j^* = b_j/\nu_{\min}$ or $b_j/\nu_{\max}$, $j = 0, 1, 2$) and

$$\begin{gathered} bP_t = K(\nu_{\min} - bP_{\min})P_{xx} + Kb(PP_x)_x - h_0 + h_1 U, \\ (\nu_{\min} - bP_{\min})U_t + b(UP)_t = D(\nu_{\min} - bP_{\min})U_{xx} + Db(U_x P)_x + \\ + KS(\nu_{\min} - bP_{\min})(UP_x)_x + KSb(UPP_x)_x - \\ - S\sigma_1(\nu_{\min} - bP_{\min})(UU_x)_x - S\sigma_1 b(PUU_x)_x + b_2 U^2 - b_1 U + b_0 \end{gathered} \tag{14}$$

in the case $\nu(P) = \nu_{\min} + b(P - P_{\min})$.

Let us consider the nonlinear system of equations (13). It can be noted that the nonlocal substitution

$$V = -K\nu_m P_x, \tag{15}$$

where $\nu_m = \nu_{\min}$ or $\nu_m = \nu_{\max}$ (see formula (12)), reduces this system to the form

$$\begin{gathered} V_x = h_1 U - \sigma_1 \nu_m U_{xx} - h_0, \\ \nu_m U_t = D\nu_m U_{xx} - S\sigma_1 \nu_m U_x^2 - SVU_x - b_{20}U^2 - b_{10}U + b_0, \end{gathered} \tag{16}$$

where $b_{20} = Sh_1 - b_2$, $b_{10} = b_1 - Sh_0$.

Theorem 1. *The maximal algebra of invariance (MAI) of the nonlinear system* (16) *with arbitrary non-zero coefficients is the infinite-dimensional Lie algebra generated by the operators*

$$P_t = \frac{\partial}{\partial t}, \qquad G^\infty = f(t)\frac{\partial}{\partial x} + \frac{\nu_m}{S} f_t \frac{\partial}{\partial V} \tag{17}$$

where $f(t)$ *is an arbitrary smooth function and* $f_t \equiv \dfrac{df}{dt}$.

The ***proof*** of the theorem is based on the classical Lie scheme (see [4, 5]). Here the relevant calculations are omitted because of their awkwardness.

It should be noted that the operator G^∞ for $f(t) = t$ takes the form

$$G = t\frac{\partial}{\partial x} + \frac{\nu_m}{S}\frac{\partial}{\partial V}. \tag{18}$$

Such a type of invariance operators is known as the Galilei operator because it produces Galilei transformations of the form

$$\begin{gathered} t' = t, \qquad x' = x + \varepsilon t, \\ V' = V + \frac{\nu_m}{S}\varepsilon, \quad U' = U, \end{gathered} \tag{19}$$

where ε is a group parameter. Now one can note that the first two formulas in (19) produce the classical Galilei transformations (see, e.g., [7, 8]).

It seems reasonable to construct exact solutions of system (16) using its Lie symmetry operators (17). According to the general procedure (see [4, 5]) it is necessary to construct the general solution of the linear equation

$$X(\Phi(t,x,U,V))=0 \tag{20}$$

where the operator X is a linear combination of the operators of MAI (17) and Φ is an unknown function. One can easily prove that there are three different types of solutions that can be found by solving (20) for (i) $X=P_t$, (ii) $X=G^\infty$ and (iii) $X=\alpha P_t+$ $+G^\infty,\ 0\neq\alpha\in\mathbf{R}$. Let us consider each of them.

(i) Putting $X=P_t$, we immediately obtain the ansatz

$$V=V(x),\qquad U=U(x). \tag{21}$$

One can see that ansatz (21) is the most general form of steady-state solutions. Substituting (21) into (16), we arrive at the system of ordinary differential equations (ODEs)

$$\begin{gathered} V_x=h_1U-\sigma_1\nu_mU_{xx}-h_0,\\ 0=D\nu_mU_{xx}-S\sigma_1\nu_mU_x^2-SVU_x-b_{20}U^2-b_{10}U+b_0. \end{gathered} \tag{22}$$

This system is not integrable if $S\neq0$ or $b_{20}\neq0$ (the case $S=b_{20}=0$ is unrealistic). However taking into account formulas (11) one can note that

$$U=\frac{h_0}{h_1}\equiv C_{GB},\qquad V=V_0,\quad V_0\in\mathbf{R}, \tag{23}$$

is a steady-state constant solution of (22). Obviously, the initial conditions (9) can be obtained from (23) as a particular case at $V_0=0$.

Consider the second case (ii) $X=G^\infty$. Solving the equation (20) for $X=G^\infty$, we obtain the following ansatz for the functions U and V:

$$U=\varphi_1(t),\qquad V=\varphi_0(t)+\varphi_2(t)x, \tag{24}$$

where $\varphi_0,\ \varphi_1,\ \varphi_2\equiv\dfrac{\nu_mf_t}{f}$ are unknown functions that should be found. Using ansatz (24), we can reduce the nonlinear system (16) to the ODE equation

$$\nu_m\frac{d\varphi_1}{dt}=-(Sh_1-b_2)\varphi_1^2-(b_1-Sh_0)\varphi_1+b_0, \tag{25}$$

and

$$\varphi_2=h_1\varphi_1-h_0.$$

One can see that equation (25) is easily integrated, and, substituting its general solution into ansatz (24), we obtain the following solution of the nonlinear system (16):

$$\begin{gathered} U=\frac{1}{2(Sh_1-b_2)}\left[Sh_0-b_1+\sqrt{H}\tanh\left(\frac{\sqrt{H}}{2\nu_m}(t+t_0)\right)\right],\\ V=\varphi_0(t)+\\ +\frac{h_1x}{2(Sh_1-b_2)}\left[(Sh_0-b_1)+\frac{2(b_2-Sh_1)h_0}{h_1}+\sqrt{H}\tanh\left(\frac{\sqrt{H}}{2\nu_m}(t+t_0)\right)\right], \end{gathered} \tag{26}$$

where $\varphi_0(t)$ and t_0 are an arbitrary function and constant, respectively, and

$$H = (b_1 - Sh_0)^2 + 4b_0(Sh_1 - b_2).$$

Finally, consider the case (iii) $X = \alpha P_t + G^\infty,\ \alpha \neq 0$. Solving the equation (20) for this operator, we obtain the ansatz

$$U = \varphi_1(\omega), \qquad V = \frac{\nu_m}{\alpha S} f(t) + \varphi_2(\omega), \tag{27}$$

where $\omega = \alpha x - \int f(t)dt$. Using this ansatz, the nonlinear system (16) is reduced to the following ODEs system:

$$\begin{gathered} \alpha^2 \sigma_1 \nu_m \frac{d^2\varphi_1}{d\omega^2} + \alpha \frac{d\varphi_2}{d\omega} = h_1\varphi_1 - h_0, \\ \alpha^2 D\nu_m \frac{d^2\varphi_1}{d\omega^2} - \alpha S\varphi_2 \frac{d\varphi_1}{d\omega} = (Sh_1 - b_2)\varphi_1^2 + (b_1 - Sh_0)\varphi_1 - b_0. \end{gathered} \tag{28}$$

Unfortunately, this system is not integrable. Moreover, we have not found any non-constant particular solution. Of course, one can solve (28) using numerical methods.

It turns out there is another possibility to obtain exact solutions of system (16). One observes that system (16) contains only quadratic nonlinearities. Several new approaches were recently suggested to find exact solutions for evolution equations with quadratic nonlinearities (see, e.g., [9] and references cited therein). Those methods lead to the so called non-Lie ansätze which cannot be found using the classical Lie method.

Following [10], let us consider the ansatz

$$\begin{gathered} U = \psi_0(t) + \ldots + \psi_n(t)x^n, \\ V = \varphi_0(t) + \ldots + \varphi_m(t)x^m, \end{gathered} \tag{29}$$

where $\psi_i,\ i = 0, \ldots, n,$ and $\varphi_j,\ j = 0, \ldots, m,$ are unknown functions. Obviously, this ansatz is a generalisation of the Lie ansatz (24). Substituting this ansatz into (16) one can easily show that the expression obtained is reduced to a ODE system only under the restriction $m = n + 1$. In the particular case $n = 2$, the corresponding ODE system takes the form

$$\begin{gathered} \nu_m \frac{d\psi_0}{dt} = 2\nu_m D\psi_2 - b_{20}\psi_0^2 - b_{10}\psi_0 + b_0, \\ \nu_m \frac{d\psi_2}{dt} = (3Sh_0 - b_1)\psi_2 + b_{20}\psi_0\psi_2. \end{gathered} \tag{30}$$

Simultaneously, we obtain the expressions

$$\varphi_0 = \varphi_2 = 0, \quad \varphi_1 = h_1\psi_0 - 2\nu_m\sigma_1\psi_2 - h_0, \quad \varphi_3 = \frac{h_1}{3}\psi_2 \tag{31}$$

for the other functions arising in (29) with $n = 2,\ m = 3$ and the additional restriction $b_2 = \dfrac{5Sh_1}{3}$. Finally, taking into account formulae (15), (29) and (31), we obtain the exact solution of (16) with $b_2 = \dfrac{5Sh_1}{3}$

$$\begin{gathered} U = \psi_0(t) + \psi_2(t)x^2, \\ V = (h_1\psi_0 - 2\nu_m\sigma_1\psi_2 - h_0)x + \frac{h_1}{3}\psi_2(t)x^3, \end{gathered} \tag{32}$$

where $(\psi_0(t), \psi_2(t))$ is a solution of (30).

It turns out that ansatz (29) with $n > 2$ works only under two additional restrictions on the coefficients of system (16). Omitting the relevant calculations, we present here only the result. System (16) with $b_2 = \dfrac{2n+1}{n+1}Sh_1$ $\left(\text{i.e., } b_{20} = -\dfrac{n}{n+1}Sh_1\right)$ and $D = 0$ has the exact solution

$$\begin{gathered} U = \psi_0(t) + \psi(t)x^n, \\ V = (h_1\psi_0 - h_0)x - n\nu_m\sigma_1\psi(t)x^{n-1} + \frac{h_1}{n+1}\psi(t)x^{n+1}, \end{gathered} \tag{33}$$

where $(\psi_0(t), \psi(t))$ is a solution of the ODE system

$$\begin{aligned} \nu_m\frac{d\psi_0}{dt} &= \frac{n}{n+1}Sh_1\psi_0^2 - b_{10}\psi_0 + b_0, \\ \nu_m\frac{d\psi}{dt} &= \left((n+1)Sh_0 - b_1 - \frac{n^2-n}{n+1}Sh_1\psi_0\right)\psi. \end{aligned} \tag{34}$$

Obviously, the general solution of the nonlinear system (34) can be constructed in explicit form (see, e.g., [9]).

***Remark* 1.** It can be easily checked by direct calculations that formulae (33), (34) with arbitrary *real* value $n \neq -1$ represent the exact solution of the nonlinear system (16) with $b_2 = \dfrac{2n+1}{n+1}Sh_1$ and $D = 0$.

4. Applications and interpretation. Let us use the particular exact solution (26) for solving the boundary-value problem (7)–(10) under additional restrictions. The restrictions are: 1) we assume $t >> 0$, i.e., the initial conditions play no essential role, and 2) we consider the process of dialysis with high P, i.e., with $\nu(P) = \nu_{\max}$. Obviously, we can construct the formula for the pressure using (15), namely:

$$\begin{gathered} P = P_0(t) - \frac{x}{K\nu_m}\varphi_0(t) - \frac{h_1x^2}{4K\nu_m(Sh_1 - b_2)}\times \\ \times\left[\left(\frac{2b_2}{h_1} - S\right)h_0 - b_1 + \sqrt{H}\tanh\left(\frac{\sqrt{H}}{2\nu_m}(t+t_0)\right)\right], \end{gathered} \tag{35}$$

where $P_0(t)$ and $\varphi_0(t)$ are arbitrary functions. To satisfy the Neunmann and Dirichlet conditions (8) for the pressure, the functions $P_0(t)$ and $\varphi_0(t)$ must be specified, and the expression

$$\begin{gathered} P = P_D - \frac{h_1}{4K\nu_m(Sh_1 - b_2)}\times \\ \times\left[b_1 + \left(S - \frac{2b_2}{h_1}\right)h_0 - \sqrt{H}\tanh\left(\frac{\sqrt{H}}{2\nu_m}(t+t_0)\right)\right](2xL - x^2) \end{gathered} \tag{36}$$

is obtained. In the similar way, we obtain the formula for the glucose concentration

$$U = \begin{cases} C_{GD}, & x = 0, \\ \dfrac{1}{2(Sh_1 - b_2)}\left(Sh_0 - b_1 + \sqrt{H}\tanh\left(\dfrac{\sqrt{H}}{2\nu_m}(t+t_0)\right)\right), & x > 0, \end{cases} \tag{37}$$

which also satisfies boundary conditions (8) for U. Thus, formulas (36), (37) present the exact solution of the balance equations (10) with $\nu(P) = \nu_m$ and the boundary conditions (8).

Consider the behaviour of (36), (37) at $t \to \infty$. Taking into account (11) it is easily calculated that

$$t \to +\infty: \quad P = P_D, \qquad U = \begin{cases} C_{GD}, & x = 0, \\ C_{GB}, & x > 0. \end{cases}$$

In quite similar way, we can construct the exact solution of the balance equations (10) with $\nu(P) = \nu_m$, $b_2 = \dfrac{2n+1}{n+1} Sh_1$, $D = 0$ and the boundary conditions (8) using the more general solution (33), (34). The final formulae for the pressure and the glucose concentration take the form

$$P = P_D - \frac{1}{2K\nu_m}\Bigg((h_1\psi_0(t) - h_0)(x^2 - 2xL) - 2\sigma_1\nu_m\Big((x-L)^n - (-L)^n\Big)\psi(t) + \\ + \frac{2h_1}{(n+1)(n+2)}\Big((x-L)^{n+2} - (-L)^{n+2}\Big)\psi(t)\Bigg)$$

and

$$U = \begin{cases} C_{GD}, & x = 0, \\ \psi_0(t) + (x-L)^n\psi(t), & x > 0, \end{cases}$$

respectively.

Let us consider a possible interpretation of the solutions obtained. With this in mind, the numerical solution of the boundary-value problem (7) – (10) was found. Omitting here the details (this will be done in another paper), we note that a numerical technique based on the finite elements method and Galerkin's method was used [6]. We have compared the numerical solution obtained for parameters given in [2] with the exact solution (36), (37) and established that it is possible to select the value of the parameter t_0 such that the numerical solution for the pressure P coincides with exact solution (36) if $t \geq t_1$ (here t_1 is a constant that depends on parameters arising in (7) – (10)) and the diffusivity $D = 0$. It was also established that the numerical solution for the concentration U tends to exact solution (37) if $t \to \infty$ and the diffusivity $D = 0$. Thus, we conclude that the exact solution (36), (37) at sufficiently large values of time describes the hydrostatic pressure and the glucose concentration in peritoneal dialysis based on the mathematical model (7) – (10) with $D = 0$.

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