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# $\lambda$-ALMOST SUMMABLE SPACES

# $\lambda$-МАЙЖЕ ПІДСУМОВАНІ ПРОСТОРИ

In this paper, we investigate some new sequence spaces which arise from the notation of generalized de la Vallée-Poussin means and introduce the spaces of strongly $\lambda$-almost summable sequences. We also consider some topological results, characterization of strongly $\lambda$-almost regular matrices.

Вивчаються деякі нові простори послідовностей, що випливають із позначення узагальнених середніх Валле Пуссена та продукують простори сильно $\lambda$-майже підсумованих послідовностей. Також розглядаються деякі топологічні результати та характеризація сильно $\lambda$-майже регулярних матриць.

**1. Introduction.** Let $w$ denote the set of all complex sequences $x = (x_k)$. By $l_\infty$ and $c$, we denote the Banach spaces of bounded and convergent sequences $x = (x_k)$ of $w$ normed by $\|x\| = \sup_k |x_k|$, respectively. A linear functional $L$ on $l_\infty$ is said to be a Banach limit [2] if it has the following properties:

$L(x) \geq 0$ if $x \geq 0$ (i.e., $x_n \geq 0$ for all $n$),

$L(e) = 1$ where $e = (1, 1, \ldots)$,

$L(Dx) = L(x)$, where $D$ denotes the sift operator on $\ell_\infty$, that is $D\colon \ell_\infty \to \ell_\infty$ defined by $D(x) = D(x_n) = \{x_{n+1}\}$.

Let $B$ be the set of all Banach limits on $l_\infty$. A sequence $x \in \ell_\infty$ is said to be almost convergent if all Banach limits of $x$ coincide. Let $\hat{c}$ denote the space of the almost convergent sequences.

It is easy to verify that if $x$ is a convergent sequence, then $L(x) = \lim_n x_n$ for any Banach limits $L$. In the other words, $L(x)$takes the same value for any Banach limits $L$. It is notable that this condition is meaningful not only for convergent sequences, but also for a certain type of bounded sequences. Lorentz [7] proved that

$$\hat{c} = \left\{ x : \lim_{m\to\infty} \frac{1}{m+1} \sum_{i=0}^{m} x_{n+i} \quad \text{exists uniformly in } n \right\}.$$

Almost convergent sequences were studied by Lorentz [8], King [7], Duran [4], Nanda [12], Savas [13 – 15] and others.

The strongly summable sequences have been systematically investigated by Hamilton and Hill [5], Kuttner [6] and some others. The spaces of strongly summable sequences were introduced and studied by Maddox [9, 11].

The goal of this paper is to study the spaces of strongly $\lambda$-almost summable sequences, which naturally come up for investigation and which will fill up a gap in the existing literature.

Let $\lambda = (\lambda_n)$ be a nondecreasing sequence of positive numbers tending to $\infty$ such that

$$\lambda_{n+1} \leq \lambda_n + 1, \qquad \lambda_1 = 1.$$

The generalized de la Vallée-Poussin mean is defined by

$$t_n(x) = \frac{1}{\lambda_n} \sum_{k \in I_n} x_k,$$

where $I_n = [n - \lambda_n + 1, n]$. A sequence $x = (x_k)$ is said to be $(V, \lambda)$-summable to a number $L$, if $t_n(x) \to L$ as $n \to \infty$.

Let $A = (a_{nk})$ be an infinite matrix of nonnegative real numbers and $p = (p_k)$ be a sequence such that $p_k > 0$. (These assumptions are made throughout.) We write $Ax = \{A_n(x)\}$ if $A_n(x) = \sum_k a_{nk} \|x_k\|^{p_k}$ converges for each $n$. We write

$$d_{mn}(A_\lambda x) = \frac{1}{\lambda_m} \sum_{i \in I_m} A_{n+i}(x) = \sum_k a(n,k,m) |x_k|^{p_k},$$

where

$$a(n,k,m) = \frac{1}{\lambda_m} \sum_{i \in I_m} a_{n+i,k}.$$

If we take $\lambda_m = m, m = 1, 2, 3, \ldots,$ the above reduces to

$$t_{mn}(Ax) = \frac{1}{m+1} \sum_{i=0}^{m} A_{n+i}(x) = \sum_k a(n,k,m) |x_k|^{p_k},$$

where

$$a(n,k,m) = \frac{1}{m+1} \sum_{i=0}^{m} a_{n+i,k}.$$

We now write

$$\big[\hat{A}_\lambda, p\big]_0 = \big\{x \colon d_{mn}(A_\lambda x) \to 0 \text{ uniformly in } n\big\},$$

$$\big[\hat{A}_\lambda, p\big] = \big\{x \colon d_{mn}(A_\lambda x - l) \to 0 \text{ for some } l \text{ uniformly in } n\big\}$$

and

$$\big[\hat{A}_\lambda, p\big]_\infty = \Big\{x \colon \sup_{mn} d_{mn}(A_\lambda x) < \infty\Big\}.$$

The sets $\big[\hat{A}_\lambda, p\big]_0$, $\big[\hat{A}_\lambda, p\big]$ and $\big[\hat{A}_\lambda, p\big]_\infty$ will be respectively called the spaces of strongly $\lambda$-almost summable to zero, strongly $\lambda$-summable and strongly $\lambda$-bounded sequences.

If $x$ is strongly $\lambda$-almost summable to $l$ we write $x_k \to l\big[\hat{A}_\lambda, p\big]$. A pair $(A, p)$ will be called strongly $\lambda$-almost regular if

$$x_k \to l \Rightarrow x_k \to l\big[\hat{A}_\lambda, p\big].$$

**2. Main results.** In this section, we give few propositions which are useful in the sequel of this paper.

**Proposition 2.1.** *If $p \in \ell_\infty$, then $\big[\hat{A}_\lambda, p\big]_0$, $\big[\hat{A}_\lambda, p\big]$ and $\big[\hat{A}_\lambda, p\big]_\infty$ are linear spaces over $\mathbb{C}$.*

***Proof.*** it is easy to prove, so we omit the detail.

We have the following proposition.

**Proposition 2.2.** $\big[\hat{A}_\lambda, p\big] \subset \big[\hat{A}_\lambda, p\big]_\infty$, *if*

$$\|A\| = \sup_m \sum_k a(n,k,m) < \infty. \tag{2.1}$$

***Proof.*** Assume that $x_k \to l\big[\hat{A}_\lambda, p\big]$ and (2.1) holds. Now we write

$$d_{mn}(A_\lambda x) = d_{mn}(A_\lambda x - l + l) \leq$$

$$\leq K d_{mn}(A_\lambda x - l) + K\sum_k a(n,k,m)|l|^{p_k} \leq$$

$$\leq K d_{mn}(A_\lambda x - l) + K\big(\sup|l|^{p_k}\big)\sum_k a(n,k,m).$$

Therefore, $x \in \big[\hat{A}_\lambda, p\big]_\infty$ and this completes the proof.

**Proposition 2.3.** *Let* $p \in \ell_\infty$, *then* $\big[\hat{A}_\lambda, p\big]_0$ *and* $\big[\hat{A}_\lambda, p\big]_\infty$ $(\inf p_k > 0)$ *are linear topological spaces paranormed by* $g$ (*see* [11]) *defined by*

$$g(x) = \sup_{m,n}\big[d_{m,n}(A_\lambda x)\big]^{1/M},$$

*where* $M = \max(1, H = \sup p_k)$. *If* (2.1) *holds, then* $\big[\hat{A}_\lambda, p\big]$ *has the same paranorm.*

***Proof.*** Obviously $g(0) = 0$ and $g(x) = g(-x)$. Since $M \geq 1$, by Minkowski's inequality it follows that $g$ is subadditive. We now show that the scalar multiplication is continuous. It follows that

$$g(\alpha x) \leq \sup|\alpha|^{p_k/M} g(x).$$

Therefore $x \to 0 \Rightarrow \alpha x \to 0$ (for fixed $\alpha$). Now let $\alpha \to 0$ and $x$ be fixed. Fot given $\varepsilon > 0$ there exists $N$ such that

$$d_{m,n}(A_\lambda \alpha x) < \varepsilon/2 \quad (\forall n\, \forall m > N). \tag{2.2}$$

Since $d_{m,n}(A_\lambda x)$ exists for all $m$, we write

$$d_{m,n}(A_\lambda x) = K(m), \quad 1 \leq m \leq N,$$

and

$$\delta = \left(\frac{\varepsilon}{2K(m)}\right)^{1/p_k}.$$

Then $|\alpha| < \delta$,

$$d_{m,n}(A_\lambda \alpha x) < \frac{\varepsilon}{2} \quad (\forall n,\ 1 \leq m \leq N). \tag{2.3}$$

It follows from (2.2) and (2.3) that

$$\alpha \to 0 \Rightarrow \alpha x \to 0 \quad (x \text{ fixed}).$$

This proves the assertion about $\big[\hat{A}_\lambda, p\big]_0$. If $\inf p_k = \theta > 0$ and $0 < |\alpha| < 1$, then

$$g^M(\alpha x) \leq |\alpha|^\theta g^M(x) \quad \forall x \in [\hat{A}_\lambda, p]_\infty.$$

Therefore $\big[\hat{A}_\lambda, p\big]_\infty$ has the paranorm $g$. If (2.1) holds it is clear from Proposition 2.2 that $g(x)$ exists for each $x \in \big[\hat{A}_\lambda, p\big]$.

Proposition 2.3 is proved.

***Remark* 2.1.** It is evident that $g$ is not a norm in general. But if $p_k = p$ for all $k$, then clearly $g$ is a norm for $1 \leq p \leq \infty$ and a $p$-norm for $0 < p < 1$.

**Proposition 2.4.** *$\left[\hat{A}_\lambda, p\right]_0$ and $\left[\hat{A}_\lambda, p\right]_\infty$ are complete with respect to their paranorm topologies $\left[\hat{A}_\lambda, p\right]$ is complete if* (2.1) *holds and*

$$\sum_k a(n,k,m) \to 0 \text{ uniformly in } n. \tag{2.4}$$

***Proof.*** Omitted.

Combining the above facts, we obtain the main result.

**Theorem 2.1.** *Let $p \in \ell_\infty$. Then $\left[\hat{A}_\lambda, p\right]_0$ and $\left[\hat{A}_\lambda, p\right]_\infty$* $(\inf p_k > 0)$ *are complete linear topological spaces paranormed by $g$. If* (2.1) *and* (2.4) *hold, then $\left[\hat{A}_\lambda, p\right]$ has the same property. If further $p_k = p$ for all $k$, they are Banach spaces for $1 \leq p < \infty$ and $p$-normed spaces for $0 < p < 1$.*

**3. Topological results.** We now study locally boundedness and $r$-convexity for the spaces of strongly almost summable sequences. For $0 < r \leq 1$ a non-void subset $W$ of a linear space is said to be absolutely $r$-convex if $x, y \in W$ and $|\gamma|^r + |\mu|^r \leq 1$ together imply that $\gamma x + \mu y \in W$. It is obvious that if $W$ is absolutely $r$-convex, then it is absolutely $t$-convex for $t < r$. A linear topological space $E$ is said to be $r$-convex if every neighbourhood of $0 \in E$ contains an absolutely $r$-convex neighbourhood of $0 \in E$. The $r$-convexity for $r > 1$ is of little interest, since $E$ is $r$-convex for $r > 1$ if and only if $E$ is the only neighbourhood of $0 \in E$ (see [10]). A subset $B$ of $E$ is said to be bounded if for each neighbourhood $W$ of $0 \in E$ there exists an integer $N > 1$ such that $B \subseteq NW$. $E$ is called locally bounded if there is a bounded neighbourhood of zero.

We first prove the following theorem.

**Theorem 3.1.** *Let $0 < p_k \leq 1$. Then $[\hat{A}_\lambda, p]_0$ and $[\hat{A}_\lambda, p]_\infty$ are locally bounded if $\inf p_k > 0$. If* (2.1) *holds, then $[\hat{A}_\lambda, p]$ has the same property.*

***Proof.*** We shall only prove for $[\hat{A}_\lambda, p]_\infty$. Let $\inf p_k = \theta > 0$. If $x \in \left[\hat{A}_\lambda, p\right]_\infty$, then there exists a constant $K' > 0$ such that

$$\sum_k a(n,k,m)|x_k|^{p_k} \leq K' \quad (\forall m, n).$$

For this $K'$ and given $\delta > 0$ choose an integer $N > 1$ such that

$$N^\theta \geq \frac{K'}{\delta}.$$

Since $\dfrac{1}{N} < 1$ and $p_k \leq \theta$ we write

$$\frac{1}{N^{p_k}} \leq \frac{1}{N^\theta} \; (\forall k).$$

Then, for all $m$ and $n$, we get

$$\sum_k a(n,k,m)\left|\frac{x_k}{N}\right|^{p_k} \leq \frac{1}{N^\theta}\sum_k a(n,k,m)|x_k|^{p_k} \leq \frac{K'}{N_\theta} \leq \delta.$$

Therefore, by taking supremum over $m$ and $n$, we get

$$\{x: g(x) \le K'\} \subseteq N\{x: g(x) \le \delta\}.$$

For every $\delta > 0$ exists $\mathbf{N} > 1$, for which the above inclusion holds, and so

$$\{x: g(x) \le K'\}$$

is bounded.

Theorem 3.1 is proved.

It is known that every locally bounded linear topological space is $r$-convex for some $r$ such that $0 < r \le 1$. But the following theorem gives exact conditions for $r$-convexity.

**Theorem 3.2.** *Let* $0 < p_k \le 1$. *Then* $\big[\hat{A}_\lambda, p\big]_0$ *and* $\big[\hat{A}_\lambda, p\big]_\infty$ *are* $r$*-convex for all* $r$ *where* $0 < r < \liminf p_k$. *Moreover, if* $p_k = p \le 1$ $\forall k$, *then they are* $p$*-convex.* $\big[\hat{A}_\lambda, p\big]$ *has the same properties if* (2.1) *holds.*

***Proof.*** We prove the theorem only for $\big[\hat{A}_\lambda, p\big]_\infty$. Let $\big[\hat{A}_\lambda, p\big]_\infty$ and $r \in \big(0, \liminf p_k\big)$. Then exists $k_0$ such that $r \le p_k$ $(\forall k > k_0)$. Now define

$$\hat{g}(x) = \sup_{m,n} \left[ \sum_{k=1}^{k_0} a(n,k,m)|x_k|^r + \sum_{k=k_0+1}^{\infty} a(n,k,m)|x_k|^{p_k} \right].$$

Since $r \le p_k \le 1$ $(\forall k > k_0)$, $\hat{g}$ is subadditive. Further, for $0 < |\gamma| \le 1$,

$$|\gamma|^{p_k} \le |\gamma|^r \quad (\forall k > k_0).$$

Therefore, for such $\gamma$, we have

$$\hat{g}(\gamma x) \le |\gamma|^r \hat{g}(x).$$

Now, for $0 < \delta < 1$,

$$U = \{x: \hat{g}(x) \le \delta\}$$

is an absolutely $r$-convex set, for $|\gamma|^r + |\mu|^r \le 1$ and $x, y \in W$ imply that

$$\hat{g}(\gamma x + \mu y) \le \hat{g}(\gamma x) + \hat{g}(\mu y) \le |\gamma|^r \hat{g}(x) + |\mu|^r \hat{g}(y) \le \big(|\gamma|^r + |\mu|^r\big)\delta \le \delta.$$

If $p_k = p$ $(\forall k)$, then, for $0 < \delta < 1$,

$$U = \{x: g(x) \le \delta\}$$

is an absolutely $p$-convex set. This can be obtained by a similar analysis and therefore we omit the details.

Theorem 3.2 is proved.

**4. Some further results.** Let $E$ and $F$ be two nonempty subsets of the space $w$ of sequences. If $x = \{x_k\} \in E$ implies that $\left\{\sum_k a_{nk} x_k\right\} \in F$, we say that $A$ defines a (matrix) transformation from $E$ into $F$, and we write $A\colon E \to F$. $(E, F.)$ denotes the class of matrices $A$ such that $A\colon E \to F$.

Let $c_0$ and $(V, \lambda)_0$ respectively denote the linear spaces of null sequences and sequences $\lambda$-almost convergent to zero.

We now characterize the class of strongly $\lambda$-almost regular matrices.

**Theorem 4.1.** *Let* $0 < \theta \le p_k \le H < \infty$. *Then* $(A,p)$ *is strongly* $\lambda$*-almost regular if and only if* $A \in \big(c_0, (\hat{V},\lambda)_0\big)$, *where*

$$(\hat{V},\lambda)_0 = \left\{ x : \lim_{m\to\infty} \frac{1}{\lambda_m} \sum_{i\in I_n} x_{n+i} = 0 \text{ uniformly in } n \right\}.$$

*It is known that* (*see* [1]) $A \in (c_0, (\hat{V},\lambda)_0)$ *if and only if*

$\|A\| < \infty$;

$\lim_{n\to\infty} a(n,k,m) = 0$ *uniformly in* $n$ $(\forall k)$.

To prove Theorem 4.1 we need the following result.

**Lemma 4.1** [9, p. 347]. *If* $p_k, q_k > 0$, *then* $c_0(q) \subset c_0(p) \Leftrightarrow \liminf \dfrac{p_k}{q_k} > 0$.

***Proof of Theorem* 4.1.** *Necessity*. Suppose that $(A,p)$ is strongly $\lambda$-almost regular. Therefore

$$|x_k - l|^{1/p_k} \to 0 \Rightarrow \sum_k a(n,k,m)|x_k - l| \to 0$$

uniformly in $n$. Since $\dfrac{1}{p^k} \ge \dfrac{1}{H} > 0$, by Lemma 4.1,

$$x_k \to l \Rightarrow |x_k - l|^{1/p_k} \to 0.$$

Thus

$$x_k \to l \Rightarrow \sum_k a(n,k,m)(x_k - l) \to 0$$

uniformly in $n$ and, therefore, $A \in \big(c_0, (\hat{V},\lambda)_0\big)$.

*Sufficiency*. Since $p_k \ge \theta > 0$, by Lemma 4.1,

$$x_k \to l \Rightarrow |x_k - l|^{p_k} \to 0.$$

Again we have $A \in \big(c_0, (\hat{V},\lambda)_0\big)$. Therefore $x_k \to l\big[\hat{A}_\lambda, p\big]$ and this concludes the proof. Note that $p_k \le H$ superfluous in the sufficiency and $\theta \le p_k$ is superfluous in the necessity.

Theorem 4.1 is proved.

We next consider the uniqueness of generalized limits.

**Theorem 4.2.** *Suppose that* $A \in \big(c_0, (\hat{V},\lambda)_0\big)$ *and* $p = \{p_k\}$ *converges to a positive limit. Then* $x = \{x_k\} \to l \Rightarrow x_k \to l[\hat{A}_\lambda, p]$ *uniquely if and only*

$$\sum_k a(n,k,m) \nrightarrow 0 \text{ uniformly in } n. \tag{4.1}$$

***Proof.*** *Necessity*. Suppose that $A \in \big(c_0, (\hat{V},\lambda)_0\big)$ and $\{p_k\}$ be bounded. Let $x_k \to l$ imply that $x_k \to l\big[\hat{A}_\lambda, p\big]$ uniquely. We have $e \to 1\big[\hat{A}_\lambda, p\big]$. Therefore the condition (4.1) must hold. For otherwise $e \to 0[\hat{A}, p]$ which contradicts the uniqueness of $l$.

Note that the restriction on $\{p_k\}$ (except boundedness) is superfluous for the necessity.

*Sufficiency*. Suppose that the condition (4.1) holds and $A \in (c_0, (\hat{V}, \lambda)_0)$ and that $p_k \to r > 0$. Further assume that $x_k \to l$ imply that $x_k \to l\big[\hat{A}_\lambda, p\big]$ and $x_k \to \acute{l}[\hat{A}, p]$ where $\big|l - \acute{l}\big| = a > 0$. Then we get

$$\lim_{n\to\infty} \sum_k a(n,k,m)u_k = 0 \quad \text{(uniformly in } n\text{)}, \tag{4.2}$$

where

$$u_k = |x_k - l|^{p_k} + \big|x_k - \acute{l}\big|^{p_k}.$$

By the assumption we have $u_k \to a^r$. Since $A \in \big(c_0, (\hat{V}, \lambda)_0\big)$, $u_k \to a^r$ implies that

$$\sum_k a(n,k,m)\big|u_k - a^r\big| \to 0 \quad \text{(uniformly in } n\text{)}. \tag{4.3}$$

But we have

$$a^r \sum_k (n,k,m) \le \sum_k a(n,k,m)u_k + \sum_k a(n,k,m)\big|u_k - a^r\big|. \tag{4.4}$$

Now by (4.2), (4.3) and (4.4) it follows that

$$\lim_{n\to\infty} \sum_k a(n,k,m) = 0 \quad \text{(uniformly in } n\text{)}.$$

Since this contradicts (4.1), we must have $l = \acute{l}$.

Theorem 4.2 is proved.

Suppose that $0 < p_k \le q_k$. We conclude this note by showing that $\big[\hat{A}_\lambda, q\big] \subset \big[\hat{A}_\lambda, p\big]$ is not true in general. However the inclusion holds for a special class. We prove the following theorem.

**Theorem 4.3.** *Suppose that* $\|A\| < \infty$ *and* $\dfrac{q_k}{p_k}$ *is bounded*, *then* $\big[\hat{A}_\lambda, q\big] \subset \big[A_\lambda, p\big]$.

***Proof.*** Write $w_k = |x_k - l|^{q_k}$ and $p_k/q_k = \gamma_k$. So that $0 < \gamma \le \gamma_k \le 1$ ($\gamma$ is constant). Let $x \in \big[\hat{A}_\lambda, q\big]$. Then

$$\sum_k a(n,k,m)w_k \to 0 \quad \text{(uniformly in } n\text{)}.$$

Define $u_k = w_k$ $(w_k \ge 1) = 0$ $(w_k < 1)$ and $\upsilon_k = 0$ $(w_k \ge 1) = w_k$ $(w_k < 1)$. So that $w_k = u_k + \upsilon_k$, $w_k^{\gamma_k} = u_k^{\gamma_k} + \upsilon_k^{\gamma_k}$. Hence it follows that $u_k^{\gamma_k} \le u_k \le w_k$, $\upsilon_k^{\gamma_k} < \upsilon_k^{\gamma}$. We have the inequality

$$\sum_k a(n,k,m)w_k^{\gamma_k} \le \sum_k a(n,k,m)w_k + \left(\sum_k a(n,k,m)\upsilon_k\right)^{\gamma} \|A\|^{1-\gamma}.$$

Hence, $x \in \big[\hat{A}_\lambda, p\big]$ and this completes the proof.

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