UDC 519.21

E. Seneta (Univ. Sydney, Australia)

M. V. OSTROGRADSKY AS PROBABILIST

М. В. ОСТРОГРАДСЬКИЙ ЯК ІМОВІРНІСНИК

We review the writings on probabilistic topics of M. V. Ostrogradsky (1801–1862) in the bicentenary year of his birth from a standpoint different from the sesquicentenary article of Gnedenko. Ostrogradsky's statistical technology follows closely that of Laplace's "Théorie analytique des probabilités" in its use of Bayes' Theorem together with the Principle of Insufficient Reason. He makes more precise or modifies certain of Laplace's application-oriented conclusions. The more striking results relate to sampling for attributes without replacement in a finite population and to the probability of error by a panel of judges, anticipating Poisson.

Наведено огляд імовірнісних досліджень М. В. Остроградського (1801–1862) під іншим кутом зору, ніж у статті Б. В. Гнєденка "О работах М. В. Остроградского по теории вероятностей" (1951). Статистична техніка М. В. Остроградського, яка пов'язана з використанням теореми Байєса разом із принципом недостатньої обґрунтованості, є близькою до відповідної техніки Лапласа в його книзі "Théorie analytique des probabilités". М. В. Остроградський уточнює або модифікує деякі висновки Лапласа прикладної спрямованості. Більш вражаючі результати, що з'явилися до відповідних робіт Пуассона, стосуються вибірки за ознаками без повернення із скінченної популяції та ймовірності помилки суду присяжних.

1. Introduction. A transliteration into English from Ukrainian of his name is Mykhailo Vasyliovich Ostrograds'kyi. A transliteration from the Russian version is Mikhail Vasil'evich Ostrogradsky (or Ostrogradskii). His published more — technical articles on probability, 4 in number, were written in French under the names Ostrogradsky (which we shall use) and Ostrogradski, and appeared in the *Bulletin Scientifique, publié par l'Académie Impériale des Sciences de Saint-Pétersbourg*.

A series of 3 "popular" articles appeared in Russian in 1847 in consecutive numbers of *Finskii Vestnik [The Finnish Bulletin]*, a scientific-literary journal (ucheno-literaturnii zhurnal) also published in StPetersburg. (Neighbouring Finland was a Russian Grand Duchy upto 1917, with Helsinki [Helsingfors] a substantially Russian town.) Ostrogradsky's articles appeared in the section entitled Sciences and the Arts (Nauki i Khudozhestva) and are somewhat hard to find because of apparently separate pagination of sections within a single number of the journal.

However, all published articles of his probabilistic writings are reprinted (in Russian) in Ostrogradsky [1].

The present article is written for the 200th anniversary of Ostrogradsky's birth. 50 years have passed since Gnedenko's [2] excellent analysis of Ostrogradsky's probabilistic writings. Gnedenko was the driving force behind the 150th anniversary commemorations, leading with an analysis of Ostrogradsky's life and work (Gnedenko [3]). In this he points out that in the 50 years since the 100th anniversary not a single monograph on Ostrogradsky had appeared, not studies of his technical writings, and argues for the publication of Ostrogradsky's complete works, and for an additional volume along the lines of "Materials Towards a Biography of M. V. Ostrogradsky". Both initiatives were eventually realized with the appearance of 3 volumes (of which Ostrogradsky [1], is the third) and of Ostrogradsky [4], in both of which Gnedenko plays a leading editorial role. Further, as Maistrov [5, p. 213] points out, there are by 1967 "many" works on Ostrogradsky; he cites specifically two post-1951 items in which Gnedenko is an author. The works Ostrogradsky [1, 4] make it possible for the present author to reassess Ostrogradsky's probabilistic milieu from a standpoint different from Gnedenko [2].

Coincidentally the year 2001 also marks the 300th anniversary of the birth of Thomas Bayes (1701–1761), and there is a special session devoted to him at the meeting of the International Statistical Institute in Seoul. Bayesian methodology underpins Ostrogradsky's probabilistic technical work, which will be mentioned at the session.

2. The background. Ostrogradsky and Viktor Yakovlevich Buniakovsky (1804 – 1889) are traditionally regarded as the probabilistic predecessors of Chebyshev. The prime impetus for the initial development (from the 1820's) of probability theory in the Russian empire was the need for a proper basis for actuarial and demographic work. Possibly the first work in Russian written with a view to such applications was *O Veroiatnosti* [*On Probability*] published in 1821 by the Kharkov University professor A. F. Pavlovsky (1789–1875). Ostrogradsky studied at this university in 1817 – 1820, and was influenced in his future mathematical directions by Pavlovsky an the dominant figure of T. F. Osipovsky (1765 – 1832). To Osipovsky Ostrogradsky retained a lifelong devotion. According to Bobynin [6, p. 454] Ostrogradsky shared with his teacher a high regard for French science and culture, and an antipathy to German philosophy in its mathematical variant as influenced by Kant.

Ostrogradsky left Russian Empire in May 1822, largely for political reasons, to study in Paris, where both he hand and Biniakovsky were in contact with Pierre-Simon, Marquis de Laplace (1749–1827), although evidence of specific interest in Laplace's classic monograph on probability at the time is lacking. Laplace's *Théorie analytique des probabilités* (1812 and later editions) not only attempted to systematize and regorize probability theory, but also addressed its applications to real-world situations. The second edition (1814) had as Introduction (p.vi – cvi) Laplace's *Essai philosophique des probabilités,* which recurs in later editions, and it is one of these editions from which Ostrogradsky worked.

Buniakovsky returned to StPetersburg in 1926. Ostrogradsky came to StPetersburg in Spring, 1828, and was apparently in Paris at the time of Laplace's death, and certainly experienced the time of Parisian street disorders and barricades when he seriously damaged one eye towards the end of his stay. He revisited Paris in May, 1830. In December 1831 he was made a full member (ordinarnii akademik) of the StPetersburg Academy. There was another academic visit to Paris in Spring or Summer of 1846 (Ostrogradsky [4, p. 317 – 318, 390]), and there is advice in a letter dated 24 July, 1847 to him from Sturm to visit Paris in Spring and not Summer *(loc. cit.,* p. 367). The documents in Ostrogradsky [4] reveal ongoing support for Chebyshev's career in StPetersburg, and the visits to Paris may have served as an incentive for Chebyshev's own regular visits. Ostrogradsky had influential French academic friends: among them was Gabriel Lamé (1795 – 1870) who worked in the Russian Empire in the period 1820 – 1832. There is a letter from him to Ostrogradsky of about 1830 when Lamé was living in StPetersburg *(loc. cit.,* p. 366). In Sturm's letter mentioned above, the academicians Binet, Lamé, and Sturm are listed as fervent supporters towards Ostrogradsky's election as Corresponding Member of the Paris Academy. In a letter of thanks for this election in 1856 Ostrogradsky [4, p. 357] says he had also been "honoured by Poisson's well-disposed friendship". (Poisson had died in 1840.)

In StPetersburg the social needs of their time motivated the probabilistic work of Buniakovsky and Ostrogradsky in the direction of applications. Poisson's book of 1837 *Recherches sur la probabilité des jugements* was not to become available for some time, and in a sense Ostrogradsky anticipated it (Ostrogradsky [7]). In any case the perception of probability by both Buniakovsky and Ostrogradsky was driven by Laplace's *Théorie analytique,* especially so in respect of the inferential tools used by Laplace: Bayes' Theorem and the Principle of Insufficient Reason.

Probably Ostrogradsky's first report on a probabilistic topic to the StPetersburg Academy is a positive assessment (2 June, 1830) of a course by Revkovsky (the Polish mathematician and political economist Zygmund Rewkowski, 1807 – 1893) on probability taught at Vilnius University. Vilnius (or Vil'no, or Wilno) was then in the Russian Empire. This led to an official proposal by Ostrogradsky [4, p. 276 – 278] through the Academy for introduction of such courses in all Empire universities and even high schools (gymnasia). The report shows Ostrogradsky's regard for the manner of exposition of Laplace's monograph.

In addition to the 7 published papers already mentioned there is evidence that in 1858 a course of 20 lectures on probability was published in lithographic form (Gnedenko [2, p. 101, 118, 119]). There are also summaries in Ostrogradsky [4, p. 293 – 296, 348, 384, 385]) of several unpublished papers. The first entitled "Moral Expectation" (and also "Moral Satisfaction") was read 12 June, 1835. In this Ostrogradsky takes issue with Daniel Bernoulli's work, and "expresses moral satisfaction by an arbitrary function of physical satisfaction which permits him to resolve the chief problems". The second mentions (in a letter of 1 December, 1837) a work "on the probability of future events on the basis of past events". As the editors remark (p. 379) this likely became, at least in part, the two papers Ostrogradsky [8, 9]. The third summary announces, on 29 January, 1858, the not-to-be-realized publication of "Sur la problème des parties". This was concerned with finding the probability of one player winning, when a game between two players is interrupted before completing the agred number of rounds. This seems rather to the even-then classical problem of points, which had been resolved by Pascal and Fermat in 1654.

Buniakovsky dedicated much more of his oeuvre to probability, and there is a study of it in English in Sheynin [10]. Buniakovsky's book *Osnovanie matematicheskoi teorii veroiatnostei* [*Foundations of the Mathematical Theory of Probabilities*] (StPetersburg, 1846), is very much a response to the needs of the time and for such a treatise iin Russian. Buniakovsky thought of himself as the Russian successor to Laplace in the realm of probability theory. From about 1858, Buniakovsky was the chief government expert on statistics and insurance, although this kind of work seems to have been carried out largely in cooperation with Ostrogradsky. In passing, it is worth nothing that Chebyshev's Moscow magisterial thesis published in 1845 was also a response to the needs of the Yaroslav Demidov Lycée for a textbook on probability theory. In StPetersburg Chebyshev was closely associated with Buniakovsky (Seneta [11]).

Similar concerns about retirement funds and insurance were being addressed in France by Irenée Jules Bienaymé (1796 – 1878), a friend of Lamé, and in England by Augustus De Morgan (1806 – 1871). It is possible that Ostrogradsky learned belatedly of the existence of the book of de Morgan: *An Essay on Probabilities and their Application to Life Contingencies and Insurance Offices* (London, 1838). In any case, Ostrogradsky abandoned his series in *Finskii Vestnik* of 1847: *O strakhovanii* before completing it, as he had initially proposed, with a discussion on how the beginnings of the theory of insurance derive from the study of probabilities. We recall also (Seneta, [11]) that Bienaymé learned Russian. Although Bienaymé's probabilistic interaction was with the younger Chebyshev, a connection with the similarly-aged Ostrogradsky is not unlikely.

3. The mathematical papers. These were analyzed by Gnedenko [2], the leading probabilist in Ukraine at the time. Gnedenko's analysis was essentially restated in Maistrov's book [5] and through the English translation of it (Maistrov [12]) became accessible in part to an English-reading audience. The history of Dale [13] reflects the strong resurgence in western statistical thinking of Bayesian methodology, but the section on Ostrogradsky is based entirely on Maistrov [12].

Bayes' Theorem in its purely probabilistic form is a simple statement about conditional probabilities. If $B_1, B_2, \ldots, B_n$ are mutually exclusive and exhaustive events and A is an event, then

$$P(B_i \mid A) = \frac{P(A \mid B_i)P(B_i)}{\sum_{j=1}^{n} P(A \mid B_j)P(B_j)}, \quad i = 1, \ldots, n. \tag{1}$$

If the B_j's are interpreted as causes and A as an effect, the left-hand can be interpreted as a methods of updating a prior probability of B_i, $P(B_i)$, to a posterior probability $P(B_i \mid A)$ when A is observed as an experimental outcome. The B_i's however may be some initial hypotheses and the $P(B_j)$, $j = 1, \dots, n$, then express subjective initial degrees of belief of the hypotheses, in which case the updated "probabilities" $P(B_i \mid A)$, $i = 1, \dots, n$, do not admit of a frequency interpretation. In such a setting when there is no reason to believe that any B_i is as likely as any other, one may put $P(B_1) = P(B_2) = \dots = P(B_n)$ (this is the Principle of Insufficient Reason) in which case (1) becomes

$$P(B_i \mid A) = \frac{P(A \mid B_i)}{\sum_{j=1}^{n} P(A \mid B_j)}, \quad i = 1, \dots, n. \tag{2}$$

This was the chief tool for inference used by Laplace in his *Théorie analytique des probabilités* and was used in the same way by Buniakovsky and Ostrogradsky in their probabilistic writings in Russia after their sojourns in France. This "subjectivist" inference methodology came in for heavy attack in the Soviet Union in the attempts sof ideologues to develop "materialist" mathematics. In the western statistical world the "Bayesian" and "frequentist" statistical ideologies have long been in competition and in recent decades Bayesian methodology has been prolific, possibly dominant.

There are two papers emphasized by Gnedenko and Maistrov, and through Maistrov Dale: namely Ostrogradsky [7, 8].

In Ostrogradsky [8] disregarding the practical motiovation, an urn contains a known number, s, of balls, some of which are white and some of which are black, but these numbers are not known. l balls are selected without replacement from the urn. Of these l, n are white and m are black. What is the probability that in the urn originally there were x white and y black balls $(x + y = s)$? Ostrogradsky applies (2), taking B_x as the event that there were x white balls in the urn initially. If A is the event that there were n white balls in the sample of l,

$$P(A \mid B_x) = \frac{\binom{x}{n}\binom{y}{m}}{\binom{s}{l}} = \frac{\binom{x}{n}\binom{s-x}{l-n}}{\binom{s}{l}}.$$

So from (2)

$$P(B_x \mid A) = \frac{\binom{x}{n}\binom{s-x}{l-n}}{\sum_{j=n}^{s-m}\binom{j}{n}\binom{s-j}{l-n}} \tag{3}$$

and using the identity

$$\sum_{j=n}^{s-m}\binom{j}{n}\binom{s-j}{l-n} = \binom{s+1}{l+1} \tag{4}$$

we obtain for the final answer

$$P(B_x \mid A) = \frac{\binom{x}{n}\binom{s-x}{l-n}}{\binom{s+1}{l+1}}, \quad x = n, \dots, s-m. \tag{5}$$

Ostrogradsky's reasoning is somewhat different although he certainly uses the Principle of Insufficient reason by taking $P(B_x) = 1/(s+1)$, $x = 0, 1, 2, \dots, s$. In effect he uses the primitive form of (1): $P(B_x) = P(A \mid B_x)P(B_x) / P(A)$, nothing (by something of an intuitive leap) that $P(A) = 1/(l+1)$. This produces the answer (5), and the identity (4) as a corollary, in view of (3) and (5). Gnedenko's [2] original account, Maistrov [5] emphasize the role of identity (4), while the accounts in Maistrov [12] and following it, Dale [13], are closer in spirit to Ostrogradsky's Bayesian reasoning, but omit reference to (4). Denoting by W the number of white balls in the urn,

$$P(W \le w \mid A) = \sum_{x=n}^{w} \frac{\binom{x}{n}\binom{s-x}{l-n}}{\binom{s+1}{l+1}} \qquad (6)$$

for $n \le w \le s-m$.

If we review the mathematical structure in modern terms, and put $j = x-n$ in (5) it becomes

$$\frac{\binom{j+n}{n}\binom{s-n-j}{l-n}}{\binom{s+1}{m+n+1}}, \quad j = 0, 1, \ldots, s-m-n, \qquad (7)$$

which is the *negative hypergeometric* (or beta-binomial). The expression (7) would arise if we sample without replacement from a totality of $s+1$ balls of which $n+m+1 = l+1$ are red and the remainder blue, and ask for the number of trials beyond $n+1$ needed to obtain $n+1$ red balls (Johnson and Kotz, 1969). There is a well-known probabilistic relationship between the negative and positive hypergeometric distributions emanating from the probabilistic duality (e. g. Bolshev, 1964) $P(U \ge u) = P(N \le n)$ where U is the number of trials until the $n+1$th red ball is selected, $s-m+1 \ge u \ge n+1$, $n \ge 0$, and N is the number of red balls selected in $u-1$ trials. This leads to

$$P(N \le n) = \sum_{r=0}^{n} \frac{\binom{l+1}{r}\binom{s-l}{u-1-r}}{\binom{s+1}{u-1}}$$

and

$$P(U \ge u) = \sum_{k=u}^{k=s+1-m} \frac{\binom{k-1}{n}\binom{s-k+1}{m}}{\binom{s+1}{l+1}}.$$

Putting $x = k-1$, $v = u-1$

$$P(U \ge u) = \sum_{k=v}^{x=s-m} \frac{\binom{x}{n}\binom{s-x}{m}}{\binom{s+1}{l+1}} = \sum_{r=0}^{n} \frac{\binom{l+1}{r}\binom{s-l}{v-r}}{\binom{s+1}{l+1}}$$

for $n \le v \le s-m$. Subtracting both from unity

$$\sum_{x=n}^{v-1} \frac{\binom{x}{n}\binom{s-x}{m}}{\binom{s+1}{l+1}} = 1 - \sum_{r=0}^{n} \frac{\binom{l+1}{r}\binom{s-l}{v-r}}{\binom{s+1}{v}}. \qquad (8)$$

The equivalence is valid also at $v = n+1$ and $v = s-m+1$, so putting $w = -1$ in (8)

$$\sum_{x=n}^{w} \frac{\binom{x}{n}\binom{s-x}{m}}{\binom{s+1}{l+1}} = 1 - \sum_{r=0}^{n} \frac{\binom{l+1}{r}\binom{s-l}{w+1-r}}{\binom{s+1}{w+1}} \tag{9}$$

for $n \le w \le s-m$. Thus we see from (9) that the cumulative sum for a negative hypergeometric distribution (6) can be expressed in terms of a tail of an ordinary (positive) hypergeometric distribution. This kind of expression was central to Liebermeister's completion in 1877 of a statistical argument originating in Laplace's Bayesian analysis (Seneta [14]), and in his numerical evaluation of cumulative probabilities such as (6). The same probabilistic duality as above can be used to write down (4).

The context of Ostrogradsky's [8] probability model above has been interpreted as an early instance of quality control; more to the point may be sampling theory.

In frequentist terms, the problem would be approached as follows. If W is the number of white balls in l balls drawn without replacement from an urn containing x white and $s-x$ black balls, the distributions of W is hypergeometric and

$$EW = \frac{lx}{s}$$

and so an *unbiased* estimator of x is $\hat{x} = sW/l$ whose statistical properties are well-known. The notion of expectation, although essentially a French invention going back to Pascal, did not seem to play a role in the Bayesian methodology of Laplace, nor, hence, of Ostrogradsky, remaining confined to essentially wagering context. Ostrogradsky was more interested in the most probable value (the mode).

Ostrogradsky's [7] memoir considers the problem of erroneous judgement by a panel of judges, following on from writings of Condorcet in 1785 and Laplace in his *Théorie analytique.* In this Ostrogradsky anticipates the treatise of Poisson on this topic, published in 1837: *Recherches sur la probabilité des jugements en matière criminelle et en matière civile.* The probability content of Ostrogradsky [7] has not been well-studied. Gnedenko [2] devotes only a few sentences and no mathematics. The argument below is what the present author believes was Ostrogradsky's reasoning in modern terms, leading to his own final expression.

The probability that the i-th condemning judge makes a correct decision (S_G^i) when a person on trial is guilty (G) is taken as:

$$P^c(S_G^i \mid G) = 2\int_{A_i}^{B_i} x\,dx = B_i^2 - A_i^2 = (B_i - A_i)(B_i + A_i)$$

where $0 \le A_i \le B_i \le 1$ are constants appropriate to the ith such judge, $i = 1, \ldots, m$.

Every condemning judges' propensity to render a guilty verdict when a person on trial is guilty is described by the triangular probability density function $2x$, $0 \le x \le 1$. For the condemning judges (still) the propensity to render a guilty verdict (S_G) when the person on trial is innocent (I) is described by the triangular density $2(1-x)$, $0 \le$ $\le x \le 1$ and

$$P^c(S_G^i \mid I) = 2\int_{A_i}^{B_i} (1-x)\,dx = (B_i - A_i)(2 - (B_i + A_i))$$

for the ith such judge.

For the acquitting judges, for the j th such judge, $j = 1, \dots, n$, by the same reasoning

$$P^A(S_I^j \mid I) = (B_j - A_j)(B_j + A_j),$$

$$P^A(S_I^j \mid G) = (B_j - A_j)\big(2 - (B_j + A_j)\big).$$

The probability of erroneous majority judgement when $m > n$ is

$$P(I \mid S_G^i, i = 1, \dots, m;\ S_I^j, j = 1, \dots, n)$$

which by Bayes' Theorem (2), using the Principle of Insufficient Reason to put $P(I) = P(G) = \dfrac{1}{2}$:

$$= \frac{P\big(S_G^i,\ i=1,\dots,m;\ S_I^j,\ j=1,\dots,n|I\big)}{P\big(S_G^i,\ i=1,\dots,m;\ S_I^j,\ j=1,\dots,n|I\big) + P\big(S_G^i,\ i=1,\dots,m;\ S_I^j,\ j=1,\dots,n|G\big)} \quad (10)$$

and assuming independence of judges,

$$P\big(S_G^i,\ i=1,\dots,m;\ S_I^j,\ j=1,\dots,n|I\big) =$$

$$= \prod_{i=1}^{m} (B_i - A_i)(2 - (B_i + A_i)) \prod_{j=1}^{n} (B_j - A_j)(B_j + A_j),$$

$$P\big(S_G^i,\ i=1,\dots,m;\ S_I^j,\ j=1,\dots,n|G\big) =$$

$$= \prod_{i=1}^{m} (B_i - A_i)(B_i + A_i) \prod_{j=1}^{n} (B_j - A_j)\big(2 - (B_j + A_j)\big).$$

Clearly, because of cancellation if we take $z = B_i + A_i = B_j + A_j$, $i = 1, \dots, m$, $i = 1, \dots, n$, the probability (10) is

$$\frac{1}{1 + (z/(2-z))^{m-n}}$$

which depends only on the majority $m - n$.

The use of sensible triangular probability densities to describe variability of judgement, and the use of densities other that the uniform on $[0, 1]$ was innovative.

Ostrogradsky [15] corrects some minor errors in Laplace's treatment of generating functions and gains a place in Gnedenko's [2] analysis. Ostrogradsky [9] issues the important warning for its time that care should be taken with the use of form (2) of Bayes' Theorem (for an instance of inappropriate use, see Jongmans and Seneta [16]), since this form is dependent on the Principle of Insufficient Reason. He exposits the general form (1), remarking that:

"Laplace ne considéra que le cas particulier quand les hypotheses sont egalement possibles a priori … il eût fallu en constater l'exactitude avant d'en faire usage, ce que l'illustré géométre n'a pas fait".

In summary, Ostrogradsky's few probabilistic papers were in keeping with his times, and addressed practical problems of the day. Except for Ostrogradsky [7], the probability writings are exceptionally technically and computationally meticulous. Since probability constituted only a small part of his mathematical interests, to address issues of asymptotic behaviour of sums of independent random variables, already prominent in Laplace's *Théorie analytique,* was left to Chebyshev, influenced in part by another disciple of Laplace, Bienaymé. The probabilistic writings which begin with Chebyshev may be seen as a jump to frequentist statistical thinking from the Bayesian methodology of his predecessors. It is appropriate, then, that Ostrogradsky takes his place in the early history of that methodology.

4. The popular writings. Notions of probability. This section initially relates to the 3 articles in *Finskii Vestnik.* The first two occur together in Ostrogradsky [1, p. 238 – 244] under the title *O strakhovanii* [*On insurance*], with the reference to the original journal given as vol. 13, No. 1 p. 29 – 34; No. 2, p. 40 – 44, 1847. The present author has seen the first of these in the original. The third article is titled *Igra v kosti* [*The game of dice*] and is reffered to the original source as vol. 13, No. 3, p. 29 – 32, 1847. It appears in Ostrogradsky [1, p. 245 – 247], although the tables which made up the bulk of this third note have been removed. We refer below to the papers as Nos. 1, 2, 3 respectively.

No. 1 attempts to give a clear explanation of probability, without which the theory of insurance, Ostrogradsky says, cannot be understood. His idea of it is heavily influenced both by Laplace's determinism and Laplace's classical definition of probability as the ratio of the number of favourable outcomes to the total number of equally likely outcomes. This definition is viewed in a subjective light as reflecting equal ignorance about outcomes. A telling paragraph (No. 1, p. 32; Ostrogradsky [1, p. 240]) reads as follows:

"In nature there is no probability. Everything that happens in the world is determined and not subject to doubt. Probability is consequence of human inadequacy; it relates to us and exists for us, and only for us personally. Its investigation is an important, even a necessary, supplementation to those few truths which we know to hold with relative certainty".

We will not dwell on the philosophical aspects; there is no lack of discussion of the erroneous nature of this concept from a materialist point of view, especially by Maistrov [5, p. 218 – 220, 17, p. 175 – 178] following on from Gnedenko's [2, p. 123] more diplomatic assessment:

"Although through his definition of probability Ostrogradsky erred methodologically, tending to a subjectivist position, the general direction of his work in the theory of probability should be regarded as intrinsically materialist".

There is one point relating to No. 1 which does however need reassessment. To illustrate that few people have a clear notion of probability, Ostrogradsky cites in the original French (No. 1, p. 30; [1, p. 238, 239]) a passage of an unknown author of Saint – Simonian ideology which criticizes Laplace, reffering implicity to his Essai philosophique. (The followers of the philosopher Henri de Saint-Simon (1760 – 1825) were a Christian sect, which died out in about 1832.) Gnedenko [2] (Section 8) regards this passage as coming from Buniakovsky's (1846) *magnum opus* which had just appeared, and thus construes it as an attack by Osrogradsky on Buniakovsky's understanding of probability. In this opinion Gnedenko is imitated by Maistrov [5, 12, 17] and also by Sheyinin [10] (Section 2.2). The inference is, however, unlikely, since Buniakovsky's (1846) book is in Russian, and because of the Saint-Simonian context of the French quotation. Further, a long editorial footnote in reprinting of No. 1 (Osrogradsky [1, p. 239]), begins:

"This citation in French is given by M. V. Ostrogradsky without any bibliographic sources. We have not succeeded in determining its origins".

A commentary by Gnedenko (*loc. cit.,* p. 365), however, repeats his original interpretation of the quotation's origin. Possibly an examination of Saint-Simon's (1825) *Nouveau Christianisme* or other writings will supply the answer.

In No. 2, Ostrogradsky [1, p. 244] actually gets to insurance only towards the very end, advising on the premium that it is fair to pay. Preceding that, he spends several paragraphs warning of the unrealistic hopes of the buyer of lottery tickets. His main point is that number of tickets sold is never published, so it is impossible to calculate the (very small) probability of a win. In No. 3 he is concerned with the calculation of probabilities and of expected gain in the dice game known as craps. (He writes it in English as "creps", a reflection of Russian pronunciation.) This game is played, he explains, with a specific number of six-sided dice, with the gambler's bet being on the sum of faces showing when the dice are thrown. He focusses in particular on the most probable sums, and appends tables for probabilities when the number of dice is from 2 upto 12. His especial focus is the version played in Pavlovsk (near StPetersburg) with 8 dice. The careful construction of these numerical tables, like the large portion of Ostrogradsky [12] given over to the methodology of numerical calculation, testifies (as do his actuarial writings) to the importance he attached to putting theory into practice.

We conclude this section with his view of the importance probability in his report to the Academy on 2 June, 1830:

"The study of probability is one of the most important applications of mathematical analysis. The philosophical study of nature is indebted to probability for methods by which, from a large number of observations, one may specify constants on which are based the main astronomical theories. Probability created the framework for those necessary social institutions known to us as insurance companies. Using it we have the means to examine the effects of causes lesser than outright errors, in observed phenomena. With each passing day, the influence of this branch of analysis, usable today even in political and moral sciences, grows".

5. The man. Like several important mathematicians of his time, for example Buniakovsky and James Joseph Sylvester (1814 – 1897), Ostrogradsky had a love of poetry (in all 3 of his languages). As for the frequently encountered capacity for music among mathematicians, he was tonedeaf, but enjoyed singing, as his son recollects (Ostrogradsky [9, p. 368 – 370]). He had an affection from childhood for the poetry of Taras Hryhorovych Shevchenko (1814 – 1861), who came to assume the role of national poet of Ukraine as awakener of Ukrainian consciousness through the Ukrainian language. Ostrogradsky's background links with Poltava region, a mainspring of Ukrainian culture, were maintained throughout his life, and it was there that he died. We close this article with a citation from Shevchenko's diary of 13 April, 1858 to illustrate these points:

"From N. D. Starov's we went together with Semen to M. V. Ostrogradsky's. A great mathematician, he greeted me with open arms as a countryman and, apparently, a long-lost relative. Bless him. Ostrogradsky with his family goes to Malorossia [Ukraine] in summer. He says he would invite Semen to come with him, but fears that in the whole of the Poltava Guberniia there would not be enough bacon [salo] for his needs".

The "Semen", a Ukrainian form of Simon, pronounced "Seh-men", is Semen Stepanovych Hulak-Artemovsky (1813 – 1873), a star of Petersburg Imperial Opera, composer and author of operettas with Ukrainian libretto, including the brilliant *Zaporozhets za Dunaiem.* [*The Zaporozhian Cossak beyond the Danube.*]

Acknowledgement. The author thanks Halyna Syta and the Mathematical Institute of the National Academy of Sciences of Ukraine for available Ostrogradsky [1, 4], and for editorial assistance.

1. *Ostrogradsky M. V.* Polnoe sobranie trudov [Complete collected works]. – Kiev: Izd. Akad. Nauk. Ukrainskoi SSR, 1961. – T. 3. – 393 s.
2. *Gnedenko B. V.* O robotakh M. V. Ostrogradskogo po teorii veroiatnostei // Istoriko-Matematicheskie Issledovaniia. – 1951. – **4.** – S. 99–123.
3. *Gnedenko B. V.* Mikhail Vasilevich Ostrogradsky // Uspekhi Matematicheskikh Nauk. – 1951. – **6.** – S. 1–25.
4. *Ostrogradsky M. V.* Pedagogicheskoe nasledie. Dokumenty o zhizni i deiatelnosti. [Pedagogical heritage. Documents on life and activities / Pod red. I. B. Pogrebisskii, A. P. Yushkevich. – Moscow: Izd. Fiz. Lit., 1961. – 397 s.
5. *Maistrov L. E.* Teoriia veroiatnostei. Istoricheskii ocherk. [Probability theory. A historical sketch]. – Moscow: Nauka, 1967. – 318 s.
6. *Bobynin V. V.* Ostrogradsky, Mikhail Vasilievich // *Russkii Biograficheskii Slovar* / Obezianinov–Ochkin. – StPetersburg, 1905. – S. 152–457. (Reprinted by Kraus Reprint Corporation, NYC, 1962. An earlier and slightly different entry is in the Brockgauz – Efron *Entsiklopedicheskii Slovar,* Opeka–Outsider. – StPetersburg, 1987. – T. 2. – S. 360–362.

7. *Ostrogradsky M. V.* Extrait d'n mémoire sur la probabilité des erreurs des tribunaux // Bull. Sci, № 3. Sci. Math. et Phys. L'Académie Impériale des Sciences de StPétersbourg. – 1834 (published 1938). – P. XIV–XXV. (In Russian translation in [1, p. 65–70].)
8. *Ostrogradsky M. V.* Sur on question des probabilés: Extrait // Bulletin de la Classe Physico-Mathématique de l'Académie Impériale des Scices de St Pétersbourg. – 1846 (published 1848). – № 141.142. – **6**, № 21.22. – P. 321–346. (In Russian translation in [1, p. 215–237].)
9. *Ostrogradsky* (here: *Ostrogradski*) *M. V.* Sur la probabilé des hypothèses d'après les évènements // Bulletin de la Classe Physico-Mathématique de l'Académie Impériale des Sciences de StPétersbourg. – 1859. – **17**. – P. 516–522. (In Russian translation in [1, p. 301–305].)
10. *Sheynin O. B.* On V. Ya. Buniakovsky's work in the theory of probability // Arch. Hist. Exact Sci. – 1991. – **43**. – P. 199–223.
11. *Seneta E.* Early influences on probability and statistics in the Russian Empire // Ibid. – 1998. – **53**. – P. 201–213.
12. *Maistrov L. E.* Probability theory: a historical sketch. – New York: Acad. Press, 1974. (Translated and edited from Maistrov [5], by S. Kotz.)
13. *Dale A. I.* T A history of inverse probability from Thomas Bayes to Karl Pearson. – New York: Springer, 1991.
14. *Seneta E.* Carl Liebermeister's hypergeometric tails // Historia Mathematica. – 1994. – **21**. – P. 453–462.
15. *Ostrogradsky M. V.* Mémoire sur le calcul des fonctions géneratrices // Bull. Sci, № 10. L'Académie Impériale des Sciences de StPétersbourg. – 1836. – **1**. – P. 73–75. (In Russian translation in [1, p. 106–108].)
16. *Jongmans F., Seneta E.* A probabilistic "New Principle" of the 19th century // Arch. Hist. Exact Sci. – 1994. – **47**. – P. 93–102.
17. *Maistrov L. E.* Rozvitie poniatiia veroiatnosti. [The development of the concept of probability.] – Moscow: Nauka, 1980. – 270 p.

Recieved 04.05.2001