UDC 517.9
A. I. Gerko (Univ. Moldova)

ON SOME DISJOINTNESS CLASSES OF EXTENSIONS OF MINIMAL TOPOLOGICAL TRANSFORMATION SEMIGROUPS

ПРО ДЕЯКІ КЛАСИ НЕСУМІСНОСТІ РОЗШИРЕНЬ МІНІМАЛЬНИХ ТОПОЛОГІЧНИХ НАПІВГРУП ПЕРЕТВОРЕНЬ

We study disjointness classes of extensions of minimal topological transformation semigroups.

Вивчаються класи несумісності розширень мінімальних топологічних напівгруп перетворень.

Basic definitions and auxiliary propositions. We use terminology and denotations generally accepted at present in theory of topological transformation groups. We give only necessary, in our view, definitions of concepts and facts; for more detailed discussions, the reader is referred to [1 – 9].

A topological transformation semigroup (abbreviation: TTS) is a triple (X, S, π), where X is a nonempty compact Hausdorff topological space with unique uniformity $\mathcal{U}[X]$ (phase space), S is a topological semigroup with unit element e (phase semigroup), and $\pi : X \times S \to X$ is a continuous mapping satisfying the following conditions:

1) $\forall x \in X : (x, e)\pi = x$;

2) $\forall x \in X,\ \forall s, t \in S : ((x, s)\pi, t)\pi = (x, st)\pi$.

We shall refer to the TTS (X, S) rather than (X, S, π).

Let (X, S, π) be a TTS, $s \in S$, $A \subset X$. Usually we shall write π^s for the map $X \to X$ defined by $x\pi^s = (x, s)\pi$ $(x \in X)$; $xs = x\pi^s$ and $xS = \{xs \mid s \in S\}$ $(x \in X)$. A is called minimal if $A \neq \varnothing$ and $\overline{xS} = A$ for every $x \in A$. A TTS (X, S) is minimal if X is minimal. If for $x \in X$ $\overline{xS}$ is minimal, x is called an almost periodic point. AJ is called the set of all almost periodic points from A.

The closure in the topology direct product X^X of the set $\{\pi^s \mid s \in S\}$ will be denoted by $E(X, S, \pi)$ (or simply $E(X, S)$). $E(X, S, \pi)$ is the compact subsemigroup of the semigroup X^X (by composition $\circ$) and it is called Ellis enveloping semigroup of (X, S). The TTS $(E(X, S, \pi), S, \pi^*)$ where $\overline{\pi} : E(X, S, \pi) \times S \to E(X, S, \pi)$ with $(p, s)\overline{\pi} = p \circ \pi^s$ is defined too.

The class of minimal TTSs with fixed phase semigroup S will be denoted by $K(S)$.

An extension (a homomorphism) $\varphi : (X, S, \pi) \to (Y, S, \rho)$ of TTSs is a continuous surjection $\varphi : X \to Y$ such that $x\pi^s\varphi = x\varphi\rho^s$ $\forall x \in X$, $\forall s \in S$.

Let $(Y, S) \in K(S)$ be a fixed TTS and $K(Y, S) = \{\varphi : (X, S) \to (Y, S) \mid (X, S) \in \in K(S)\}$, $M(Y, S) \subset K(Y, S)$, $\varphi : (X, S) \to (Y, S)$, $\psi : (Z, S) \to (Y, S) \in K(Y, S)$. Define:

$$\Delta(X) = \{(x, x) \mid x \in X\}; \qquad R_{\varphi\psi} = \{(x, z) \mid (x, z) \in X \times Z \wedge x\varphi = z\psi\},$$

$$P(R_{\varphi\varphi}) = \bigcap_{\alpha \in \mathcal{U}[X]} \overline{\bigcup_{s \in S} \{(x, y) \mid (x, y) \in R_{\varphi\varphi} \wedge (xs, ys) \in \alpha\}},$$

$$Q(R_{\varphi\varphi}) = \bigcap_{\alpha \in \mathcal{U}[X]} \bigcup_{s \in S} \{(x, y) \mid (x, y) \in R_{\varphi\varphi} \wedge (xs, ys) \in \alpha\}.$$

An extension φ is called distal (proximal, regionally distal), if $P(R_{\varphi\varphi}) = \Delta(X)$

$(P(R_{\varphi\varphi}) = R_{\varphi\varphi},\ Q(R_{\varphi\varphi}) = \Delta(X))$.

The class of all distal extensions from $K(Y, S)$ will be denoted by $D(Y, S)$.

The class of all regionally distal extensions from $K(Y, S)$ will be denoted by $RD(Y, S)$.

The class of all proximal extensions from $K(Y, S)$ will be denoted by $P(Y, S)$.

An extension $\psi: (Z, S) \to (Y, S)$ is called factor of the extension $\varphi: (X, S) \to (Y, S)$, if there exists an extension $\eta: (X, S) \to (Z, S)$ such that $\varphi = \eta \circ \psi$.

A factor ψ of φ is called M-factor, if $\psi \in M(Y, S)$.

A M-factor ψ of φ is called maximal, if every M-factor of φ is the factor of ψ too.

An extension $\varphi \in K(Y, S)$ is called M-prime, if every its M-factor ψ, $\psi \neq \varphi$, is an isomorphism. The class of all M-prime extensions from $K(Y, S)$ will be denoted by $MP(Y, S)$.

An extension φ is called universal for $M(Y, S)$, if every extension from $M(Y, S)$ is the factor of φ.

In our research, we will use some algebraic technique and the τ-topology [5, 6]. Developing them [6], we will follow the idea of the article [5]. But the way of realization of this idea will not be the same. The constructions in [5] are based on Stone – Cech compactification of phase (discrete) group. We are starting from the Ellis enveloping semigroup of universal minimal topological transformation semigroup for the class $K(S)$.

Henceforth it is assumed that (U, S, σ) is the universal minimal TTS for $K(S)$, $E = E(U, S, \sigma)$ is the Ellis enveloping semigroup of (U, S, σ), I is a fixed minimal right ideal of E; $u \in I$ is a fixed idempotent. It is known that $(I, S) \in K(S)$. For $(X, S) \in K(S)$, there exists a commutative diagram

$$\begin{array}{ccc} (U, S) & \xleftarrow{\Gamma} & (E, S) \\ \Phi \downarrow & & \Theta \downarrow \\ (X, S) & \xleftarrow{\rho_{x_0}} & (E(X, S), S), \end{array}$$

where $E(X, S)$ is the Ellis enveloping semigroup of (X, S), Φ is a homomorphism taken from the definition of universality of (U, S), Θ is a homomorphism induced by Φ, $p\rho_{x_0} = x_0 p$ $(p \in E(X, S))$, $x_0 \in X$ is a fixed point, Γ is a map defined analogously to ρ_{x_0}. E acts naturally on X: $xp = x_0(p\Theta)$ $(x \in X,\ p \in E)$.

Let $(X, S, \pi) \in K(S)$ and 2^X be the collection of nonempty closed subsets of X endowed with the Vietoris topology. Then $(2^X, S, \pi^*)$ defined by $(A, s)\pi^* = A\pi^s$ also TTS $(A \in 2^X, s \in S)$ and E acts on 2^X too. Let $p \in E$ and $\lim_i \sigma^{s_i} = p$ for any net $\{s_i\} \subset S$. For $A \in 2^X$, define $A \odot p = \lim_i A\sigma^{s_i} = \lim_i \{a\sigma^{s_i} \mid a \in A\}$, where the limit is understood in the Vietoris topology. If $A \subset X$ is a not necessarily closed nonempty subset of X, we define $A \odot p = \overline{A} \odot p$. For $A = \varnothing$, we define $A \odot p = \varnothing$.

The operation $\mathbf{c}$: $\mathbf{c}(A) = A \odot u \cap Xu$ $(A \subset Xu)$ defines a closure operator on Xu. The topology associated with closure operator $\mathbf{c}$ is called a τ-topology.

Let $G = Iu$. Then (G, τ) is a T_1 compact semitopological group (with unit element u).

Let C be a τ-closed subset of G, $u \in C$, $N(C)$ be the neighbourhoods filter for the τ-topology on C at u, $H(C) = \bigcap_{V \in N(C)} cls_\tau V$. We now define inductively the set $H^\alpha(C)$ for all ordinals α:

1) $H^0(C) = C$;

2) let $H^{\alpha}(C)$ be defined for every ordinal α, $\alpha < \beta$; if $\beta = \alpha + 1$, then we consider $H^{\beta}(C) = H\big(H^{\alpha}(C)\big)$; if β is a limit ordinal, then we consider $H^{\beta}(C) = \bigcap_{\alpha<\beta} H^{\alpha}(C)$.

If C is τ-closed subgroup of G, A and B are τ-closed subgroups of C, then for every ordinal α, $H^{\alpha}(C)$ is a τ-closed normal subgroup of G. Moreover, $C = ABH(C)$ implies $C = ABH^{\alpha}(C)$ [6].

For $(X, S) \in K(S)$ and $x_0 \in Xu$, we define the Ellis group of (X, S): $\mathcal{G}(X, x_0) \equiv \mathcal{G}(X) = \{ p \mid p \in G \wedge x_0 p = x_0 \}$. $\mathcal{G}(X)$ is the τ-closed subgroup of G.

For $D(Y, S)$, there exists an universal extension $(W_1, S) \to (Y, S)$. The Ellis group $D_1 \equiv \mathcal{G}(W_1)$ of (W_1, S) is an invariant subgroup of the Ellis group $\mathcal{G}(Y)$. Analogously, for $RD(Y, S)$, there exists an universal extension $(W_2, S) \to (Y, S)$. The Ellis group $D_2 \equiv \mathcal{G}(W_2)$ of (W_2, S) is an invariant subgroup of the Ellis group $\mathcal{G}(Y)$, moreover, $D_2 = D_1 H(\mathcal{G}(Y))$ [7].

Henceforth it is assumed that $D_1 (D_2)$ is the Ellis group of (W_1, S) $((W_2, S))$, where $(W_1, S) \to (Y, S)$ $((W_2, S) \to (Y, S))$ is the universal extension for $D(Y, S)$ $(RD(Y, S))$.

For every extension $\varphi: (X, S) \to (Y, S) \in K(Y, S)$, there exists its maximal D-factor $\psi_1: (Z_1, S) \to (Y, S)$. Analogously, for every extension $\varphi: (X, S) \to (Y, S) \in K(Y, S)$, there exists its maximal RD-factor $\psi_2: (Z_2, S) \to (Y, S)$. Moreover, the following lemma is proved in [7]:

Lemma 1. *$\mathcal{G}(Z_1) = \mathcal{G}(X)D_1$ and $\mathcal{G}(Z_2) = \mathcal{G}(X)D_2 = \mathcal{G}(X)D_1 H(\mathcal{G}(Y))$. Consequently, $\varphi: (X, S) \to (Y, S) \in DP(Y, S)$ iff $\mathcal{G}(X)D_1 = \mathcal{G}(Y)$ and $\varphi: (X, S) \to (Y, S) \in RDP(Y, S)$ iff $\mathcal{G}(X)D_2 = \mathcal{G}(X)D_1 H(\mathcal{G}(Y)) = \mathcal{G}(Y)$.*

In [9], the following two propositions are proved:

Lemma 2. *Let $\{(X_i, S) \mid i \in L\} \subset K(S)$, $x_i \in X_i u$ $(i \in L)$, $x = (x_i)_{i\in L}$ and $X = \overline{\{(x_i s)_{i\in L} \mid s \in S\}}$, where the closure is understood in the topology direct product X^L. Then TTS (X, S) is defined which is the subTTS of direct product $\Pi_{i\in L}(X_i, S)$ of TTSs (X_i, S) $(i \in L)$. In addition, $(X, S) \in K(S)$ and $\mathcal{G}(X, x) = \bigcap_{i\in L} \mathcal{G}(X_i, x_i)$.*

Lemma 3. *Assume that $\varphi: (X, S) \to (Y, S)$, $\psi: (Z, S) \to (Y, S) \in K(Y, S)$; $x_0 \in Xu$, $y_0 \in Yu$, $z_0 \in Zu$ with $x_0\varphi = z_0\psi = y_0$; $\mathcal{G}(X, x_0)$ and $\mathcal{G}(Z, z_0)$ are the Ellis groups of (X, S) and (Z, S) correspondently. The distal extension ψ is a factor of φ iff $\mathcal{G}(X, x_0) \subset \mathcal{G}(Z, z_0)$.*

Let θ be some limit ordinal, $\{\varphi_\alpha \mid \alpha < \theta\} \equiv \{\varphi_\alpha: (X_\alpha, S) \to (Y, S) \mid \alpha < \theta\}$ be a transfinite sequence of extensions of TTSs; $x_\alpha^0 \in X_\alpha u$, $y^0 = x_\alpha^0 \varphi_\alpha$ $(\alpha < \theta)$; and let $\{\varphi_\alpha^\beta: (X_\beta, S) \to (X_\alpha, S) \mid \alpha \le \beta < \theta\}$ be a family of extensions of TTSs with

$$x_\beta^0 \varphi_\alpha^\beta = x_\alpha^0, \quad \varphi_\alpha^\beta \circ \varphi_\alpha = \varphi_\beta, \quad \varphi_\alpha^\beta \circ \varphi_\gamma^\alpha = \varphi_\gamma^\beta, \tag{1}$$

$$\varphi_\alpha^\alpha \text{ is the identity map}, \quad \alpha \le \beta \le \gamma < \theta.$$

Now let $x_\theta^0 \in \prod_{\alpha<\theta} X_\alpha$ with $\mathrm{Pr}_{X_\alpha} x_\theta^0 = x_\alpha^0$ $(\alpha < \theta)$. There exist the TTS $\left(\overline{x_\theta^0 S}, S\right)$ which is a subTTS of direct product $\left(\prod_{\alpha<\theta} X_\alpha, S\right)$ of TTSs (X_α, S) $(\alpha <$

$< \theta)$, and the extension $\varphi_\theta : \left(\overline{x_\theta^0 S}, S\right) \to (Y, S)$ defined by $x_\theta^0 p \varphi_\theta = y^0 p \quad (p \in I)$. We denote:

$$\left(\overline{x_\theta^0 S}, S\right) = \lim_{\leftarrow}\left((X_\alpha, S), \varphi_\alpha^\beta\right)_0^\theta, \qquad \varphi_\theta = \lim_{\leftarrow}\left(\varphi_\alpha, \varphi_\alpha^\beta\right)_0^\theta.$$

Let μ be an ordinal, $\{\varphi_\alpha : (X_\alpha, S) \to (Y, S) | \alpha \le \mu\} \subset K(Y, S)$ be a family of extensions of TTSs, and let $\{\varphi_\alpha^\beta | \alpha \le \beta \le \mu\}$ be a family of its morphisms satisfying condition (1). Assume also that, for each limit ordinal θ, $\theta \le \mu$, we have

$$\varphi_\theta = \lim_{\leftarrow}\left(\varphi_\alpha, \varphi_\alpha^\beta\right)_0^\theta, \qquad (X_\theta, S) = \lim_{\leftarrow}\left((X_\alpha, S), \varphi_\alpha^\beta\right)_0^\theta.$$

Then the system $\left\{\varphi_\alpha, \varphi_\alpha^\beta\right\}_0^\mu$ is called a projective system of extensions.

An extension φ is called F-extension (PRD-extension, correspondently), if there exists a projective system $\left\{\varphi_\alpha : (X_\alpha, S) \to (Y, S),\ \varphi_\alpha^\beta : (X_\beta, S) \to (X_\alpha, S)\right\}_0^\mu$ such that:

1) $X_0 = Y$;

2) $\varphi_\mu = p \circ \varphi$ for some proximal homomorphism $p : (X_\mu, S) \to (X, S) \in K(X, S)$;

3) $\varphi_\alpha^{\alpha+1}$ is a regionally distal extension ($\varphi_\alpha^{\alpha+1}$ is a composition proximal extension and regionally distal extension, correspondently) $(\alpha < \mu)$.

If the above-mentioned proximal homomorphism p is an isomorphism, then φ is called strictly F-extension or strictly PRD-extension, correspondently.

The class of all F-extensions from $K(Y, S)$ will be denoted by $F(Y, S)$.

The class of all PRD-extensions from $K(Y, S)$ will be denoted by $PRD(Y, S)$.

Henceforth it is assumed that $(Y, S) \in K(S)$, $y_0 \in Yu$, $C = \mathcal{G}(Y, y_0)$ is the Ellis group of (Y, S), $A \subset C$ is a τ-closed subgroup of C and $M(Y, S) \subset K(Y, S)$.

The class of all extensions $\varphi : (X, S) \to (Y, S) \in K(Y, S)$ with $A \subset \mathcal{G}(X, x_0)$ $(x_0 \in Xu,\ x_0\varphi = y_0)$ will be denoted by $PS_A(Y, S)$.

The class of all extensions $\varphi : (X, S) \to (Y, S) \in K(Y, S)$ with $\varphi^{-1}(y_0 p) = x_0 A \odot p$ for some point $x_0 \in Xu$, $x_0\varphi = y_0$ and $\forall p \in I$, will be denoted by $RIC_A(Y, S)$.

An extension φ is called RIC-extension, if $\varphi \in RIC_C(Y, S)$.

The class of all RIC-extensions from $K(Y, S)$ will be denoted by $RIC(Y, S)$.

Lemma 4. *$PRD(Y, S) = PS_{H^\alpha(C)}(Y, S)$ for some ordinal α.*

Proof. There exists an ordinal α such that $H^\mu(C) = H^\alpha(C)$ for each $\mu \ge \alpha$. Then the inclusion $PRD(Y, S) \subset PS_{H^\alpha(C)}(Y, S)$ follows from definition of the $PRD(Y, S)$ by principle of transfinite induction. Let $\varphi : (X, S) \to (Y, S) \in PS_{H^\alpha(C)}(Y, S)$. For the extension φ, by theorem 4 [9], there exists the commutative diagram

$$\begin{array}{ccc} (X, S) & \xleftarrow{p} & (X^*, S) \\ \varphi\downarrow & & \varphi^*\downarrow \\ (Y, S) & \xleftarrow{q} & (Y^*, S), \end{array}$$

where $p \in P(X, S)$, $\varphi^* \in RIC(Y^*, S) \cap RDP(Y^*, S)$ and q is a strictly PRD-extension. Since $\varphi^* \in RIC(Y^*, S) \cap RDP(Y^*, S)$ and $p \in P(X, S)$, we have $\mathcal{G}(Y^*) = \mathcal{G}(X^*)H(\mathcal{G}(Y^*))$, hence, $\mathcal{G}(Y^*) = \mathcal{G}(X^*)H^\alpha(\mathcal{G}(Y^*))$. The relations

$H^{\alpha}(C) \subset \mathcal{G}(X) \subset \mathcal{G}(X^*)$ and $H^{\alpha}(\mathcal{G}(Y^*)) \subset H^{\alpha}(C)$ imply $\mathcal{G}(Y^*) = \mathcal{G}(X^*)$. Therefore, the extension φ^* is proximal, hence, isomorphism. By definition, the extension φ is PRD-extension and the inclusion $PS_{H^{\alpha}(C)}(Y, S) \subset PRD(Y, S)$ is proved.

In [9], the following two propositions are proved:

Lemma 5. *Let A be a τ-closed subgroup of G and $[A] = \{A \odot p \mid p \in I\}$. Then the set $[A]$ is a minimal subset of $(2^I, S, \pi^*)$, in addition, $\mathcal{G}([A], A \odot u) = A$ and the RIC-extension $\psi: (I, S) \to ([A], S)$ with $p\psi = A \odot p$ $(p \in I)$ is defined too.* If $(X, S) \in K(S)$, $x_0 \in Xu$, *and $A = \mathcal{G}(X, x_0)$ is the Ellis group of (X, S), then the extension $\varphi: ([A], S) \to (X, S)$ with $(A \odot p)\varphi = x_0 p$ $(p \in I)$ is defined; in addition, φ is the universal extension for $P(X, S)$.*

Lemma 6. *Let $(Z, S) \to (Y, S)$ be a maximal RD-factor of the RIC-extension $\varphi: (X, S) \to (Y, S)$. Then $\mathcal{G}(Z) = \mathcal{G}(X)H(C)$. Consequently, the extension $\varphi: (X, S) \to (Y, S) \in RIC(Y, S) \cap RDP(Y, S)$ iff $C = \mathcal{G}(X)H(C)$.*

The class of all extensions $\varphi: (X, S) \to (Y, S) \in K(Y, S)$ with $C = \mathcal{G}(X, x_0)A$ $(x_0 \in Xu,\ x_0\varphi = y_0)$ we denote also by $K_A(Y, S)$.

We recall that two extensions φ and ψ from $K(Y, S)$ are called disjoint (and denote $\varphi \perp \psi$), if $R_{\varphi\psi}$ is minimal.

The collection of all extensions from $K(Y, S)$ which are disjoint with every member of $M(Y, S)$ is denoted by $M(Y, S)^{\perp}$. $M(Y, S)^{\perp}$ is called the disjointness class.

$$M(Y, S)^{\perp\perp} \equiv (M(Y, S)^{\perp})^{\perp}.$$

In [8], the following proposition is proved:

Lemma 7. *If $\varphi: (X, S) \to (Y, S)$, $\psi: (Z, S) \to (Y, S) \in K(Y, S)$, then $\varphi \perp \psi$ iff $\overline{R_{\varphi\psi}J} = R_{\varphi\psi}$ and $C = \mathcal{G}(X)\mathcal{G}(Z)$.*

Results.

Theorem 1.

1. $D(Y, S)^{\perp} = K_{D_1}(Y, S) = DP(Y, S)$.

2. $RD(Y, S)^{\perp} = K_{D_2}(Y, S) = RDP(Y, S)$.

3. $F(Y, S)^{\perp} = RD(Y, S)^{\perp}$.

Proof. 1. Let $\varphi: (X, S) \to (Y, S) \in D(Y, S)^{\perp}$. Then φ and the universal extension for $D(Y, S)$ are disjoint, hence, by Lemma 7, $C = \mathcal{G}(X)D_1$ and $D(Y, S)^{\perp} \subset K_{D_1}(Y, S)$. Suppose that $\varphi: (X, S) \to (Y, S) \in K_{D_1}(Y, S)$, $\psi: (Z, S) \to (Y, S) \in D(Y, S)$ and $\varphi = \eta \circ \psi$ for some extension $\eta: (X, S) \to (Z, S)$. Since $C = \mathcal{G}(X)D_1$, $D_1 \subset \mathcal{G}(Z)$ and $\mathcal{G}(X) \subset \mathcal{G}(Z)$, we have $C = \mathcal{G}(X)D_1 \subset \mathcal{G}(Z)$. Hence, $C = \mathcal{G}(Z)$ and the extension ψ is proximal. At this point, ψ is isomorphism. Therefore, $\varphi \in DP(Y, S)$, $K_{D_1}(Y, S) \subset DP(Y, S)$, and we have proved the inclusions $D(Y, S)^{\perp} \subset K_{D_1}(Y, S) \subset DP(Y, S)$. Let $\varphi: (X, S) \to (Y, S) \in DP(Y, S)$. Since $\mathcal{G}(X)D_1$ is τ-closed subgroup of C, the minimal transformation semigroup $([\mathcal{G}(X)D_1], S)$ and the extension $\psi: ([\mathcal{G}(X)D_1], S) \to (Y, S)$ with $(\mathcal{G}(X)D_1 \odot p)\psi = y_0 p$ $(p \in I)$ are defined by Lemma 5. Here, $\mathcal{G}([\mathcal{G}(X)D_1], \mathcal{G}(X)D_1 \odot u) = \mathcal{G}(X)D_1$. Let $\delta: (W, S) \to (Y, S)$ be the maximal D-factor of the extension

ψ. Then $\mathcal{G}(W) = (\mathcal{G}(X)D_1)D_1 = \mathcal{G}(X)D_1$ by Lemma 1. Since $\mathcal{G}(X) \subset \mathcal{G}(X)D_1 = \mathcal{G}(W)$, we have that, by Lemma 3, the distal extension δ is a factor of the extension φ which belongs to $DP(Y, S)$. Hence, δ is an isomorphism. Therefore, $\mathcal{G}(X)D_1 = \mathcal{G}(W) = C$, $\varphi \in K_{D_1}(Y, S)$ and $DP(Y, S) \subset K_{D_1}(Y, S)$. If $\varphi : (X, S) \to (Y, S) \in K_{D_1}(Y, S)$, then $\mathcal{G}(X)D_1 = C$ and, by Lemma 7, φ and the universal extension for $D(Y, S)$ are disjoint. Hence, $\varphi \in D(Y, S)^{\perp}$, $K_{D_1}(Y, S) \subset D(Y, S)^{\perp}$ and the inclusions $DP(Y, S) \subset K_{D_1}(Y, S) \subset D(Y, S)^{\perp}$ are proved. The statement 1 is proved.

2. The statement 2 is proved by analogy.

3. The inclusion $F(Y, S)^{\perp} \subset RD(Y, S)^{\perp}$ is obvious, since $RD(Y, S) \subset F(Y, S)$. Let $\varphi : (X, S) \to (Y, S) \in RD(Y, S)^{\perp}$ and $\psi : (Z, S) \to (Y, S) \in F(Y, S)$. If $\varphi \in RD(Y, S)^{\perp}$, then $\varphi \in RDP(Y, S)$ and $C = \mathcal{G}(X)D_1H(C)$ by Lemma 1. Since $\psi \in F(Y, S) \subset D(Y, S) \cap PRD(Y, S)$, we have that $D_1 \subset \mathcal{G}(Z)$ and, by Lemma 4, $H^{\alpha}(C) \subset \mathcal{G}(Z)$ for some ordinal α. Therefore, $C = \mathcal{G}(X)D_1H(C) = \mathcal{G}(X)D_1H^{\alpha}(C) \subset \mathcal{G}(X)D_1\,\mathcal{G}(Z) \subset \mathcal{G}(X)\,\mathcal{G}(Z) \subset C$, hence, $C = \mathcal{G}(X)\,\mathcal{G}(Z)$. The fact that ψ is a distal extension implies that $R_{\varphi\psi} = R_{\varphi\psi}J$ and $\varphi \perp \psi$ by Lemma 7. Thus, $\varphi \in F(Y, S)^{\perp}$ and we have proved the inclusion $RD(Y, S)^{\perp} \subset F(Y, S)^{\perp}$. The statement 3 is proved.

***Corollary* 1.** *If S is a group or the semigroup S is σ-compact and $Ss \subset sS$ $(s \in S)$, then $D(Y, S)^{\perp} = RDP(Y, S)$.*

Proof. This is a consequence of the Theorem 1 and of the theorems on the structure of distal extensions transformation groups [10] and semigroups [11] (here, $D(Y, S) = F(Y, S)$).

Theorem 2. $PS_A(Y, S)^{\perp} = RIC_A(Y, S) = K_A(Y, S) \cap RIC(Y, S)$.

Proof. Let $(W, S) \to (Y, S)$ be a Whitney sum of the extensions from $PS_A(Y, S)$, $v_0 \in W$ be a point such that $\mathrm{Pr}_X v_0 = x^0$, where $(X, S) \to (Y, S) \in PS_A(Y, S)$ and let $x^0 \in Xu$ be a point which is used in definition of the $\mathcal{G}(X, x^0)$. If $V^* = \overline{v_0 S}$, then, by Lemma 2, V^* is the minimal subset of W and $\mathcal{G}(V^*, v_0) = \bigcap\,[\,\mathcal{G}(X, x^0) \mid (X, S) \to (Y, S) \in PS_A(Y, S)]$. If $(X, S) \to (Y, S) \in PS_A(Y, S)$, then $A \subset \mathcal{G}(X)$ and, hence, $A \subset \mathcal{G}(V^*)$. Therefore, the extension $\beta : ([A], S) \to ([\,\mathcal{G}(V^*)], S)$ with $(A \odot p)\beta = \mathcal{G}(V^*) \odot p$ $(p \in I)$ is defined correct. The extension $\delta : ([A], S) \to (Y, S)$ with $(A \odot p)\delta = y_0 p$ $(p \in I)$ is defined too. Since $\mathcal{G}([A]) = A$, $\delta \in PS_A(Y, S)$, and, for every $(X, S) \to (Y, S) \in PS_A(Y, S)$, there exists commutative diagram

$$\begin{array}{ccccc} ([A], S) & \xrightarrow{\beta} & \big([\mathcal{G}(V^*)], S\big) & \to & (V^*, S) \\ \delta\downarrow & & & & \downarrow \\ (Y, S) & & \leftarrow & & (X, S), \end{array}$$

we have that the extension δ is universal for $PS_A(Y, S)$. Suppose that $\psi : (Z, S) \to (Y, S) \in PS_A(Y, S)^{\perp}$, $z_0 \in Zu$, $z_0\psi = y_0$. Then $\psi \perp \delta$ and $R_{\psi\delta} = (z_0, A \odot u)I$. Let r be a projection of $R_{\psi\delta}$ onto $[A]$. Then, in view of the fact that $\forall p \in I$

$$\psi^{-1}(y_0 p) \times \{A \odot p\} = r^{-1}(A \odot p) = \big\{(z_0 q, A \odot q) \mid q \in I \wedge A \odot q = A \odot p\big\} =$$

$$= \big\{(z_0 q, A \odot p) \mid q \in A \odot p\big\} = x_0 A \odot p \times \{A \odot p\},$$

i.e. $\psi^{-1}(y_0p)\times\{A\odot p\} = z_0A\odot p\times\{A\odot p\}$, we have that $\psi^{-1}(y_0p) = z_0A\odot p$ and $\psi\in RIC_A(Y,S)$. The inclusion $PS_A(Y,S)^{\perp}\subset RIC_A(Y,S)$ is proved. Let $\psi\colon (Z,S)\to(Y,S)\in RIC_A(Y,S)$, $z_0\in Zu$, $z_0\psi=y_0$ and $\psi^{-1}(y_0p)=z_0A\odot p$ for $\forall p\in I$. And let r be a projection of $R_{\psi\delta}$ onto $[A]$. Since

$$\forall p\in I \quad r^{-1}(A\odot p) = \psi^{-1}(y_0p)\times\{A\odot p\} =$$
$$= z_0A\odot p\times\{A\odot p\} = (z_0, A\odot u)A\odot p,$$

relation $r^{-1}(A\odot p) = (z_0, A\odot u)A\odot p$ holds $\forall p\in I$. At this point, $R_{\psi\delta}$ is minimal, consequently, $\psi\perp\delta$ and $\psi\in PS_A(Y,S)^{\perp}$. The inclusion $RIC_A(Y,S)\subset$ $\subset PS_A(Y,S)^{\perp}$ (and the equality $PS_A(Y,S)^{\perp}=RIC_A(Y,S)$) is proved. Let $\psi\colon(Z,S)\to(Y,S)\in\in RIC_A(Y,S)$. The relation $\psi\in PS_A(Y,S)$ implies $\psi\perp\delta$ and, thus, $C=\mathcal{G}(Z)A$ by Lemma 7. Consequently, $\psi\in K_A(Y,S)$. Obviously, $RIC_A(Y,S)\subset$ $\subset RIC(Y,S)$, so the inclusion $RIC_A(Y,S)\subset K_A(Y,S)\bigcap RIC(Y,S)$ is proved too. Suppose that $\psi\colon(Z,S)\to(Y,S)\in K_A(Y,S)\bigcap RIC(Y,S)$, $z_0\in Zu$, $z_0\psi=y_0$. Then $C=\mathcal{G}(Z)A$ and $\psi^{-1}(y_0p)=z_0C\odot p$ $\forall p\in I$. At this point, $\psi^{-1}(y_0p) =$ $= z_0A\odot p$ $(p\in I)$ and $\psi\in RIC_A(Y,S)$. The inclusion $K_A(Y,S)\bigcap RIC(Y,S)\subset$ $\subset RIC_A(Y,S)$ is also proved and the proof is complete.

***Corollary* 2.**

1. $P(Y,S)^{\perp} = RIC(Y,S)$.
2. $RIC_{D_1}(Y,S) = DP(Y,S)\bigcap RIC(Y,S)$.
3. $RIC_{D_2}(Y,S) = RDP(Y,S)\bigcap RIC(Y,S)$.

Theorem 3. $PS_{H(C)}(Y,S)^{\perp} = RDP(Y,S)\bigcap RIC(Y,S)$.

Proof. Since $RD(Y,S)\subset PS_{H(C)}(Y,S)$ and $P(Y,S)\subset PS_{H(C)}(Y,S)$, we have $PS_{H(C)}(Y,S)^{\perp}\subset RD(Y,S)^{\perp}\bigcap P(Y,S)^{\perp}$. Therefore, by Theorem 1 and by Corollary 2, $PS_{H(C)}(Y,S)^{\perp} = RDP(Y,S)\bigcap RIC(Y,S)$. Suppose that $\varphi\colon(X,S)\to(Y,S)\in$ $\in RDP(Y,S)\bigcap RIC(Y,S)$ and $\psi\colon(Z,S)\to(Y,S)\in PS_{H(C)}(Y,S)$. Then, by Lemma 6, $C=\mathcal{G}(X)H(C)$ and $H(C)\subset\mathcal{G}(Z)$. Therefore, $C=\mathcal{G}(X)\mathcal{G}(Z)$. Since φ is a RIC-extension, we have $R_{\varphi\psi}=\overline{R_{\varphi\psi}J}$ and, by Lemma 7, $\varphi\perp\psi$. Hence, the inclusion $RDP(Y,S)\bigcap RIC(Y,S)\subset PS_{H(C)}(Y,S)^{\perp}$ is proved.

Theorem 4. *Let* $\alpha>0$ *be an ordinal. Then:*

1. $K_{H^{\alpha}(C)}(Y,S) = K_{H(C)}(Y,S)$.
2. $PS_{H^{\alpha}(C)}(Y,S)^{\perp} = PS_{H(C)}(Y,S)^{\perp} = RIC_{H(C)}(Y,S) = RIC_{H^{\alpha}(C)}(Y,S) =$ $= RIC_{D_2}(Y,S)$.

Proof. The statement 1 is obvious. The statement 2 is a consequence of the statement 1 and of the Theorems 2 and 3.

Theorem 5.

1. $RIC_A(Y,S)\subset PS_AP(Y,S)$.
2. $RIC_{D_1}(Y,S) = PS_{D_1}P(Y,S)\bigcap RIC(Y,S)$.
3. $RIC_{D_2}(Y,S) = PS_{D_2}P(Y,S)\bigcap RIC(Y,S)$.
4. *For every ordinal* α, $RIC_{H^{\alpha}(C)}(Y,S) = PS_{H^{\alpha}(C)}(Y,S)\bigcap RIC(Y,S)$.

Proof. 1. Let $\varphi\in RIC_A(Y,S)$. Then $\varphi\in PS_A(Y,S)^{\perp}$ by Theorem 2. Suppose that $\psi\in PS_A(Y,S)$ is a factor of φ. Since φ is RIC-extension, ψ is RIC-

extension too. Hence, ψ is an isomorphism (since $\varphi \perp \psi$). Therefore, $\varphi \in PS_A P(Y, S)$. The statement 1 is proved.

2. Obviously, $PS_{D_1}P(Y, S) \subset DP(Y, S)$, therefore the statement 2 is a consequence of following series of inclusions

$$PS_{D_1}P(Y, S) \bigcap RIC(Y, S) \subset DP(Y, S) \bigcap RIC(Y, S) =$$
$$= RIC_{D_1}(Y, S) \subset PS_{D_1}P(Y, S) \bigcap RIC(Y, S).$$

3. The statements 3 and 4 are proved by analogy the proof of 2.

***Corollary* 3.** $PS_{D_i}P(Y, S)^{\perp} = PS_{D_i}P(Y, S) \bigcap RIC(Y, S)$, $i = 1, 2$.

Theorem 6.

1. $PRDP(Y, S) \bigcap RIC(Y, S) = PS_{D_2}P(Y, S) \bigcap RIC(Y, S)$.
2. $DP(Y, S) \bigcap PP(Y, S) = PS_{D_1}P(Y, S) \bigcap PP(Y, S)$.
3. $RDP(Y, S) \bigcap PP(Y, S) = PS_{D_2}P(Y, S) \bigcap PP(Y, S)$.

Proof. 1. Let $\varphi: (X, S) \to (Y, S) \in PRDP(Y, S) \bigcap RIC(Y, S)$ and let $\psi: (Z, S) \to (Y, S) \in PS_{D_2}(Y, S)$ be its factor. Suppose also that $\eta: (W, S) \to (Y, S)$ be a maximal RD-factor of ψ. Then $\psi \in RIC(Y, S)$ too and, by Lemma 6, $\mathcal{G}(W) = \mathcal{G}(Z)H(C)$. Since $\varphi \in PRDP(Y, S)$ and $\eta \in PRD(Y, S)$, η is an isomorphism. Therefore, $C = \mathcal{G}(W)$, whence $\mathcal{G}(Z)H(C) = C$. Hence, by virtue of $H(C) \subset D_2$ and $D_2 \subset \mathcal{G}(Z)$, we have $C = \mathcal{G}(Z)$, whence $\psi \in P(Y, S)$. And since $\psi \in RIC(Y, S)$, ψ is an isomorphism. Therefore, $\varphi \in PS_{D_2}P(Y, S)$ and the inclusion $PRDP(Y, S) \bigcap RIC(Y, S) \subset PS_{D_2}(Y, S) \bigcap RIC(Y, S)$ is proved.

Inverse, let $\varphi: (X, S) \to (Y, S) \in PS_{D_2}(Y, S) \bigcap RIC(Y, S)$ and $\psi: (Z, S) \to (Y, S) \in PRD(Y, S)$ be its factor. Suppose also that $\eta: (W, S) \to (Y, S)$ is a maximal RD-factor of ψ. Then $\mathcal{G}(W) = \mathcal{G}(Z)H(C)$. Since $h \in PRD(Y, S)$, we have that η is an isomorphism and $C = \mathcal{G}(W)$, whence $\mathcal{G}(Z)H(C) = C$. Since $\psi \in PRD(Y, S)$, $H^{\alpha}(C) \subset \mathcal{G}(Z)$ for some ordinal α. Hence, $C = \mathcal{G}(Z)H(C) = \mathcal{G}(Z)H^{\alpha}(C) = \mathcal{G}(Z)$, i.e., $C = \mathcal{G}(Z)$. Thus, ψ is an isomorphism. Therefore, $\varphi \in PS_{D_2}P(Y, S)$, $\varphi \in PRDP(Y, S)$, and the inclusion $PS_{D_2}P(Y, S) \bigcap RIC(Y, S) \subset PRDP(Y, S) \bigcap RIC(Y, S)$ is proved. The statement 1 is proved.

2. To prove the statement 2, it suffices to show the implication

$$\varphi \in DP(Y, S) \bigcap PP(Y, S) \Rightarrow \varphi \in PS_{D_1}P(Y, S).$$

Let $\varphi \in DP(Y, S) \bigcap PP(Y, S)$ and let $\psi: (Z, S) \to (Y, S)$ with $D_1 \subset \mathcal{G}(Z)$ be a factor of φ. Suppose that $\eta: (W, S) \to (Y, S)$ is the maximal D-factor of ψ. Then $\mathcal{G}(W) = \mathcal{G}(Z)D_1 = \mathcal{G}(Z)$. Since η is D-factor of φ, we have that η is an isomorphism and, consequently, $C = \mathcal{G}(W)$. Therefore, $C = \mathcal{G}(Z)$ and the extension ψ is proximal. And since $\varphi \in PP(Y, S)$, we have, that ψ is an isomorphism, and, consequently, $\varphi \in PS_{D_1}P(Y, S)$. The necessary implication is proved.

3. The statement 3 is proved by analogy.

Theorem 7. $PRD(Y, S)^{\perp} = PRDP(Y, S) \bigcap RIC(Y, S)$.

Proof. Since, by Theorems 4 – 6,

$$PRDP(Y, S) \bigcap RIC(Y, S) = PS_{D_2}P(Y, S) \bigcap RIC(Y, S) =$$
$$= RIC_{D_2}(Y, S) = RIC_{H(C)}(Y, S),$$

to prove our theorem it suffices to show the equality

$$PRD(Y, S)^{\perp} = RIC_{H(C)}(Y, S). \tag{2}$$

If $H(C)=C$, then $PRD(Y,S)=P(Y,S)$ (by Lemma 4, since $H^{\alpha}(C)=C$ for all ordinals α) and $RIC_{H(C)}(Y,S)=RIC(Y,S)$. At this point, the equality (2) is proved by Corollary 2. Let $H(C)\neq C$. Then, by Lemma 4, $PRD(Y,S)=PS_{H^{\alpha}(C)}(Y,S)$ for some ordinal $\alpha>0$. And at this point, the equality (2) is proved by Theorem 4.

Theorem 8.

1. $PRD(Y,S)\bigcap RIC(Y,S)\subset RD(Y,S)^{\perp\perp}\subset D(Y,S)^{\perp\perp}$.
2. $RDP(Y,S)=(PRD(Y,S)\bigcap RIC(Y,S))^{\perp}$.
3. $PRD(Y,S)\bigcap RIC(Y,S)=PRD(Y,S)\bigcap RD(Y,S)^{\perp\perp}=PRD(Y,S)\bigcap D(Y,S)^{\perp\perp}$.

Proof. 1. Obviously, $RD(Y,S)^{\perp\perp}\subset D(Y,S)^{\perp\perp}$. Let $\varphi:(X,S)\to(Y,S)\in PRD(Y,S)\bigcap RIC(Y,S)$ and $\psi:(Z,S)\to(Y,S)\in RD(Y,S)^{\perp}$. To prove the statement 1, it suffices to show that $\varphi\perp\psi$. Suppose that $\eta:(W,S)\to(Y,S)$ is a maximal RD-factor of φ. Then $D_2\subset\mathcal{G}(W)=\mathcal{G}(X)H(C)$. Since $\psi\in RD(Y,S)^{\perp}$ and $RD(Y,S)^{\perp}=RDP(Y,S)$, we have $C=\mathcal{G}(Z)D_2$. Therefore, $C=\mathcal{G}(Z)\,\mathcal{G}(W)=\mathcal{G}(Z)\,\mathcal{G}(X)H(C)$, i.e., $C=\mathcal{G}(Z)\,\mathcal{G}(X)H(C)$. Since $\varphi\in PRD(Y,S)$, we have that by Lemma 4, $H^{\alpha}(C)\subset\mathcal{G}(X)$ for some ordinal α. At this point, $C=\mathcal{G}(Z)\,\mathcal{G}(X)H(C)=\mathcal{G}(Z)\,\mathcal{G}(X)H^{\alpha}(C)=\mathcal{G}(Z)\,\mathcal{G}(X)$, i.e., $C=\mathcal{G}(Z)\,\mathcal{G}(X)$. In view of the relation $\varphi\in RIC(Y,S)$, we have $R_{\varphi\psi}=\overline{R_{\varphi\psi}J}$ and, by Lemma 7, $\varphi\perp\psi$.

2. Statement 1 and the equality $RD(Y,S)^{\perp}=RD(Y,S)^{\perp\perp\perp}$ imply the relation

$$RDP(Y,S)=RD(Y,S)^{\perp}\subset(PRD(Y,S)\bigcap RIC(Y,S))^{\perp}.$$

Since $RD(Y,S)\subset PRD(Y,S)\bigcap RIC(Y,S)$, we have

$$(PRD(Y,S)\bigcap RIC(Y,S))^{\perp}\subset RD(Y,S)^{\perp}=RDP(Y,S)\subset$$
$$\subset(PRD(Y,S)\bigcap RIC(Y,S))^{\perp}.$$

Hence, the statement 2 is proved.

3. Since $D(Y,S)\subset RIC(Y,S)$ and $RIC(Y,S)=P(Y,S)^{\perp}=(P(Y,S)^{\perp})^{\perp\perp}=RIC(Y,S)^{\perp\perp}$, i.e., $RIC(Y,S)=RIC(Y,S)^{\perp\perp}$, we have

$$D(Y,S)^{\perp\perp}\subset RIC(Y,S)^{\perp\perp}=RIC(Y,S).$$

Therefore, the statement 3 is a consequence of the following series of inclusions:

$$PRD(Y,S)\bigcap RIC(Y,S)\subset PRD(Y,S)\bigcap RD(Y,S)^{\perp\perp}\subset$$
$$\subset PRD(Y,S)\bigcap D(Y,S)^{\perp\perp}\subset PRD(Y,S)\bigcap RIC(Y,S).$$

Theorem 9.

1. $PRD(Y,S)\bigcap RDP(Y,S)\subset RIC(Y,S)^{\perp}$.
2. $RIC(Y,S)=(PRD(Y,S)\bigcap RDP(Y,S))^{\perp}$.
3. $PRD(Y,S)\bigcap RIC(Y,S)^{\perp}=PRD(Y,S)\bigcap D(Y,S)^{\perp}=PRD(Y,S)\bigcap RD(Y,S)^{\perp}$.

Proof 1. Let $\varphi:(X,S)\to(Y,S)\in PRD(Y,S)\bigcap RDP(Y,S)$. Then, by Lemma 4, $H^{\alpha}(C)\subset\mathcal{G}(X)$ for some ordinal α, and, by Theorem 1, φ and every regionally distal extension from $K(Y,S)$ are disjoint. Let $\psi:(Z,S)\to(Y,S)\in RIC(Y,S)$

and let $\psi_0 : (W, S) \to (Y, S)$ be its maximal regionally distal factor. Then, by Lemma 6, $\mathcal{G}(W) = \mathcal{G}(Z)H(C)$. This fact, the relation $\varphi \perp \psi_0$, and Lemma 7, $C = \mathcal{G}(X)\mathcal{G}(Z)H(C) = \mathcal{G}(X)\mathcal{G}(Z)H^{\alpha}(C) = \mathcal{G}(X)\mathcal{G}(Z)$, i.e., $C = \mathcal{G}(X)\mathcal{G}(Z)$. At this point, in view of $\psi \in RIC(Y, S)$, we have $\varphi \perp \psi$ by Lemma 7. The statement 1 is proved.

2. By virtue of $RIC(Y, S) = RIC(Y, S)^{\perp\perp}$, statement 1 implies

$$RIC(Y, S) \subset (PRD(Y, S) \cap RDP(Y, S))^{\perp}.$$

Now, since $P(Y, S) \subset PRD(Y, S) \cap RDP(Y, S)$, we have

$$(PRD(Y, S) \cap RDP(Y, S))^{\perp} \subset P(Y, S)^{\perp} = RIC(Y, S) \subset (PRD(Y, S) \cap RDP(Y, S))^{\perp}$$

and the statement 2 is proved.

3. Since $RD(Y, S) \subset D(Y, S) \subset RIC(Y, S)$, we have $RIC(Y, S)^{\perp} \subset D(Y, S)^{\perp} \subset RD(Y, S)^{\perp}$. Hence, by statement 1 and by Theorem 1, we have inclusions

$$PRD(Y, S) \cap RDP(Y, S) \subset PRD(Y, S) \cap RIC(Y, S)^{\perp} \subset PRD(Y, S) \cap D(Y, S)^{\perp} \subset PRD(Y, S) \cap RD(Y, S)^{\perp} = PRD(Y, S) \cap RDP(Y, S).$$

Hence, the statement 3 is proved.

In the case where $Y = \{*\}$ is a singleton, the semigroup S is commutative, and $M(*, S) \equiv M(S)$, we have:

Theorem 10.

1. $PS_A(S)^{\perp} = RIC_A(S)^{\perp} = K_A(S)$.
2. $D(S)^{\perp} = PS_{D_1}(S)^{\perp} = PS_{D_1}P(S) = RIC_{D_1}(S) = DP(S)$.
3. $RD(S)^{\perp} = PS_{D_2}(S)^{\perp} = PS_{H(G)}(S)^{\perp} = PS_{D_2}P(S) = RIC_{D_2}(S) = RDP(S)$.
4. $PRD(S)^{\perp} = PRDP(S) = RDP(S)$.
5. $(PRD(S) \cap RD(S)^{\perp})^{\perp} = (PRD(S) \cap K(S)^{\perp})^{\perp} = K(S)$.

Proof. This follows from Theorems 1 – 9 since in our case $RIC(*, S) = K(S)$ and $C = G$.

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Received 10.03.98