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TOPOLOGY OF NON-STATIONARY ATTRACTORS IN SPACES OF CONTROL PROCESSES AND A SYNERGETIC MODEL IN FLIGHT DYNAMICS

ТОПОЛОГІЯ НЕСТАЦІОНАРНИХ АТРАКТОРІВ У ПРОСТОРІ ПРОЦЕСІВ КЕРУВАННЯ ТА ОДНА СИНЕРГЕТИЧНА МОДЕЛЬ У ДИНАМІЦІ ПОЛЬОТУ

A definition of non-stationary attractors which can originally exist in spaces of control processes is given. Topological conditions for an arbitrary set to belong to a class of non-stationary attractors is formulated. A certain synergetic model for the airplane ascent is given.

Дається означення нестаціонарних атракторів, які можуть початково існувати у просторах процесів керування. Наводяться топологічні умови, за яких довільна множина належить до класу нестаціонарних атракторів. Розглядається деяка синергетична модель у задачі виведення літаків.

Let us assume that motion of an initial dynamical system with control (denote it by IDS) is described by an indicatrix of velocities $f(x, t, u)$, where $f = (f_1, \dots, f_n) \in C^s$ is a vector function, $x = (x_1, \dots, x_n)$ is a phase vector, $t \in T = [t_0; +\infty[$ is time, $u = (u_1, \dots, u_p)$ is a vector of controls, $u = \bar{u}(t) \in C^s$, $s \geq 0$.

Denote by

a) $x(t, x_0)_u$ the phase trajectory of IDS corresponding to some admissible control law $u = \bar{u}(t)$ and starting from the point x_0 at the instant t_0, $x(t_0, x_0)_u = x_0$;

b) $x_t = (x(t, x_0)_u, t)$ the integral curve of IDS;

c) $z_t = (x_t, \bar{u}(t))$ the control process of IDS.

In the field of designing control algorithms, the major phenomenon of interest is not only the attracting process of integral curves by attractors each of which corresponds to its admissible control, but also the collection of attractors as a locus in the given control processes space irrespective of every possible admissible controls. Suppose that the locus can be approximately described by one functional equation. It is this circumstance that allows to carry out the approximate reduction of IDS, decreasing by one the order of its system of differential equations.

Let us impose the constraints on the vector of controls u in the following form

$$u = \bar{u}(t) \in U, \quad u' = \bar{u}'(t) = \frac{d\bar{u}(t)}{dt} \in U' \quad \forall t \in T,$$

and consider the manifold

$$Z_k^0 = \{ x_k = \sigma_k(x^k, t, u), (u, t) \in U \times T \} \subset R^{n+p+1}_{x,t,u}$$

with the corresponding functions $\sigma_k^e, \Delta\sigma_k$:

a) $$\varphi_k(x, t, u, u') = f_k(x, t, u) - \frac{\partial \sigma_k(x^k, t, u)}{\partial t} - \frac{\partial \sigma_k(x^k, t, u)}{\partial x^k} f^k(x, t, u) -$$

$$- \frac{\partial \sigma_k(x^k, t, u)}{\partial u} u' = 0 \Rightarrow x_k = \sigma_k^e(x^k, t, u, u');$$

b) $\Delta\sigma_k(x^k, t, u, u') = \sigma_k^e(x^k, t, u, u') - \sigma_k(x^k, t, u)$, where $u' = (u'_1, \dots, u'_p)$, $\sigma_k(x^k, t, u) \in C^{s+1}$, $x^k = (x_1, \dots, x_{k-1}, x_{k+1}, \dots, x_n)$.

$$\left.\begin{array}{l}\alpha_k = \sup \Delta\sigma_k(x^k, t, u, u'), \\ -\alpha_k = \inf \Delta\sigma_k(x^k, t, u, u')\end{array}\right\} \quad \forall\,(x^k, t, u, u') \in R^{n-1}_{x^k} \times T \times U \times U', \quad \alpha_k > 0.$$

Now one may form the $n+p+1$-dimensional set

$$EZ_k^0 = \{x_k \in [\sigma_k(x^k, t, u) - \alpha_k;\ \sigma_k(x^k, t, u) + \alpha_k]\} \cap R_x^n \times T \times U.$$

Definition. *The set* EZ_k^0 *is called a homogeneous* $n+p+1$*-dimensional* ω*-attractor if the following conditions are true*

1) *there exists such a* $n+p+1$*-dimensional* V *of* EZ_k^0 *that* $z_t = (x_t, \overline{u}(t)) \subset V \ \forall\, t > t_0$ *follows from* $z_{t_0} \in V$;

2) *if* $z_{t_0} \in EZ_k^0$, *then* $z_t \subset EZ_k^0 \ \ \forall\, t > t_0$;

3) *if* $z_{t_0} \notin EZ_k^0$, *then either* $z_t \subset V \backslash EZ_k^0 \ \ \forall\, t > t_0$ *and*

$$z_t^\omega \subset \{x_k = \sigma_k(x^k, t, u) - \alpha_k\}\ (\{x_k = \sigma_k(x^k, t, u) + \alpha_k\})$$

or there is such a time instant $\hat{t}$ *that* $z_t \subset EZ_k^0 \ \ \forall\, t \geq \hat{t}$;

4) *the first or the second inequality holds true*

$$\frac{d}{dt}\|z_t - z_t^-\| < 0 \ \ \forall\, t > t_0 \ \ (\text{or } \forall\, t \in [t_0; \hat{t}\,[) \text{ if } z_{t_0} \in \{x_k < \sigma_k(x^k, t, u) - \alpha_k\},$$

$$\frac{d}{dt}\|z_t - z_t^+\| < 0 \ \ \forall\, t > t_0 \ \ (\text{or } \forall\, t \in [t_0; \hat{t}\,[) \text{ if } z_{t_0} \in \{x_k > \sigma_k(x^k, t, u) + \alpha_k\},$$

where $z_{t_0} = (x_{t_0}, \overline{u}(t_0))$, z_t^ω *is the* ω*-limit curve of* z_t, z_t^- *and* z_t^+ *are orthogonal (relatively to the hyperplane* $\{x_k = 0\}$*) projections of* z_t *on the manifolds* $\{x_k = \sigma_k(x^k, t, u) - \alpha_k\}$ *and* $\{x_k = \sigma_k(x^k, t, u) + \alpha_k\}$, *respectively.*

Denote by $[A^{\omega\text{-hom}}]$ the class of homogeneous ω-attractors.

Consider a smooth function $p_k(y)$ generating the smooth map $\omega_k : R^m \to S_k = \{q_k = p_k(y), y \in R^m\}$ with the 0-submanifold $S_k^0 = \{y_k = \sigma_k(y^k)\}$, where ω_k belongs to the class $[\omega_k]^A$ and $y = (y_1, \ldots, y_m)$. It signifies that

$$\omega_k : \{\{y_k < \sigma_k(y^k)\} \to S_k^+,\ S_k^0 \to S_k^0,\ \{y_k > \sigma_k(y^k)\} \to S_k^-\},$$

where $S_k^+ (S_k^-)$ is the stratum of the manifold S_k satisfying the inequality $q_k > 0$ ($q_k < 0$).

Theorem. *Let a smooth function* $\varphi_k(x, t, u, u')$ *generate the smooth map*

$$\omega_k : R_x^n \times T \times U \times U' \to Z_k =$$

$$= \{q_k = \varphi_k(x, t, u, u'), (x, t, u, u') \in R_x^n \times T \times U \times U'\} \in [\omega_k]^A$$

with the 0*-level submanifold*

$$Z_k^e = \{x_k = \sigma_k^e(x^k, t, u, u'), (x^k, t, u, u') \in R^{n-1}_{x^k} \times T \times U \times U'\}.$$

Then $EZ_k^0 \in [A^{\omega\text{-hom}}]$.

A small enough value of α_k often allows to carry out the approximate reduction of the initial dynamical system with control by replacing the differential equation

$$\frac{dx_k}{dt} = f_k(x, t, u)$$

with the functional one

$$x_k = \sigma_k(x^k, t, u).$$

The distance between phase trajectories of these systems can be estimated by some real value $\varepsilon > 0$ over a finite interval of t [1]. Thus we deal with the reduction of degrees of freedom, which leads to the hierarchy of simplified systems with control. This is a typical situation in synergetics when the centers of self-organization processes are non-stationary attractors [2]. A specific example of such a synergetic model in flight dynamics is presented below.

Consider the motion of an aeroplane executing the lift-accelerate manoeuvre with the slip angle being equal to zero; we neglect atmospheric disturbances, cross-wind force. The differential equations of the motion with an additional constraint on the atmospheric flight path and on conditions that $\cos\alpha \approx 1 - 0{,}5\alpha^2$ are written in the Wind-Body coordinate system as

$$\frac{d\theta}{d\overline{h}} = \frac{1{,}25}{\overline{m}V^2\sin\theta}\Big([P\sin\alpha + C_y^{\alpha}qS\alpha]\cos\gamma -$$

$$-4\cdot 10^3\overline{m}\cos\theta\left(g - \frac{V^2}{R+\overline{h}\cdot 5\cdot 10^3}\right)\Big) = Q(\overline{h}, \theta, \overline{m}, \gamma, u),$$

$$\frac{d\psi}{d\overline{h}} = -\frac{1{,}25}{\overline{m}V^2\sin\theta\cos\theta}\Big(\big[P\sin\alpha + C_y^{\alpha}qS\alpha\big]\sin\gamma +$$

$$+\frac{4\cdot 10^3\overline{m}V^2\cos^2\theta}{R+\overline{h}\cdot 5\cdot 10^3}tg\,\sigma\cos\psi\Big), \tag{1}$$

$$\frac{d\sigma}{d\overline{h}} = \frac{5\cdot 10^3}{R+\overline{h}\cdot 5\cdot 10^3}\sin\psi\,\text{ctg}\,\theta, \quad \frac{d\overline{m}}{d\overline{h}} = \frac{1{,}25G}{V\sin\theta},$$

$$\frac{d\chi}{d\overline{h}} = \frac{5\cdot 10^3}{\left(R+\overline{h}\cdot 5\cdot 10^3\right)\cos\sigma}\cos\psi\,\text{ctg}\,\theta, \quad \frac{dt}{d\overline{h}} = -\frac{5\cdot 10^3}{V\sin\theta},$$

$$\alpha = \left(\frac{P - G_{x_0}qS - 4\cdot 10^3\overline{m}\sin\theta\left(\frac{V}{5\cdot 10^3}\cdot\frac{dV}{d\overline{h}} + g\right)}{0{,}5P + AqS}\right)^{0{,}5},$$

where $V = V(\overline{h})$ is an additional constraint on the atmospheric flight path, $\frac{d}{d\overline{h}}V(\overline{h}) > 0 \ \forall\, \overline{h} \in [\overline{h}_0; \overline{h}_1]$.

It is denoted here: $\overline{m} = m\cdot 2{,}5\cdot 10^{-4}$ — reduced mass of the aeroplane, $\overline{m} \in [\overline{m}_1; \overline{m}_2]$; $\overline{h} = h\cdot 2\cdot 10^{-4}$ — reduced altitude of flight; R — Earth radius; $g = g(\overline{h})$ — free-fall acceleration; V — flight speed of the aeroplane; θ — trajectory angle; ψ — course angle; σ — geographical latitude of the aeroplane; χ — geographical longitude of the aeroplane; α — angle of attack; γ — bank angle; $P = P(\overline{h}, u)$ — engine thrust; $G = G(\overline{h}, u)$ — fuel flow rate; t — time; C_{x_0}, C_y^{α}, A — aerodynamic coefficients; q — dynamic pressure; S — aerodynamic reference area; u — engine control.

Let $\sin\alpha \approx \alpha$ in the first differential equation of the system (1). It turns out that the set

$$EZ_\theta^0 = \{\theta \in [-\alpha_\theta + \arcsin\sigma_\theta(\overline{h}, \overline{m}, \gamma, u); \alpha_\theta + \arcsin\sigma_\theta(\overline{h}, \overline{m}, \gamma, u)]\}$$

which is constructed by using the 0-level submanifold $Z_\theta^0 = \{\theta = \arcsin\sigma_\theta(\overline{h}, \overline{m}, \gamma, u)\}$ of the map

$$\omega_k = \{\overline{h} \in [\overline{h}_1; \overline{h}_2], \overline{m} \in [\overline{m}_1; \overline{m}_2], \gamma \in [-\hat{\gamma}; \hat{\gamma}], u \in [u_1; u_2]\} \to$$
$$\to Z_\theta = \{q = Q(\overline{h}, \theta, \overline{m}, \gamma, u)\} \in [\omega_k]^A$$

belongs to $[A^{\omega\text{-hom}}]$, where

$$\sigma_\theta(\overline{h}, \overline{m}, \gamma, u) = \frac{B_1\cos^2\gamma}{\overline{m}} - \left(\frac{\cos^2\gamma\left(B_1^2\cos^2\gamma - B_2\right)}{\overline{m}^2} + 1\right)^{0,5},$$

$$B_1 = \frac{\left(\dfrac{V}{5\cdot 10^3}\dfrac{dV}{d\overline{h}} + g\right)\left(P + C_y^\alpha qS\right)^2}{8\cdot 10^3\,\overline{m}\,(0{,}5P + AqS)\left(g - \dfrac{V^2}{R + \overline{h}\cdot 5\cdot 10^3}\right)^2},$$

$$B_2 = \frac{\left(P - C_{x_0}qS\right)\left(P + C_y^\alpha qS\right)^2}{1{,}6\cdot 10^7\,(0{,}5P + AqS)\left(g - \dfrac{V^2}{R + \overline{h}\cdot 5\cdot 10^3}\right)^2}.$$

It should be noted that α_θ takes values in the interval $[10^{-3}; 10^{-5}]$ and critically depends on the lift-to-drag ratio, dynamic pressure, propulsion mode. Very small value of α_θ allows to identify the set EZ_θ^0 with the manifold Z_θ^0 and to carry out the approximate reduction of the system (1) having replaced its first differential equation with the functional equation $\theta = \arcsin\sigma_\theta(\overline{h}, \overline{m}, \gamma, u)$.

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