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Yu. A. Shkol'nikov, cand. phys.-math. sci. (Inst. Math. National Acad. Sci. Ukraine, Kiev)

HANDLE DECOMPOSITIONS OF SIMPLY-CONNECTED FIVE-MANIFOLDS. III*

We prove the existence of the exact handle decomposition of a simply-connected smooth or PL 5-manifold with a standard simply-connected boundary of signature 0, the triviality of a 5-dimensional h-cobordism with the ends of such type and the uniqueness, up to diffeomorphism (PL-isomorphism), of a smooth (or PL) h-cobordism between the given simply-connected 4-manifold and the corresponding standard manifold.

Доведено існування точного розкладу на ручки однозв'язного гладкого або кусково-лінійного п'ятивимірного многовида із стандартною однозв'язною границею сигнатури 0, тривіальність п'ятивимірного h-кобордизму з кінцями такого типу, а також єдиність гладкого (або кусково-лінійного h-кобордизму між заданим однозв'язним чотиривимірним многовидом і відповідним стандартним многовидом з точністю до диффеоморфізму (кусково-лінійного ізоморфіму).

A 1-connected 4-manifold is called standard if it is diffeomorphic to $kS^2 \underset{\sim}{\times} S^2 \#$ $\# lS^2 \times S^2$.

The principal results of this paper are Theorems 2–4 and Corollary 3.

1. Some technical facts. Consider a finitely generated free $\mathbb{Z}$-module F of rank r and a nondegenerate integral symmetric bilinear form Φ on F; $\Phi(x, y)$ will be denoted by $x \cdot y$.

An element $y \in F$ is called basic if it generates a free summand of F. It is obvious that $y \in F$ is basic iff it is indivisible, i.e., it follows from $y = \alpha x$ that $|\alpha| = 1$.

Statement 1. *Let* $\{y, z_2, \dots, z_r\}$ *be a basis of* F. *Then there exist a basic element* $x \in F$ *such that* $x \cdot y = 1, x \cdot z_i = 0,\ i = 2, \dots, r$.

To prove Statement 1, we define a homomorphism $\varphi : F \to \mathbb{Z}$ as follows: $\varphi(y) = 1$, $\varphi(z_i) = 0$, $i = 2, \dots, r$. Since the form Φ is nondegenerate, there is $x \in F$ such that $\varphi(z) = x \cdot z$ for every $z \in F$ [1]. Since $x \cdot y = 1$, we conclude that x is indivisible and, hence, basic.

Statement 2. *Let a set* $\{y_1, \dots, y_m\}$ *of independent basic elements of* F *with* $m \le [r/2]$ *and* $y_i \cdot y_j = 0$, $i, j = 1, \dots, m$, *be given. Then there exist an orthogonal decomposition* $U \oplus U^{\perp}$ *and a set* $\{x_1, \dots, x_m\} \subset U$ *such that* $\{x_1, y_1, \dots, x_m, y_m\}$ *is a basis of* U, *for which*

$$\Phi|_U = k\begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} \oplus l\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \ k + l = m. \tag{1}$$

If $r = 2m$, *then* $U^{\perp}$ *is trivial.*

Proof. Let us construct an arbitrary basis of F containing $\{y_1, \dots, y_m\}$. By Statement 1, there exist a basic element $x_1' \in F$ such that $x_1' \cdot y_1 = 1$ and $x_1' \cdot z = 0$ for each other element z of the basis. Then $x_1 = x_1' - [x_1'/2, x_1']\, y_1$ satisfies the same conditions as x_1' and $\alpha = x_1 \cdot x_1$ is either 0 or 1. The restriction of Φ to the submodule $\langle x_1, y_1 \rangle$ can be represented by the matrix

$$\begin{pmatrix} \alpha & 1 \\ 1 & 0 \end{pmatrix}.$$

Since it is nondegenerate on the submodule $\langle x_1, y_1 \rangle$, we may consider an orthogonal decomposition $F = \langle x_1, y_1 \rangle \oplus F'$. Since $y_1 \cdot y_i = 0$, $i = 2, \dots, r$, all elements

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$y_2, \ldots, y_r$ are in F'. Since Φ' is nondegenerate on F', we can continue the process for $y_2 \in F'$ and so on. Thus, we have the desired decomposition. If $r = 2m$, then $U = F$ and $U^{\perp} = 0$. Note that the basis $\{x_1, y_1, \ldots, x_m, y_m\}$ can be modified so that k be equal to 0 or 1. We do this as follows:

$$x_1' = x_1, \;\; y_1' = y_1 + \ldots + y_m, \;\; x_j' = x_j - x_1 + y_1, \;\; y_j' = y_j, \quad j = 2, \ldots, m.$$

The set $\{y_1', \ldots, y_m'\}$, as we can easily see, satisfies the same conditions as $\{y_1, \ldots \ldots, y_m\}$.

Consider a closed 1-connected 4-manifold M, which bounds a 1-connected 5-manifold W. Then $H_1(M) = H_3(M) = 0$, $H_2(M)$ is free of rank $r = 2m$ and $\sigma(M) = 0$. The natural embedding $i: M \to W$ induces a homomorphism $i_*: H_2(M) \to H_2(W)$.

Theorem 1 [2, 3]. *The following statements are valid*:

1) $\operatorname{Ker} i_*$ *is a free submodule of* $H_2(M)$ *of* rank $m = r/2$;

2) *for any* y, z *from* $\ker i_*$, *it is true that* $y \cdot z = 0$;

3) *if* $y \in \ker i_*$, *then, for any* $x \in H_2(M)$ *with* $x \cdot y \neq 0$, *it is true that* $i_*(x) \neq 0$.

Corollary 1. *There exists a basis* $\{x_1, y_1, \ldots, x_m, y_m\}$ *of* $H_2(M)$, *in which the intersection form* $Q(M)$ *is represented by a matrix of form* (1) *with* $k = 0$ *or* $k = 1$, $\ker i_* = \langle y_1, \ldots, y_m \rangle$, *and* $i_*|_{G_x}$ *is an embedding of* $G_x = \langle x_1, \ldots, x_m \rangle$ *into* $H_2(W)$ *such that* $i_*(G_x)$ *is a free summand of* $H_2(W)$.

Proof. Let $\{y_1, \ldots, y_m\}$ be a basis of $\ker i_*$. By statement 2) of Theorem 1, we have $y_i \cdot y_j = 0$, $i, j = 1, \ldots, m$. By Statement 2, there exists a basis $\{x_1, y_1, \ldots, x_m, y_m\}$, in which $Q(M)$ is represented by a matrix of form (1). Let us modify this basis by analogy with the proof of Statement 2 to set k to be equal to 0 or 1. Since $\{y_1', \ldots, y_m'\}$ is a basis of $\ker i_*$, we may consider the basis $\{x_j, y_j, j = 1, \ldots, m\}$ replaced with $\{x_j', y_j';\, j = 1, \ldots, m\}$. Then $H_2(M) = G_x \oplus G_y$ (not an orthogonal decomposition), where $G_x = \langle x_1, \ldots, x_m\}$ and $G_y = \langle y_1, \ldots, y_m \rangle$ and $i_*|_{G_x}$ is an embedding of G_x into $H_2(W)$. Since M is 1-connected, the end of the exact homological sequence of the pair (W, M) is $\ldots \to H_2(M) \xrightarrow{i_*} H_2(W) \xrightarrow{j_*} H_2(W, M) \to 0$.

Consider $T = \operatorname{tors} H_2(W)$. By Poincaré duality, $T \simeq \operatorname{tors} H_2(W, M)$. Therefore,

$$H_2(W) \simeq FrH_2(W) \oplus T, \qquad H_2(W, M) \simeq FrH_2(W, M) \oplus T.$$

Since j_* is an epimorphism and $\ker j_* = i_*(G_x) \subset FrH_2(W)$, $i_*|_T$ is an isomorphism and $i_*|_{Fr(H_2(W))}$ is an epimorphism of $FrH_2(W)$ onto $FrH_2(W, M)$. Thus, both the modules $i_*(G_x)$ and $FrH_2(W)/i_*(G_x)$ are free and we have $FrH_2(W) \simeq\simeq i_*(G_x) \oplus FrH_2(W, M)$, which completes the proof.

2. Simply connected 5-manifolds with a standard boundary. We prove here the following theorem:

Theorem 2. *Any 1-connected 5-manifold with a standard boundary admits an exact handle decomposition with geometric incidence indices.*

The proof will be performed according to the following scheme: Construct a closed 1-connected 5-manifold W from a given 5-manifold V by gluing a standard 5-manifold V' with the same boundary along $M = \partial V$. Consider a Barden handle decomposition of W. Isotope V' into a 2-skeleton of W ambiently in W. Remove V' from W to obtain a 5-manifold diffeomorphic to V, preserving the exact handle decomposition.

So, consider a 1-connected 5-manifold V with a standard boundary M. Since, for $M = S^4$, there is nothing to prove, suppose in the sequel that $M \neq S^4$, i.e., $2m = rkH_2(M) > 0$. Let $i_{1*}: H_2(M) \to H_2(V)$ be a homomorphism induced by a natural embedding $i_1 : M \to V$. By Corollary 1, there exists a basis $\{x_1, y_1, \dots, x_m, y_m\}$ of $H_2(M)$ such that $i_{1*}(y_j) = 0$, $j = 1, \dots, m$, and $i_{1*}(G_x)$ is a direct summand of $H_2(V)$, where $G_x = \langle x_1, \dots, x_m \rangle$.

The standard 4-manifold M determines the standard 5-manifold V' with $\partial V' = M$. Consider an exact handle decomposition $V' = h^0 \cup h_1^2 \cup \dots \cup h_m^2$ and the induced canonical handle decomposition of M with a canonical basis $\{a_1, b_1, \dots, a_m, b_m\}$. Let $i'_{2*}: H_2(M) \to H_2(V')$ be a homomorphism induced by the natural embedding $i'_2 : M \to V'$. Then $i'_{2*}(b_j) = 0$, $j = 1, \dots, m$, and $i'_{2*}(G_a) = H_2(V')$, where $G_a = \langle a_1, \dots, a_m \rangle$. Consider an automorphism φ_* of $H_2(M)$, which acts on the basis $\{a_1, b_1, \dots, a_m, b_m\}$ as follows: $\varphi_*(a_j) = x_j$, $\varphi_*(b_j) = y_j$, $j = 1, \dots, m$. By the Wall theorem [4], the homomorphism φ_* can be realized by a diffeomorphism φ of M. Consider a closed 1-connected 5-manifold $W = V \cup_\varphi (-V')$. Let $n = rkH_3(V)$.

Lemma 1. *$H_3(W)$ is a free group of rank $m + n$.*

Proof. Since the embedding of pairs $(V, M) \to (W, V')$ is cutting out and $W = V \cup (-V')$ with $\partial V' = M$, one can write out the Mayer–Vietoris sequence for $W = V \cup (-V')$. Since $H_3(M) = 0$ and $H_3(V') = 0$, this sequence is as follows:

$$0 \to H_3(V) \to H_3(W) \to H_2(M) \xrightarrow{i_*} H_2(V) \oplus H_2(V') \to H_2(W) \to 0. \qquad (2)$$

The homomorphism i_* in sequence (2) is defined as a pair $(i_{1*}, -i_{2*})$, where $i_{2*} = i'_{2*}\varphi_*^{-1}$. From the definition of the basis $\{x_j, y_j;\ j = 1, \dots, m\}$ of $H_2(M)$, we see that $H_2(M) = G_x \oplus G_y$ and $\ker i_{1*} = G_y$. Since

$$\varphi_*^{-1}\{x_j, y_j;\ j = 1, \dots, m\} = \{a_j, b_j;\ j = 1, \dots, m\}$$

and $\operatorname{Ker} i'_{2*} = \langle b_1, \dots, b_m \rangle$, we see also that $\ker i_{2*} = G_y$. As a result, we have a short exact sequence obtained from (2): $0 \to H_3(V) \to H_3(W) \to G_y \to 0$, where $H_3(V)$ is a free group of rank n and G_y is a free group of rank m. Thus, $H_3(W) = H_3(V) \oplus G_y$ is a free group of rank $n + m$ and the lemma is proved.

Since V' is given with an exact handle decomposition $V' = h^0 \cup h_1^2 \cup \dots \cup h_m^2$, we have $W = V \cup_\varphi (-V') = V \cup \overline{h_1^3} \cup \dots \cup \overline{h_m^3} \cup \overline{h^5}$, where $\overline{h_j^k}(W) = h_j^{5-k}(V')$, $j = 1, \dots, m$, $k = 3, 5$. By Lemma 1, each $\overline{h_i^3}$ determines a free generator of $H_3(W)$. Consider the dual decomposition $W \simeq -W = V' \cup_\varphi (-V) = h^0 \cup h_1^2 \cup \dots \cup h_m^2 \cup_\varphi \cup_\varphi (-V)$, where each 2-handle h_j^2, $j = 1, \dots, m$, determines a free generator of $H_2(W)$. This defines naturally an embedding $\varphi': V' \to W$ with $\overline{W \setminus \psi'(V)} \simeq V$. The induced homomorphism $\psi'_*: H_2(V') \to H_2(W)$ is an embedding, which maps isomorphically $H_2(V')$ with a basis $\{u'_1, \dots, u'_m\}$ onto a summand of $FrH_2(W)$. For $M = kS^2 \underset{\sim}{\times} S^2 \# (m-k) S^2 \times S^2$ with $k = 0, 1$, we can prove the following lemma:

Lemma 2. *If $k = 1$, then there exists a free generator $u \in H_2(W)$ with $w^2(u) \neq 0$. If $k = 0$, then there exist m free generators $\{u_1, \ldots, u_m\}$ of $H_2(W)$ with $w^2(u_j) = 0$, $j = 1, \ldots, m$.*

Proof. If $k = 1$, then, in the exact handle decomposition of V', there is a 2-handle attached along the embedding of its α-tube, which corresponds to the element $1 \in \pi_1(SO_3) \simeq \mathbb{Z}_2 = \{0, 1\}$. The core of this handle determines a cycle $u' \in \in H_2(V')$, which is realized by a 2-sphere $\tilde{u}'$ with a nontrivial normal bundle in V'. Then u' is a free generator of $H_2(V')$ with $w^2(u') \neq 0$. The embedding $\psi' \colon V' \to \to W$ defined above maps $\tilde{u}'$ into 2-sphere $\tilde{u} \subset W$, which realizes a free generator u of $H_2(W)$. The tubular neighborhood of $\tilde{u}$ can be chosen to be lying in $\psi'(V')$. Therefore, this neighborhood also is a nontrivial normal bundle in W and $w^2(\tilde{u}) \neq 0$. In the case where $k = 0$, the proof is similar and can be omitted.

For a closed 1-connected 5-manifold W, consider a minimal w^2-b-basis of $H_2(W)$ [5]. This basis can be chosen so that it contains all free generators $\{u_j = \psi'_*(u'_j) \colon j = 1, \ldots, m\}$ of $H_2(W)$. The Barden decomposition theorem [5] determines an exact handle decomposition of W corresponding to a given minimal w^2-b-basis. By Lemma 1, there are $m + n \geq m$ summands of type M_∞ or X_∞ in this decomposition. By Lemma 2, for $k = 0$, the number of summands M_∞ is not less than m and, for $k = 1$, there are the only summand X_∞ and not less than $m - 1$ summands M_∞. Thus, $W = W' \# W''$, where W' and W'' admit exact handle decompositions and W' is $X_\infty \# (m-1)M_\infty$ if $k = 1$ (i.e., $M = S^2 \underset{\sim}{\times} S^2 \# (m-1)S^2 \times S^2$) or mM_∞ if $k = 0$ (i.e., $M = mS^2 \times S^2$). It is easy to see that $W' = V'_1 \cup_M (-V'_2)$, where $V'_1 \simeq \simeq V'_2 \simeq V'$ and V'_2 is glued to V'_1 along a diffeomorphism of M. W' admits an exact handle decomposition with the 2-skeleton V'_1 and dual 2-skeleton V'_2. $H_2(V'_2) = = \langle u_1, \ldots, u_m \rangle$, where $u_j = \psi'_*(u'_j)$; $j = 1, \ldots, m$, and $\{u'_i, i = 1, \ldots, m\}$ are free generators of $H_2(V')$ determined by the exact handle decomposition of V'.

Define an embedding $\psi'' \colon V' \to W$ as follows: $\psi''(V') = V'_1 \subset W' \subset W$. It induces an embedding $\psi''_* \colon H_2(V') \to H_2(W)$ such that $\psi''_*(H_2(V')) = H_2(V'_1)$. The manifold W and, hence,

$$\overline{W \setminus \psi''(V')} = \overline{W \setminus V'_1} = \overline{(W'' \# W') \setminus V'_1} = V'_2 \natural \overline{W'' \setminus D^5} \simeq V' \natural \overline{W'' \setminus D^5}$$

admits an exact handle decomposition with geometric incidence indices [6]. If we find a diffeomorphism θ of W such that $\psi'' = \theta\psi'$, the proof of Theorem 2 will be completed, because $V \simeq \overline{W \setminus \psi'(V')} \simeq \overline{W \setminus \psi''(V)}$.

Each of the manifolds V', V'_1 can be regarded as a tubular neighborhood in W of a boquet of m 2-spheres. By the Haefliger theorem [7], every two 2-spheres embedded into a closed 1-connected 5-manifold W are isotopic in W iff they are homotopic in W. Since

$$\psi'_*(H_2(V')) = \psi''_*(H_2(V')) = H_2(V'_1) = \langle u_1, \ldots, u_m \rangle$$

and W is 1-connected, the boquets of m 2-spheres for V' and V'_1 are homotopic and, therefore, isotopic in W. Then V' and V'_1 are isotopic in W. Each isotopy of W in codimension ≥ 3 can be made ambient [8]. This provides the desired diffeomorphism θ of W. Thus, the proof of Theorem 2 is completed.

Corollary 2. *Each 1-connected 5-manifold V with a standard boundary M*

is diffeomorphic to the band connected sum of a closed 1*-connected* 5*-manifold (without the standard* D^5*) and a standard* 5*-manifold. The second summand is determined by the boundary* M *and the first one is determined by the linking form*

$$b: H_2(V)\times H_2(V,M)\to \mathbb{Q}/\mathbb{Z}.$$

Theorem 3. *Every* 1*-connected* 5*-dimensional cobordism* (V, M_0, M_1) *with standard ends admits an exact handle decomposition with geometric incidence indices.*

Proof. The natural embedding $i_1: M_1 \to V$ induces the homomorphism i_{1*}: $H_2(M_1)\to H_2(V, M_0)$. First, we prove that there is the decomposition $H_2(M_1) = G_x \oplus G_y$, the same as in the previous case, with

$$G_x = \langle x_1, \dots, x_{m_1}\rangle \quad\text{and}\quad G_y = \langle y_1, \dots, y_{m_1}\rangle$$

such that $i_{1*}(G_y)=0$ and the intersection form $Q(M_1)$ in the basis $\langle x_1, y_1, \dots, x_{m_1}, y_{m_1}\rangle$ is

$$k\begin{pmatrix}1 & 1\\ 1 & 0\end{pmatrix}\oplus l\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix},$$

where $k+l=m_1$ and k is either 0 or 1. To show this, consider a manifold $\hat{V} = V_0 \cup_{id_{M_0}} V$, where V_0 is a standard 5-manifold with the boundary M_0. Using Corollary 1, we choose G_x, G_y for $H_2(M_1)$ and a homomorphism $\hat{i}_*: H_2(M_1)\to H_2(\hat{V})$ induced by the embedding $\hat{i}: M_1\to \hat{V}$. Since the embedding $(V, M_0)\to (\hat{V}, V_0)$ is cutting out, there is an isomorphism $\chi: H_2(V, M_0)\to H_2(\hat{V}, V_0)$. We have $i_{1*}=\chi^{-1}j\hat{i}_*$, where $j: H_2(\hat{V})\to H_2(\hat{V}, V_0)$ is a homomorphism from the exact homological sequence of the pair $(\hat{V}, V_0)$. Therefore, we have $i_{1*}(G_y) = 0$. Thus, we can consider a manifold $\hat{W} = V\cup_{\varphi_1}(-V_1)$, where V_1 is a standard 5-manifold with the boundary M_1 and the diffeomorphism φ_1 of M_1 is constructed in the same way as in the proof of Theorem 2. We have the embedding $\psi_1: V_1\to \hat{W}$ determined by the construction of $\hat{W}$. By using M_1 instead of M and the pair $(\hat{W}, M_0)$ instead of W in the relative Mayer–Vietoris sequence (2), we can prove Lemma 1 for $H_3(\hat{W}, M_0)$. Assuming that $-\hat{W}\simeq\hat{W}$, we see that $\psi_1(V_1)$ determines a free summand of $H_2(-\hat{W})$. Since $-\hat{W}$ is a 1-connected 5-manifold with the standard boundary M_0, by Theorem 2, it admits an exact handle decomposition. We choose a canonical embedding $\psi_1': V_1\to\hat{W}$ such that $\psi_1'(V_1)$ determines the same free summand of $H_2(-\hat{W})$ as that determined by $\psi_1(V_1)$. By using the Haefliger theorem in the same way as in the proof of Theorem 2, we can prove that imbeddings ψ_1 and ψ_1' are ambiently isotopic in $\hat{W}$. So, we can remove V_1 from $\hat{W}$ preserving the exact handle decomposition. Thus, the proof is completed.

Corollary 3. *Every* 5*-dimensional* h*-cobordism with standard ends is trivial.*
Corollary 3 immediately follows from Theorem 3.

This gives a series of smooth h-cobordisms between 1-connected smooth 4-manifolds satisfying the h-cobordism conjecture. In general, as was proved by Donaldson

[9], this conjecture for smooth 1-connected 4-manifolds is not true.

3. An example of applications.

Lemma 3 [8]. *Every* 1-*connected cobordism* (W, M_0, M_1) *of dimension* $n \geq 5$ *is invertible. The inverse cobordism is* $(-W, M_1, M_0)$, *i.e.*, $(-W) \cup_{id_{M_0}} W \simeq M_1 \times \times [0, 1]$.

Let M be an arbitrary 1-connected 4-manifold of signature 0. There is a standard 4-manifold M' with the same intersection form. By the Wall theorem [10], there is an h-cobordism (W, M', M).

Theorem 4. *The* h-*cobordism* (W, M', M) *between a* 1-*connected* 4-*manifold* M *of signature* 0 *and the corresponding standard* 4-*manifold* M' *is unique up to a diffeomorphism.*

Proof. Let (W_1, M', M) and (W_2, M', M) be h-cobordisms between M and M'. Consider the h-cobordism $V = W_2 \cup_{id_M} (-W_1) \cup_{id_{M'}} W_1$. By Lemma 3, $(-W_1) \cup_{id_{M'}} W_1 \simeq M \times [0, 1]$, therefore, we have $V \simeq W_2$. On the other hand, $V' = W_2 \cup_{id_M} (-W_1)$ is an h-cobordism with standard ends diffeomorphic to M'. By using Corollary 3, we obtain $V' \simeq M' \times [0, 1]$. Hence, $V \simeq W_1$ and, thus, $W_2 \simeq W_1$.

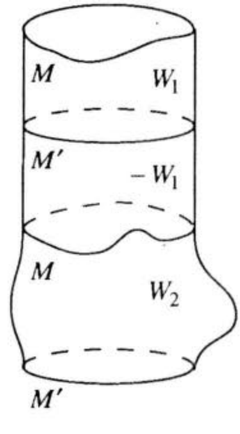

Fig. 1.

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