DOI: 10.37863/umzh.v73i8.588

UDC 512.5

S. Maheshwari (KIET Group of Institutions Delhi-NCR, Ghaziabad, India),
R. K. Sharma (Indian Inst. Technology Delhi, India)

A NOTE ON UNITS IN $\mathbb{F}_q SL(2, \mathbb{Z}_3)$
ПОВІДОМЛЕННЯ ПРО ОДИНИЦІ У $\mathbb{F}_q SL(2, \mathbb{Z}_3)$

Let R be a ring, and $SL(2, R)$ be the special linear group of 2×2 matrices with determinant 1 over R. We obtain the Wedderburn decomposition of $\dfrac{\mathbb{F}_q SL(2,\mathbb{Z}_3)}{J(\mathbb{F}_q SL(2,\mathbb{Z}_3))}$ and show that $1 + J(\mathbb{F}_q SL(2, \mathbb{Z}_3))$ is a non-Abelian group, where $\mathbb{F}_q$ is a finite field with $q = p^k$ elements of characteristic 2 and 3.

Нехай R — кільце, а $SL(2, R)$ — спеціальна лінійна група (2×2)-матриць над R з детермінантом 1. Отримано декомпозицію Веддербурна для $\dfrac{\mathbb{F}_q SL(2,\mathbb{Z}_3)}{J(\mathbb{F}_q SL(2,\mathbb{Z}_3))}$ і показано, що $1 + J(\mathbb{F}_q SL(2, \mathbb{Z}_3))$ є неабелевою групою, де $\mathbb{F}_q$ — скінченне поле з $q = p^k$ елементами та характеристикою 2 або 3.

1. Introduction. Let $\mathbb{F}G$ be a group algebra of a group G over a field $\mathbb{F}$ and $\mathcal{U}(\mathbb{F}G)$ denotes the unit group of $\mathbb{F}G$. It is a classical problem to study units and their properties in group ring theory. The case, when G is a finite Abelian group and characteristic of $\mathbb{F}$ does not divide order of G, the structure of $\mathbb{F}G$ is studied by Perlis and Walker in [18]. If characteristic of $\mathbb{F}$ divides order of G, the structure of $\mathcal{U}(\mathbb{F}G)$ is studied by Makhijani [12, p. 10 – 12]. Hurley introduced a correspondence between group ring and certain ring of matrices [6]. As an application of units of a group ring, Hurley gave a method to construct convolutional codes from units in group rings [7].

Many authors have found the unit group of group algebra $\mathbb{F}_q G$, where G is a finite non-Abelian group and $\mathbb{F}_q$ denotes a finite field with $q = p^k$ elements. Monaghan [17] has found $\mathcal{U}(\mathbb{F}_q G)$, for some non-Abelian groups G of order 24 over a field of characteristic 3. In this paper, we have obtained the Wedderburn decomposition of $\mathbb{F}_q G / J(\mathbb{F}_q G)$ for $G = SL(2, \mathbb{Z}_3)$ over a finite field of characteristic 2 and 3. When characteristic of $\mathbb{F}_q$ does not divide order of G, then the structure of $\mathcal{U}(\mathbb{F}_q G)$ for $G = SL(2, \mathbb{Z}_3)$ has been obtained by Maheshwari et al. [11]. Here we are providing some literature survey for the same. For dihedral group, the structure of the unit group $\mathcal{U}(\mathbb{F}_q G)$ has been discussed in [1, 5, 13, 14]. Gildea et al. [4] and Sharma et al. [19] have given the structure of the unit group $\mathcal{U}(\mathbb{F}_q G)$, where G is alternating group A_4. The unit group of group algebras of some non-Abelian groups of small orders have been studied in [9, 20 – 22].

2. Preliminaries. We are summarizing some results that provide useful information about the decomposition of $A/J(A)$, where $A = \mathbb{F}_q G$ and $J(A)$ be its Jacobson radical. For basic definitions and results, see [16]. We briefly introduce some definitions and notations those will be needed subsequently.

Let G be a finite group and $\mathbb{F}_q$ be a finite field with characteristic p. We have some definitions due to Ferraz.

Definition 2.1. *An element $g \in G$ is said to be p-regular if p does not divide order of g. Let l be the $l.c.m.$ of the orders of the p-regular elements of G, η be a primitive lth root of unity over $\mathbb{F}_q$. Then $T_{G,\mathbb{F}_q}$ be the multiplicative group consisting of those integers t, taken modulo s, for which $\zeta \mapsto \eta^t$ defines an automorphism of $\mathbb{F}_q(\eta)$ over $\mathbb{F}_q$.*

Note that if q is a power of a prime such that $(q,l)=1$ and $d=ord_l\ (q)$ is the multiplicative order of q modulo l, then

$$T_{G,F_q}=\left\{1,q,\ldots,q^{d-1}\right\} \bmod s$$

and $F_q(\zeta)\cong F_{q^d}$ follow using [10] (Theorem 2.21).

Definition 2.2. *If $g\in G$ is a p-regular element, then the sum of all conjugates of $g\in G$ is denoted by γ_g and the cyclotomic $\mathbb{F}_q$-class of g is defined to be the set*

$$S_{\mathbb{F}_q}(\gamma_g)=\left\{\gamma_{g^t}\ \mid t\in T_{G,\mathbb{F}_q}\right\}.$$

Proposition 2.1 ([3], Theorem 1.2)**.** *The number of simple components of $\mathbb{F}_qG/J(\mathbb{F}_qG)$ is equal to the number of cyclotomic $\mathbb{F}_q$-classes in G.*

Theorem 2.1 ([3], Theorem 1.3)**.** *Suppose that $Gal(\mathbb{F}_q(\zeta)/\mathbb{F}_q)$ is cyclic. Let w be the number of cyclotomic $\mathbb{F}_q$-classes in G. If $K_1,K_2,\ldots,K_w$ are the simple components of $Z(\mathbb{F}_qG/J(\mathbb{F}_qG))$ and $S_1,S_2,\ldots,S_w$ are the cyclotomic $\mathbb{F}_q$-classes of G, then with a suitable reordering of indices*

$$\mid Si\mid=[K_i:\mathbb{F}_q].$$

Proposition 2.2 ([8, p. 31], Proposition 6.24)**.** *Let $f:R\to S$ be a surjective homomorphism of rings. Then $f(J(R))\subseteq J(S)$ with equality if* $\ker, f\subseteq J(R)$.

Proposition 2.3 ([8, p. 108], Proposition 1.7)**.** *Let G be a finite group and $\mathbb{F}_q$ be a finite field. Then $G\cap(1+J(\mathbb{F}_qG))=O_pG$, where O_pG denotes the maximal normal p-subgroup of G.*

Theorem 2.2 ([13], Lemma 3.2)**.** *Let $\mathbb{F}$ be a perfect field, G be a finite group and $J(\mathbb{F}G)$ be the Jacobson radical of $\mathbb{F}G$. Then*

$$\mathcal{U}(\mathbb{F}G)=(1+J(\mathbb{F}G))\rtimes\mathcal{U}(\mathbb{F}G/J(\mathbb{F}G)).$$

Lemma 2.1 ([15], Lemma 3.1)**.** *Let $\mathfrak{B}_1$, $\mathfrak{B}_2$ be two finite dimensional F-algebras such that $\mathfrak{B}_2$ is semisimple. If $f:\mathfrak{B}_1\to\mathfrak{B}_2$ is an onto homomorphism of F-algebras, then there exists a semisimple F-algebra ℓ such that*

$$\mathfrak{B}_1/J(\mathfrak{B}_1)\cong\ell\oplus\mathfrak{B}_2.$$

Theorem 2.3 ([2, p. 146], Theorem 7.9(i))**.** *Let q be a power of a prime. If E is a finite field extension of $\mathbb{F}_q$, then*

$$E\otimes_{\mathbb{F}_q}(\mathbb{F}_qG/J(\mathbb{F}_qG))\cong\left(E\otimes_{\mathbb{F}_q}\mathbb{F}_qG\right)/\left(E\otimes_{\mathbb{F}_q}J(\mathbb{F}_qG)\right),$$

$$J\left(E\otimes_{\mathbb{F}_q}\mathbb{F}_qG\right)=E\otimes_{\mathbb{F}_q}J(\mathbb{F}_qG).$$

Theorem 2.4 ([8, p. 110], Proposition 1.9)**.** *Let N be a normal subgroup of G such that G/N is p-solvable. If $|G/N|=np^a$, where $(n,p)=1$, then*

$$J(\mathbb{F}_qG)^{p^a}\subseteq\mathbb{F}_qGJ(\mathbb{F}_qN)\subseteq J(\mathbb{F}_qG).$$

In particular, if G is p-solvable of order np^a, where $(n,p)=1$, then $J(\mathbb{F}_qG)^{p^a}=0$.

***Corollary* 2.1** ([15], Corollary 3.3)**.** *Let q be a power of prime. Then, for any $k,m\in\mathbb{N}$,*

$$\mathbb{F}_{q^k}\otimes_{\mathbb{F}_q}\mathbb{F}_{q^m}\cong\left(\mathbb{F}_{q^{l_{m,k}}}\right)^{(m,k)}$$

as $\mathbb{F}_{q^k}$-algebras, where $l_{m,k}=l.c.m.(m,k)$ and $(m,k)=g.c.d.(m,k)$.

Throughout this paper, $\mathbb{F}_q$ is a field of characteristic p, where q is a power of positive prime integer. The conjugacy class of $g \in G$ is denoted by $[g]$.

We can see that G has 7 conjugacy classes as follows:

Representative	Elements in the class	Order of element
$[x]$	$x, (yx)^4, (xy)^4, y^{-1}xy$	3
$[x^{-1}]$	$x^{-1}, (yx)^2, (xy)^2, xyx$	3
$[y]$	$y, y^{-1}, x^2yx, xyx^2, xy^{-1}x^2, x^2y^{-1}x$	4
$[y^2]$	y^2	2
$[xy]$	xy, yx, x^2yx^2, xy^2	6
$[(xy)^{-1}]$	$(xy)^{-1}, x^2y^{-1}, xy^{-1}x, x^2y^2$	6

Theorem 2.5. *Let $p=2$, $q=p^k$ and $G=SL(2,\mathbb{Z}_3)$. Then*

$$\frac{\mathbb{F}_qG}{J(\mathbb{F}_qG)} \cong \mathbb{F}_q \oplus \mathbb{F}_{q^2}.$$

Proof. Suppose k is odd. Hence there exists an element of order 3, say $\eta \in \mathbb{F}_{q^2} \setminus \mathbb{F}_q$. We define the $\mathbb{F}_q$-algebra homomorphism

$$\theta : \mathbb{F}_qG \longrightarrow \mathbb{F}_q \oplus \mathbb{F}_{q^2}$$

by the assignment

$$x \mapsto (1,\eta), \quad y \mapsto (1,1).$$

By using Table 1, we see that θ is onto.

Table 1. Ontoness of θ

Basis element	Pre-image under θ
$(1,0)$	$x^{-1}+x+y$
$(0,1)$	$x+x^{-1}$
$(0,\eta)$	$x^{-1}+y$

$|S_{\mathbb{F}_q}(\gamma_x)|=2$, now by using Theorem 2.1 and Lemma 2.1, we get

$$\frac{\mathbb{F}_qG}{J(\mathbb{F}_qG)} \cong \mathbb{F}_q \oplus \mathbb{F}_{q^2}.$$

Now suppose k is even, then we have

$$\frac{\mathbb{F}_qG}{J(\mathbb{F}_qG)} \cong \mathbb{F}_q \otimes_{\mathbb{F}_2} \frac{\mathbb{F}_2G}{J(\mathbb{F}_2G)} \cong \mathbb{F}_q \otimes_{\mathbb{F}_2} (\mathbb{F}_2 \oplus \mathbb{F}_4) \cong \mathbb{F}_q \oplus \mathbb{F}_{q^2}. \tag{2.1}$$

Theorem 2.5 is proved.

***Corollary* 2.2.** *The structure of $\mathcal{U}\left(\frac{\mathbb{F}_qG}{J(\mathbb{F}_qG)}\right)$ is given by*

$$\mathcal{U}\left(\frac{\mathbb{F}_qG}{J(\mathbb{F}_qG)}\right) \cong C_{q-1} \oplus C_{q^2-1}$$

and $1+J(\mathbb{F}_qG)$ is a non-Abelian group of exponent 8.

Proof. We know that G has unique 2-Sylow subgroup of order 8. By using Proposition 2.3, we have $G \cap 1 + J(\mathbb{F}_qG) = O_pG$. Suppose that $X = xy + x$ and $Y = y^2 + y$, then $X, Y \in J(\mathbb{F}_qG)$. We see that $XY \neq YX$, this proves that $1 + J(\mathbb{F}_qG)$ is a non-Abelian subgroup of $\mathcal{U}(\mathbb{F}_qG)$. Now by using Theorem 2.4, we have $(1 + J(\mathbb{F}_qG))^8 = 1$. Since $X^4 \neq 0$, it implies that $(1 + J(\mathbb{F}_qG))$ has exponent 8.

Theorem 2.6. *Let* $p = 3$, $q = p^k$ *and* $G = SL(2, \mathbb{Z}_3)$. *Then*

$$\frac{\mathbb{F}_qG}{J(\mathbb{F}_qG)} \cong \mathbb{F}_q \oplus \mathcal{M}(2, \mathbb{F}_q) \oplus \mathcal{M}(3, \mathbb{F}_q).$$

Proof. Suppose k is odd. We define the $\mathbb{F}_q$-algebra homomorphism

$$\theta' : \mathbb{F}_qG \longrightarrow \mathbb{F}_q \oplus \mathcal{M}(2, \mathbb{F}_q) \oplus \mathcal{M}(3, \mathbb{F}_q)$$

by an assignment

$$x \mapsto \left(1, \begin{bmatrix} -1 & -1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 1 & -1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}\right)$$

and

$$y \mapsto \left(1, \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 1 & -1 & 0 \\ 0 & -1 & 0 \\ -1 & -1 & -1 \end{bmatrix}\right).$$

Since

$$\left|S_{\mathbb{F}_q}(\gamma_y)\right| = \left|S_{\mathbb{F}_q}\left(\gamma_{y^2}\right)\right| = \left|S_{\mathbb{F}_q}(\gamma_1)\right| = 1.$$

Now by using Table 2, Theorem 2.1 and Lemma 2.1, we see that

$$\frac{\mathbb{F}_qG}{J(\mathbb{F}_qG)} \cong \mathbb{F}_q \oplus \mathcal{M}(2, \mathbb{F}_q) \oplus \mathcal{M}(3, \mathbb{F}_q).$$

Now suppose k is even, then apply the same argument as in equation (2.1). We get

$$\frac{\mathbb{F}_qG}{J(\mathbb{F}_qG)} \cong \mathbb{F}_q \oplus \mathcal{M}(2, \mathbb{F}_q) \oplus \mathcal{M}(3, \mathbb{F}_q).$$

Theorem 2.6 is proved.

***Corollary* 2.3.** *The structure of* $\mathcal{U}\left(\frac{\mathbb{F}_qG}{J(\mathbb{F}_qG)}\right)$ *is given by*

$$\mathcal{U}\left(\frac{\mathbb{F}_qG}{J(\mathbb{F}_qG)}\right) \cong C_{q-1} \oplus GL(2, \mathbb{F}_q) \oplus GL(3, \mathbb{F}_q)$$

and $1 + J(\mathbb{F}_qG)$ *is a non-Abelian group of exponent* 3.

Proof. We can directly obtain the structure of $\mathcal{U}\left(\frac{\mathbb{F}_qG}{J(\mathbb{F}_qG)}\right)$, by Theorem 2.6. Observe that G is p-solvable, so we have $(1+J(\mathbb{F}_qG))^3 = 1$ is a group of exponent 3 as a consequence of Theorem 2.4. Let $X = -x^{-1}+y-y^{-1}-yxy+y^2x^{-1}+x^{-1}y^{-1}x^{-1}$ and $Y = -1+y^2+xy-yx^{-1}-xy^{-1}+y^{-1}x^{-1}$, we can see that $X, Y \in J(\mathbb{F}_qG)$. Further, $XY \neq YX$, hence $1 + J(\mathbb{F}_qG)$ is a non-Abelian group.

Table 2. Ontoness of θ'

Basis element	Pre-image under θ'
$1, O, O$	$-x^{-1}-y+y^2-y^{-1}-x^{-1}y+x^{-1}y^2+x^{-1}y^{-1}-yx^{-1}+y^{-1}x^{-1}-xyx-$ $-xyx^{-1}+xy^{-1}x-xy^{-1}x^{-1}-x^{-1}yx-x^{-1}y^{-1}x$
$O, E^2_{1,1}, O$	$-x+x^{-1}-y+y^{-1}+xy-yx-xy^2+x^{-1}y+x^{-1}y^2+x^{-1}y^{-1}-y^{-1}x-$ $-y^{-1}x^{-1}-xy^{-1}x-x^{-1}yx+x^{-1}y^{-1}x^{-1}+x^{-1}y^{-1}x$
$O, E^2_{1,2}, O$	$1+x^{-1}+x^{-1}y^{-1}x+xy^2+x^{-1}y+xy-y^{-1}-yx-x^{-1}yx^{-1}+y^2+x^{-1}y^{-1}x^{-1}+$ $+y^{-1}x-xyx^{-1}+y^{-1}x^{-1}+x^{-1}y^{-1}x-x+xy^{-1}x-xy^{-1}$
$O, E^2_{2,1}, O$	$-yx+y^{-1}x^2-y^2-xy^{-1}x^{-1}-x-x^{-1}y^{-1}+xy^{-1}x+xyx-xy^2-x^{-1}+$ $+yx^{-1}+x^{-1}yx^{-1}+1+xy-y^{-1}x+xyx^{-1}$
$O, E^2_{2,2}, O$	$-1+x-y^{-1}-yx-y^2-xy^2-x^{-1}-y^{-1}-x^{-1}y+y^{-1}x-y^{-1}x^{-1}-xy^{-1}x-$ $-xyx^{-1}-xy^{-1}x^{-1}+x^{-1}y^{-1}x+x^{-1}yx^{-1}-x^{-1}y^{-1}x^{-1}$
$O, O, E^3_{1,1}$	$-1-x+x^{-1}+y+y^2-y^{-1}+xy-xy^{-1}-x^{-1}y^2-x^{-1}y^{-1}-yx-yx^{-1}+$ $+y^{-1}x+y^{-1}x^{-1}-xy^{-1}x^{-1}-x^{-1}yx^{-1}+x^{-1}y^{-1}x^{-1}$
$O, O, E^3_{1,2}$	$-1-x-x^{-1}+y+y^2-y^{-1}-xy-xy^{-1}-x^{-1}y^2-x^{-1}y^{-1}+yx+yx^{-1}-y^{-1}x-$ $-y^{-1}x^{-1}-xyx-xyx^{-1}+xy^{-1}x-x^{-1}yx+x^{-1}yx^{-1}-x^{-1}y^{-1}x-x^{-1}y^{-1}x^{-1}$
$O, O, E^3_{1,3}$	$1+x^{-1}-y+y^2+y^{-1}-xy^2-x^{-1}y+x^{-1}y^{-1}+y^{-1}x-y^{-1}x^{-1}-xyx-$ $-xyx^{-1}-xy^{-1}x-xy^{-1}x^{-1}-x^{-1}yx+x^{-1}yx^{-1}-x^{-1}y^{-1}x^{-1}$
$O, O, E^3_{2,1}$	$1-x+x^{-1}+y-y^2+y^{-1}+xy+xy^2+xy^{-1}-x^{-1}y-x^{-1}y^2+yx^{-1}-$ $-y^{-1}x+xyx-xyx^{-1}-xy^{-1}x^{-1}-x^{-1}y^{-1}x+x^{-1}yx+x^{-1}y^{-1}x^{-1}$
$O, O, E^3_{2,2}$	$-1+x^{-1}-y-y^2+xy^2-xy^{-1}+x^{-1}y+x^{-1}y^{-1}-yx-yx^{-1}-y^{-1}x+$ $+y^{-1}x^{-1}-xyx+xyx^{-1}+xy^{-1}x-x^{-1}yx-x^{-1}y^{-1}x^{-1}$
$O, O, E^3_{2,3}$	$-y^{-1}-xy-xy^2+x^{-1}y^2+x^{-1}y^{-1}+x^{-1}y-yx-y^{-1}x+y^{-1}x^{-1}-xyx+$ $+xy^{-1}x^{-1}-x^{-1}yx+x^{-1}y^{-1}x+x^{-1}y^{-1}x^{-1}$
$O, O, E^3_{3,1}$	$1-y-y^{-1}-xy-xy^2+xy^{-1}-x^{-1}y^2+x^{-1}y^{-1}-yx+yx^{-1}-y^{-1}x+$ $+y^{-1}x^{-1}-xyx+xy^{-1}x-xyx^{-1}+xy^{-1}x^{-1}-x^{-1}yx+x^{-1}yx^{-1}-x^{-1}y^{-1}x^{-1}$
$O, O, E^3_{3,2}$	$1-x+y-y^{-1}-y^2+xy^{-1}-x^{-1}y+x^{-1}y^2+yx+yx^{-1}+y^{-1}x^{-1}-xyx^{-1}+$ $+xyx-xy^{-1}x^{-1}+x^{-1}yx^{-1}-x^{-1}y^{-1}x+x^{-1}y^{-1}x^{-1}$
$O, O, E^3_{3,3}$	$1+x^{-1}-y^{-1}-xy-xy^2+xy^{-1}+x^{-1}y+yx^{-1}+y^{-1}x+y^{-1}x^{-1}-xyx-$ $-xyx^{-1}-xy^{-1}x^{-1}-x^{-1}yx$

References

1. L. Creedon, J. Gildea, *The structure of the unit group of the group algebra* $\mathbb{F}_{2^k}D_8$, Canad. Math. Bull., **54**, 237–243 (2011).
2. C. W. Curtis, I. Reiner, *Methods of representation theory*, vol. I, Wiley-Intersci., New York (1981).
3. R. A. Ferraz, *Simple components of the center of* $\mathbb{F}G/J(\mathbb{F}G)$, Commun. Algebra, **36**, № 9, 3191–3199 (2008).
4. J. Gildea, *The structure of the unit group of the group algebra* $\mathbb{F}_2^k A_4$, Czechoslovak Math. J., **61**, № 136, 531–539 (2011).
5. J. Gildea, F. Monaghan, *Units of some group algebras of groups of order 12 over any finite field of characteristic 3*, Algebra and Discrete Math., **11**, 46–58 (2011).
6. T. Hurley, *Group rings and ring of matrices*, Int. J. Pure and Appl. Math., **31**, № 3, 319–335 (2006).
7. T. Hurley, *Convolutional codes from units in matrix and group rings*, Int. J. Pure and Appl. Math., **50**, № 3, 431–463 (2009).
8. G. Karpilvosky, *The Jacobson radical of group algebras*, North-Holland, Amsterdam (1987).
9. M. Khan, R. K. Sharma, J. B. Srivastava, *The unit group of* $\mathbb{F}S_4$, Acta Math. Hungar., **118**, № 1–2, 105–113 (2008).

10. R. Lidl, H. Niederreiter, *Introduction to finite fields and their applications*, Cambridge Univ. Press, New York (1986).
11. S. Maheshwari, R. K. Sharma, *The unit group of group algebra* $\mathbb{F}_q SL(2,\mathbb{Z}_3)$, J. Algebra Comb. Discrete Struct. and Appl., **3**, № 1, 1 – 6 (2016).
12. N. Makhijani, *Units in finite group algebras*, Ph.D. thesis, IIT Delhi (2014).
13. N. Makhijani, R. K. Sharma, J. B. Srivastava, *A note on units in* $\mathbb{F}_{p^m} D_{2p^n}$, Acta Math. Acad. Paedagog. Nyházi. (N. S.), **30**, 17 – 25 (2014).
14. N. Makhijani, R. K. Sharma, J. B. Srivastava, *The unit group of* $\mathbb{F}_q[D_{30}]$, Serdica Math. J., **41**, 185 – 198 (2015).
15. N. Makhijani, R. K. Sharma, J. B. Srivastava, *A note on the structure of* $\mathbb{F}_{p^k} A_5 / J(\mathbb{F}_{p^k} A_5)$, Acta Sci. Math. (Szeged), **82**, 29 – 43, (2016).
16. C. P. Milies, S. K. Sehgal, *An introduction to group rings*, Kluwer Acad. Publ. (2002).
17. F. Monaghan, *Units of some group algebras of non-abelian groups of order 24 over any finite field of characteristic 3*, Int. Electron. J. Algebra, **12**, 133 – 161 (2012).
18. S. Perlis, G. L. Walker, *Abelian group algebras of finite order*, Trans. Amer. Math. Soc., **68**, № 3, 420 – 426 (1950).
19. R. K. Sharma, J. B. Srivastava, M. Khan, *The unit group of* $\mathbb{F}A_4$, Publ. Math. Debrecen, **71**, 1 – 6 (2006).
20. R. K. Sharma, J. B. Srivastava, M. Khan, *The unit group of* $\mathbb{F}S_3$, Acta Math. Acad. Paedagog. Nyházi. (N. S.), **23**, № 2, 129 – 142 (2007).
21. R. K. Sharma, Pooja Yadav, *The unit group of* $\mathbb{Z}_p Q_8$, Algebras Groups and Geom., **24**, 425 – 430 (2008).
22. G. Tang, Y. Wei, Y. Li, *Unit groups of group algebras of some small groups*, Czechoslovak Math. J., **64**, № 1, 149 – 157 (2014).

Received 15.12.18