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GENERALIZED DERIVATIONS ACTING ON MULTILINEAR POLYNOMIALS AS A JORDAN HOMOMORPHISMS

УЗАГАЛЬНЕНІ ПОХІДНІ, ЩО ДІЮТЬ НА МУЛЬТИЛІНІЙНИХ ПОЛІНОМАХ ЯК ЖОРДАНОВІ ГОМОМОРФІЗМИ

Let R be a prime ring whose characteristic is not equal to 2, let U be the Utumi quotient ring of R, and let C be the extended centroid of R. Also let G and H be two generalized derivations on R and let $f(x_1,\ldots,x_n)$ be a noncentral multilinear polynomial over C. If $G(H(u^2)) = (H(u))^2$ for all $u = f(r_1,\ldots,r_n)$, $r_1,\ldots,r_n \in R$, then one of the following holds:

1) $H = 0$;

2) there exists $\lambda \in C$ such that $G(x) = H(x) = \lambda x$ for all $x \in R$;

3) there exist $\lambda \in C$ and $a \in U$ such that $H(x) = \lambda x$ and $G(x) = [a,x] + \lambda x$ for all $x \in R$ and $f(x_1,\ldots,x_n)^2$ is central-valued on R.

Нехай R — просте кільце з характеристикою, що не дорівнює 2, U — фактор-кільце Утумі для R, а C — продовжений центроїд для R. Крім того, припустимо, що G та H — дві узагальнені похідні на R, а $f(x_1,\ldots,x_n)$ — нецентральний мультилінійний поліном над C. Якщо $G(H(u^2)) = (H(u))^2$ для всіх $u = f(r_1,\ldots,r_n)$, $r_1,\ldots,r_n \in R$, то справджується одне з таких тверджень:

1) $H = 0$;

2) існує таке $\lambda \in C$, що $G(x) = H(x) = \lambda x$ для всіх $x \in R$;

3) існують такі $\lambda \in C$ та $a \in U$, що $H(x) = \lambda x$, $G(x) = [a,x] + \lambda x$ для всіх $x \in R$ і $f(x_1,\ldots,x_n)^2$ є центральнозначним на R.

1. Introduction. Throughout the article R denotes a prime ring of characteristic different from 2 with center $Z(R)$ and U denotes the Utumi quotient ring of R. The center of U denoted by C is called the extended centroid of R. An additive mapping d on a ring R is said to be a derivation if $d(xy) = d(x)y + xd(y)$ for all $x, y \in R$. For a fixed $a \in R$, the mapping $d_a : R \to R$, defined by $d_a(x) = [a, x]$, for all $x \in R$ is a derivation, usually called inner derivation induced by an element $a \in R$. A derivation is called outer if it is not an inner derivation. An additive mapping H on a ring R is said to be a generalized derivation associated with a derivation d if $H(xy) = H(x)y + xd(y)$ for all $x, y \in R$. For fixed $a, a' \in R$, the mapping $F_{(a,a')} : R \to R$ defined by $F_{(a,a')}(x) = ax + xa'$ is a generalized derivation on R. The mapping $F_{(a,a')}$ is usually called generalized inner derivation on R.

An additive mapping on a ring R is a homomorphism if $H(xy) = H(x)H(y)$ for all $x, y \in R$ and H is said to an anti-homomorphism if $H(xy) = H(y)H(x)$ for all $x, y \in R$. An additive mapping H is said to be a Jordan homomorphism if $H(x^2) = (H(x))^2$ for all $x \in R$. We observe that every homomorphism and anti-homomorphism is a Jordan homomorphism but the converse is not true in general. Following example justify our observation.

Example **1.1.** Suppose that $*$ is an involution on ring R and $S = R \bigoplus R$ is a ring such that $r_1 a r_2 = 0$ for all $r_1, r_2 \in R$, where $a \in Z(R)$. Define a function ζ on S such that $\zeta(r_1, r_2) =$

$= (ar_1, r_2^*)$ for all $r_1, r_2 \in R$. This example shows that ζ is a Jordan homomorphism but not a homomorphism.

Herstein [13], in 1956 proved that every Jordan homomorphism from a ring R onto a prime ring R' with $\mathrm{char}(R) \neq 2, 3$ is either a homomorphism or anti-homomorphism. Further, Smiley [20], in 1957 improve the above result by removing the restriction of characteristic is not equal to 3 in the hypothesis of the Herstein's [13].

The context of derivation, which acts as a homomorphism or as an anti-homomorphism, was first studied by Bell and Kappe [8]. More precisely, they proved that there is no nonzero derivation on prime ring which acts as a homomorphism or as an anti-homomorphism on right ideal of R. Later on many mathematician have studied the additive mapping which acts as a homomorphism, anti-homomorphism, Jordan homomorphism, Lie homomorphism on some subsets of a particular ring. For more details, we refer to reader [1 – 6, 21 – 25].

Recently, in this line of investigation De Filippis and Dhara [4], in 2019 studied the structure of prime ring R, when generalized skew derivation acts as a Jordan homomorphism on multilinear polynomial over C.

Motivated by above cited results, we would like to study the following.

Theorem 1.1. *Let R be a prime ring of characteristic not equal to* 2, *U be the Utumi quotient ring of R and C be the extended centroid of R. Let G and H be two generalized derivations on R and $f(x_1, \ldots, x_n)$ be a multilinear polynomial over C which is noncentral valued on R. If $G(H(u^2)) = (H(u))^2$ for all $u = f(r_1, \ldots, r_n)$, $r_1, \ldots, r_n \in R$, then one of the following holds*:

1) $H = 0$;

2) *there exists $\lambda \in C$ such that $G(x) = H(x) = \lambda x$ for all $x \in R$;*

3) *there exist $\lambda \in C$ and $a \in U$ such that $H(x) = \lambda x$, $G(x) = [a, x] + \lambda x$ for all $x \in R$ and $f(x_1, \ldots, x_n)^2$ is central valued on R.*

The following corollaries are an immediate application of Theorem 1.1. In particular, for $G = I$, identity mapping in Theorem 1.1, we have the following.

***Corollary* 1.1.** *Let R be a prime ring of characteristic not equal to* 2, *U be the Utumi quotient ring of R and C be the extended centroid of R. Let H be a nonzero generalized derivation on R and $f(x_1, \ldots, x_n)$ be a multilinear polynomial over C which is noncentral valued on R. If $H(u^2) = (H(u))^2$ for all $u = f(r_1, \ldots, r_n)$, $r_1, \ldots, r_n \in R$, then $H(x) = x$ for all $x \in R$.*

In particular, for $H = I$, the identity mapping on R in Theorem 1.1, we have the following.

***Corollary* 1.2.** *Let R be a prime ring of characteristic not equal to* 2, *U be the Utumi quotient ring of R and C be the extended centroid of R. Let H be a nonzero generalized derivation on R and $f(x_1, \ldots, x_n)$ be a multilinear polynomial over C which is noncentral valued on R. If $H(u^2) = u^2$ for all $u = f(r_1, \ldots, r_n)$, $r_1, \ldots, r_n \in R$, then one of the following holds*:

1) $H(x) = x$ *for all* $x \in R$;

2) *there exists $a \in U$ such that $H(x) = [a, x] + x$ for all $x \in R$ and $f(x_1, \ldots, x_n)^2$ is central valued on R.*

***Corollary* 1.3.** *Let R be a prime ring of characteristic not equal to* 2, *U be the Utumi quotient ring of R and C be the extended centroid of R. Let $q \notin Z(R)$, H be a generalized derivation on R*

and $f(x_1,\ldots,x_n)$ *be a multilinear polynomial over* C *which is noncentral valued on* R *such that* $[q,H(u^2)]=(H(u))^2$ *for all* $u=f(r_1,\ldots,r_n)$, $r_1,\ldots,r_n\in R$. *Then* $H=0$.

2. Notations and known results. Let d and δ be two derivations on R. We denote by $f^d(x_1,\ldots,x_n)$ the polynomials obtained from $f(x_1,\ldots,x_n)$ replacing each coefficients α_σ with $d(\alpha_\sigma)$. Then we have

$$d(f(x_1,\ldots,x_n))=f^d(x_1,\ldots,x_n)+\sum_i f(x_1,\ldots,d(x_i),\ldots,x_n)$$

and

$$d\delta(f(r_1,\ldots,r_n))=f^{d\delta}(r_1,\ldots,r_n)+\sum_i f^d(r_1,\ldots,\delta(r_i),\ldots,r_n)+$$

$$+\sum_i f^\delta(r_1,\ldots,d(r_i),\ldots,r_n)+\sum_i f(r_1,\ldots,d\delta(r_i),\ldots,r_n)+$$

$$+\sum_{i\neq j} f(r_1,\ldots,d(r_i),\ldots,\delta(r_j),\ldots,r_n).$$

The following facts are frequently used to prove our results.

Fact 2.1. *Let* R *be a prime ring and* I *a two-sided ideal of* R. *Then* R, I *and* U *satisfy the same generalized polynomial identities with coefficients in* U [10].

Fact 2.2. *Let* R *be a prime ring and* I *a two-sided ideal of* R. *Then* R, I *and* U *satisfy the same differential identities* [17].

Fact 2.3. *Let* R *be a prime ring. Then every derivation* d *of* R *can be uniquely extended to a derivation of* U (*see Proposition* 2.5.1 [7]).

Fact 2.4 ([15], Theorem 2)**.** *Let* R *be a prime ring,* d *a nonzero derivation on* R *and* I *a nonzero ideal of* R. *If* I *satisfies the differential identity*

$$f(r_1,\ldots,r_n,d(r_1),\ldots,d(r_n))=0$$

for any $r_1,\ldots,r_n\in I$, *then either*

(i) *I satisfies the generalized polynomial identity*

$$f(r_1,\ldots,r_n,x_1,\ldots,x_n)=0$$

or

(ii) d *is* U*-inner, i.e., for some* $q\in U$, $d(x)=[q,x]$ *and* I *satisfies the generalized polynomial identity*

$$f(r_1,\ldots,r_n,[q,r_1],\ldots,[q,r_n])=0.$$

Fact 2.5 ([6], Lemma 2.9)**.** *Let* R *be a prime ring of characteristic with* $\mathrm{char}(R)\neq 2$, a, b, c, $c'\in U$ *and* $p(x_1,\ldots,x_n)$ *be any polynomial over* C *which is not identity for* R. *If* $ap(r)+p(r)b+$ $+cp(r)c'=0$ *for all* $r\in R^n$, *then one of the following conditions holds*:

1) $b,c'\in C$ *and* $a+b+cc'=0$;

2) $a,c\in C$ *and* $a+b+cc'=0$;

3) $a+b+cc'=0$ *and* $p(x_1,\ldots,x_n)^2$ *is central valued on* R.

3. *G* and *H* are generalized inner derivations. In this section, we study the case when G and H are generalized inner derivations. Suppose that $G(x) = ax + xb$ and $H(x) = px + xq$ for all $x \in R$ and for some $a, b, p, q \in U$. From the given identity $G(H(f(r)^2)) = H(f(r))^2$ we get the expression $a'f(r)^2 + af(r)^2q + pf(r)^2b + f(r)^2b' = pf(r)pf(r) + pf(r)^2q + f(r)p'f(r) + f(r)qf(r)q$ where $a' = ap, b' = qb$ and $p' = qp$. To prove main result we prove the following propositions.

Proposition 3.1. *Let R be a prime ring of characteristic not equal to* 2, *U be the Utumi quotient ring of R and C be the extended centroid of R. Let G and H be two generalized inner derivations on R and $f(x_1, \ldots, x_n)$ be a multilinear polynomial over C which is noncentral valued on R. If $G(H(u^2)) = (H(u))^2$ for all $u = f(r_1, \ldots, r_n)$, $r_1, \ldots, r_n \in R$, then one of the following holds*:

1) $H = 0$;

2) *there exists $\lambda \in C$ such that $G(x) = H(x) = \lambda x$ for all $x \in R$*;

3) *there exist $\lambda \in C$ and $a \in U$ such that $H(x) = \lambda x$, $G(x) = [a, x] + \lambda x$ and $f(x_1, \ldots, x_n)^2$ is central valued on R.*

To prove the above proposition we need the following results.

Proposition 3.2. *Let $R = M_m(K)$ be the ring of all $m \times m$ matrices over the field K with characteristic not equal to 2 and $m \geq 2$ and $f(x_1, \ldots, x_n)$ be a noncentral multilinear polynomial over K. Let a, a', b, b', p, p', $q \in U$ such that $a'f(r)^2 + af(r)^2q + pf(r)^2b + f(r)^2b' = pf(r)pf(r) + {}$ $+ pf(r)^2q + f(r)p'f(r) + f(r)qf(r)q$ for all $r = (r_1, \ldots, r_n) \in R^n$. Then $p \in K \cdot I_m$ and $q \in K \cdot I_m$.*

Proof. Since $f(x_1, \ldots, x_n)$ be a noncentral on R. By [18] (Lemma 2, Proof of Lemma 3), there exists a sequence of matrices $r = (r_1, \ldots, r_n)$ in R such that $f(r_1, \ldots, r_n) = \gamma e_{ij}$ with $0 \neq \gamma \in K$ and $i \neq j$. Since the set $f(R) = \{f(x_1, \ldots, x_n) \mid x_i \in R\}$ is invariant under the action of all inner automorphisms of R for all $i \neq j$ there exists a sequence of matrices $r = (r_1, \ldots, r_n)$ in R such that $f(r_1, \ldots, r_n) = \gamma e_{ij}$. Thus our hypothesis

$$a'f(r_1, \ldots, r_n)^2 + af(r_1, \ldots, r_n)^2q + pf(r_1, \ldots, r_n)^2b + f(r_1, \ldots, r_n)^2b' =$$
$$= pf(r_1, \ldots, r_n)pf(r_1, \ldots, r_n) + pf(r_1, \ldots, r_n)^2q +$$
$$+ f(r_1, \ldots, r_n)p'f(r_1, \ldots, r_n) + f(r_1, \ldots, r_n)qf(r_1, \ldots, r_n)q. \tag{1}$$

Gives that

$$pe_{ij}pe_{ij} + e_{ij}p'e_{ij} + e_{ij}qe_{ij}q = 0.$$

Left multiplying above relation by e_{ij}, we obtain $e_{ij}pe_{ij}pe_{ij} = 0$. It implies that $p_{ij}^2 = 0$ and hence $p_{ij} = 0$ with $i \neq j$. It implies that p is a diagonal matrix.

Right multiplication by e_{ij} in above expression we get $q_{ij} = 0$ with $i \neq j$. This implies that q is a diagonal matrix.

For any K-automorphism θ of R, p^θ enjoy the same property as p does, p^θ must be diagonal. Write $p = \sum_{i=1}^m p_{ii}e_{ii}$; then, for $s \neq t$, we have

$$(1 + e_{ts})p(1 - e_{ts}) = \sum_{i=1}^m p_{ii}e_{ii} + (p_{ss} - p_{tt})e_{ts}$$

diagonal. Hence, $p_{ss} = p_{tt}$ and so p is a scalar matrix, that is, $p \in K \cdot I_m$. Similarly, we can show that q is diagonal and hence central.

Proposition 3.2 is proved.

Lemma 3.1. *Let R be a prime ring of characteristic not equal to 2. Let U be the Utumi ring of quotients and C be the extended centroid of ring R. Suppose that $f(x_1,\ldots,x_n)$ be a multilinear polynomial over C which is not central valued on R such that $a'f(r)^2 + af(r)^2q + pf(r)^2b + + f(r)^2b' = pf(r)pf(r) + pf(r)^2q + f(r)p'f(r) + f(r)qf(r)q$ for all $r \in R^n$ and for some a, a', b, b', p, p', $q \in U$. Then p and q are central.*

Proof. On contrary suppose that both p and q are not central. By hypothesis, we have

$$\mathrm{h}(x_1,\ldots,x_n) = a'f(x_1,\ldots,x_n)^2 + af(x_1,\ldots,x_n)^2q + pf(x_1,\ldots,x_n)^2b + \\ + f(x_1,\ldots,x_n)^2b' - pf(x_1,\ldots,x_n)pf(x_1,\ldots,x_n) - \\ - pf(x_1,\ldots,x_n)^2q - f(x_1,\ldots,x_n)p'f(x_1,\ldots,x_n) - \\ - f(x_1,\ldots,x_n)qf(x_1,\ldots,x_n)q$$

for all $x_1,\ldots,x_n \in R$, that is,

$$\mathrm{h}(x_1,\ldots,x_n) = a'f(x_1,\ldots,x_n)^2 + \Big\{af(x_1,\ldots,x_n)^2 - pf(x_1,\ldots,x_n)^2 - \\ - f(x_1,\ldots,x_n)qf(x_1,\ldots,x_n)\Big\}q + pf(x_1,\ldots,x_n)^2b + \\ + f(x_1,\ldots,x_n)^2b' - pf(x_1,\ldots,x_n)pf(x_1,\ldots,x_n) - \\ -f(x_1,\ldots,x_n)p'f(x_1,\ldots,x_n) \tag{2}$$

for all $x_1,\ldots,x_n \in R$.

Since R and U satisfy same generalized polynomial identity (GPI) (see [10]), U satisfies $\mathrm{h}(x_1,\ldots,x_n) = 0_T$. Suppose that $\mathrm{h}(x_1,\ldots,x_n)$ is a trivial GPI for U. Let $T = U *_C C\{x_1,\ldots,x_n\}$, the free product of U and $C\{x_1,\ldots,x_n\}$, the free C-algebra in non commuting indeterminates $x_1,\ldots,x_n$. Then $\mathrm{h}(x_1,\ldots,x_n)$ is zero element in $T = U *_C C\{x_1,\ldots,x_n\}$. It implies that $\{b, b', q, 1\}$ is linearly C-dependent. Then there exist $\alpha_1,\alpha_2,\alpha_3$ and $\alpha_4 \in C$ such that $\alpha_1 b + + \alpha_2 b' + \alpha_3 q + \alpha_4 1 = 0$. If $\alpha_1 = 0 = \alpha_2$, then $\alpha_3 \neq 0$ and so $q = -\alpha_3^{-1}\alpha_4 \in C$, gives a contradiction. Therefore either $\alpha_1 \neq 0$ or $\alpha_2 \neq 0$. Without loss of generality, we assume that $\alpha_1 \neq 0$. Then $b = \alpha b' + \beta q + \gamma$, where $\alpha = -\alpha_1^{-1}\alpha_2$, $\beta = -\alpha_1^{-1}\alpha_3$ and $\gamma = -\alpha_1^{-1}\alpha_4$. Then U satisfies

$$(a' + p\gamma)f(x_1,\ldots,x_n)^2 + \Big\{af(x_1,\ldots,x_n)^2 - pf(x_1,\ldots,x_n)^2 - \\ - f(x_1,\ldots,x_n)qf(x_1,\ldots,x_n) + p\beta f(x_1,\ldots,x_n)^2\Big\}q + \\ + \Big\{p\alpha + 1\Big\}f(x_1,\ldots,x_n)^2b' - pf(x_1,\ldots,x_n)pf(x_1,\ldots,x_n) - \\ - f(x_1,\ldots,x_n)p'f(x_1,\ldots,x_n). \tag{3}$$

This implies that $\{b', q, 1\}$ is linearly C dependent. Then there exist $\beta_1,\beta_2,\beta_3 \in C$ such that $\beta_1 b' + \beta_2 q + \beta_3 1 = 0$. Again using similar argument as we have used above, since $q \notin C$, we get $\beta_1 \neq 0$ and, hence, $b' = \alpha' q + \beta'$, where $\alpha' = -\beta_1^{-1}\beta_2$ and $\beta' = -\beta_1^{-1}\beta_3$. Thus equation (3) reduces to

$$(a' + p\gamma + p\alpha\beta' + \beta')f(x_1, \ldots, x_n)^2 +$$

$$+ \Big\{(a - p + p\beta + p\alpha\alpha' + \alpha')f(x_1, \ldots, x_n)^2 - f(x_1, \ldots, x_n)qf(x_1, \ldots, x_n)\Big\}q -$$

$$- pf(x_1, \ldots, x_n)pf(x_1, \ldots, x_n) - f(x_1, \ldots, x_n)p'f(x_1, \ldots, x_n).$$

Since $\{q, 1\}$ is linearly C-independent, hence, U satisfies

$$\Big\{(a - p + p\beta + p\alpha\alpha' + \alpha')f(x_1, \ldots, x_n)^2 - f(x_1, \ldots, x_n)qf(x_1, \ldots, x_n)\Big\}q = 0,$$

that is, U satisfies

$$\Big\{(a - p + p\beta + p\alpha\alpha' + \alpha')f(x_1, \ldots, x_n) - f(x_1, \ldots, x_n)q\Big\}f(x_1, \ldots, x_n)q = 0.$$

Since $\{q, 1\}$ is linearly C-independent, hence, U satisfies

$$f(x_1, \ldots, x_n)qf(x_1, \ldots, x_n)q = 0.$$

This gives that $q \in C$, a contradiction.

Next, suppose that $\mathrm{h}(x_1, \ldots, x_n)$ is a non trivial GPI for U. In case C is infinite, we have $\mathrm{h}(x_1, \ldots, x_n) = 0$ for all $x_1, \ldots, x_n \in U \otimes_C \overline{C}$, where $\overline{C}$ is the algebraic closure of C. Since both U and $U \otimes_C \overline{C}$ are prime and centrally closed [11] (Theorems 2.5 and 3.5), we may replace R by U or $U \otimes_C \overline{C}$ according to C finite or infinite. Then R is centrally closed over C and $\mathrm{h}(x_1, \ldots, x_n) = 0$ for all $x_1, \ldots, x_n \in R$. By Martindale's theorem [19], R is then a primitive ring with nonzero socle $\mathrm{soc}(R)$ and with C as its associated division ring. Then, by Jacobson's theorem [14, p. 75], R is isomorphic to a dense ring of linear transformations of a vector space V over C.

Assume first that V is finite dimensional over C, that is, $\dim_C V = m$. By density of R, we have $R \cong M_m(C)$. Since $f(r_1, \ldots, r_n)$ is not central valued on R, R must be non commutative and so $m \geq 2$. In this case, by Proposition 3.2, we get that $p \in C$, a contradiction.

Next we suppose that V is infinite dimensional over C. By Martindale's theorem [19] (Theorem 3), for any $e^2 = e \in \mathrm{soc}(R)$ we have $eRe \cong M_t(C)$ with $t = \dim_C Ve$. Since p and q are not central, there exist $h_1, h_2 \in \mathrm{Soc}(R)$ such that $[p, h_1] \neq 0$ and $[q, h_2] \neq 0$. By Litoff's theorem [12], there exists an idempotent $e \in \mathrm{soc}(R)$ such that $ph_1, h_1p, qh_2, h_2q, h_1, h_2 \in eRe$. Since R satisfies generalized identity

$$e\{a'f(ex_1e, \ldots, ex_ne)^2 + af(ex_1e, \ldots, ex_ne)^2q + pf(ex_1e, \ldots, ex_ne)^2b +$$

$$+ f(ex_1e, \ldots, ex_ne)^2b' - pf(ex_1e, \ldots, ex_ne)pf(ex_1e, \ldots, ex_ne) -$$

$$- pf(ex_1e, \ldots, ex_ne)^2q - f(ex_1e, \ldots, ex_ne)p'f(ex_1e, \ldots, ex_ne) -$$

$$- f(ex_1e, \ldots, ex_ne)qf(ex_1e, \ldots, ex_ne)q\}e,$$

the subring eRe satisfies

$$ea'ef(x_1, \ldots, x_n)^2 + eaef(x_1, \ldots, x_n)^2eqe + epef(x_1, \ldots, x_n)^2ebe +$$

$$+ f(x_1, \ldots, x_n)^2eb'e - epef(x_1, \ldots, x_n)epef(x_1, \ldots, x_n) -$$

$$- epef(x_1,\ldots,x_n)^2eqe - f(x_1,\ldots,x_n)ep'ef(x_1,\ldots,x_n) -$$
$$- f(x_1,\ldots,x_n)eqef(x_1,\ldots,x_n)eqe. \tag{4}$$

Then by the above finite dimensional case, epe and eqe are central elements of eRe. Thus, $ph_1 = (epe)h_1 = h_1epe = h_1p$ and $qh_2 = (eqe)h_2 = h_2(eqe) = h_2q$, a contradiction.

Lemma 3.1 is proved.

Now we prove Proposition 3.1.

***Proof of Proposition* 3.1.** By the hypothesis, we have

$$a\Big(pf(r_1,\ldots,r_n)^2 + f(r_1,\ldots,r_n)^2q\Big) + \Big(pf(r_1,\ldots,r_n)^2 + f(r_1,\ldots,r_n)^2q\Big)b =$$
$$= \Big(pf(r_1,\ldots,r_n) + f(r_1,\ldots,r_n)q\Big)^2,$$

that is,

$$apf(r_1,\ldots,r_n)^2 + af(r_1,\ldots,r_n)^2q + pf(r_1,\ldots,r_n)^2b + f(r_1,\ldots,r_n)^2qb =$$
$$= pf(r_1,\ldots,r_n)pf(r_1,\ldots,r_n) + f(r_1,\ldots,r_n)qf(r_1,\ldots,r_n)q +$$
$$+ pf(r_1,\ldots,r_n)^2q + f(r_1,\ldots,r_n)qpf(r_1,\ldots,r_n).$$

By Lemma 3.1 we have that $p \in C$ and $q \in C$. Then $H(x) = (p+q)x = \lambda x$, where $\lambda = p+q \in C$. From the given hypothesis we get $\lambda\{(a-\lambda)f(r)^2 + f(r)^2b\} = 0$. If $\lambda = 0$, then $H(x) = \lambda x = 0$ for all $x \in R$, which is the conclusion 1. Let $\lambda \neq 0$. Then we obtain $(a-\lambda)f(r)^2 + f(r)^2b = 0$. From Fact 2.5 we have one of the following:

$b \in C$ and $a - \lambda + b = 0$, which gives $a \in C$. Therefore, $G(x) = (a+b)x = \lambda x = H(x)$ for all $x \in R$, which is the conclusion 2.

$a - \lambda \in C$ and $a - \lambda + b = 0$ which gives $b \in C$, $a \in C$. Therefore, $G(x) = (a+b)x = \lambda x = H(x)$ for all $x \in R$, which is the conclusion 2.

$a + b = \lambda$ and $f(x_1,\ldots,x_n)^2$ is central valued on R which gives $b = \lambda - a$.
In this case, we get $G(x) = ax + xb = ax + \lambda x - xa = [a,x] + \lambda x$ for all $x \in R$ and $f(x_1,\ldots,x_n)^2$ is central valued on R, which is the conclusion 3.

4. Proof of Theorem 1.1. If $H = 0$, then we are done. Suppose that $H \neq 0$. In view of [16] (Theorem 3), we may assume that, for some $a, p \in U$, there exist derivations d and δ on U such that $G(x) = ax + d(x)$ and $H(x) = px + \delta(x)$ for all $x \in R$. Then by the hypothesis, we have

$$a\Big(pf(x_1,\ldots,x_n)^2 + \delta(f(x_1,\ldots,x_n)^2)\Big) + d\Big(pf(x_1,\ldots,x_n)^2 +$$
$$+ \delta(f(x_1,\ldots,x_n)^2)\Big) = \Big(pf(x_1,\ldots,x_n) + \delta(f(x_1,\ldots,x_n))\Big)^2.$$

By simplifying above relation, we obtain

$$apf(x_1,\ldots,x_n)^2 + a\delta(f(x_1,\ldots,x_n)^2) + d(p)f(x_1,\ldots,x_n)^2 +$$
$$+ pd(f(x_1,\ldots,x_n))f(x_1,\ldots,x_n) + pf(x_1,\ldots,x_n)d(f(x_1,\ldots,x_n)) +$$
$$+ d(\delta(f(x_1,\ldots,x_n)^2)) = pf(x_1,\ldots,x_n)pf(x_1,\ldots,x_n) +$$

$$+\,\delta(f(x_1,\ldots,x_n))pf(x_1,\ldots,x_n)+$$
$$+\,pf(x_1,\ldots,x_n)\delta(f(x_1,\ldots,x_n))+(\delta(f(x_1,\ldots,x_n)))^2, \tag{5}$$

that is,

$$apf(x_1,\ldots,x_n)^2+a\delta(f(x_1,\ldots,x_n)^2)+d(p)f(x_1,\ldots,x_n)^2+$$
$$+\,pd(f(x_1,\ldots,x_n))f(x_1,\ldots,x_n)+pf(x_1,\ldots,x_n)d(f(x_1,\ldots,x_n))+$$
$$+\,(d\delta(f(x_1,\ldots,x_n)))f(x_1,\ldots,x_n)+\delta(f(x_1,\ldots,x_n))d(f(x_1,\ldots,x_n))+$$
$$+\,d(f(x_1,\ldots,x_n))\delta(f(x_1,\ldots,x_n))+f(x_1,\ldots,x_n)(d\delta(f(x_1,\ldots,x_n)))=$$
$$=pf(x_1,\ldots,x_n)pf(x_1,\ldots,x_n)+\delta(f(x_1,\ldots,x_n))pf(x_1,\ldots,x_n)+$$
$$+\,pf(x_1,\ldots,x_n)\delta(f(x_1,\ldots,x_n))+(\delta(f(x_1,\ldots,x_n)))^2. \tag{6}$$

If d and δ both are inner derivations then the result follows from Proposition 3.1. So assume that both d and δ are not an inner derivations. Now we have the following cases.

Case I. Let d be inner derivation and δ be an outer derivation. Then, for some $q\in U$, $d(x)=$ $=[q,x]$ for all $x\in R$. From equation (5), we get

$$apf(x_1,\ldots,x_n)^2+a\delta(f(x_1,\ldots,x_n))f(x_1,\ldots,x_n)+$$
$$+\,af(x_1,\ldots,x_n)\delta(f(x_1,\ldots,x_n))+[q,p]f(x_1,\ldots,x_n)^2+$$
$$+\,p\big[q,f(x_1,\ldots,x_n)\big]f(x_1,\ldots,x_n)+pf(x_1,\ldots,x_n)\big[q,f(x_1,\ldots,x_n)\big]+$$
$$+\,\big[q,\delta(f(x_1,\ldots,x_n))f(x_1,\ldots,x_n)\big]+\big[q,f(x_1,\ldots,x_n)\delta(f(x_1,\ldots,x_n))\big]=$$
$$=pf(x_1,\ldots,x_n)pf(x_1,\ldots,x_n)+pf(x_1,\ldots,x_n)\delta(f(x_1,\ldots,x_n))+$$
$$+\,\delta(f(x_1,\ldots,x_n))pf(x_1,\ldots,x_n)+(\delta f(x_1,\ldots,x_n))^2. \tag{7}$$

In (7) replace $\delta(f(x_1,\ldots,x_n))$ with $f^{\delta}(x_1,\ldots,x_n)+\sum_i f(x_1,\ldots,\delta(x_i),\ldots,x_n)$:

$$apf(x_1,\ldots,x_n)^2+af^{\delta}(x_1,\ldots,x_n)f(x_1,\ldots,x_n)+$$
$$+\,a\sum_i f(x_1,\ldots,\delta(x_i),\ldots,x_n)f(x_1,\ldots,x_n)+af(x_1,\ldots,x_n)f^{\delta}(x_1,\ldots,x_n)+$$
$$+\,af(x_1,\ldots,x_n)\sum_i f(x_1,\ldots,\delta(x_i),\ldots,x_n)+[q,p]f(x_1,\ldots,x_n)^2+$$
$$+\,p\big[q,f(x_1,\ldots,x_n)\big]f(x_1,\ldots,x_n)+pf(x_1,\ldots,x_n)\big[q,f(x_1,\ldots,x_n)\big]+$$
$$+\left[q,f^{\delta}(x_1,\ldots,x_n)f(x_1,\ldots,x_n)+\sum_i f(x_1,\ldots,\delta(x_i),\ldots,x_n)f(x_1,\ldots,x_n)\right]+$$
$$+\left[q,f(x_1,\ldots,x_n)f^{\delta}(x_1,\ldots,x_n)+f(x_1,\ldots,x_n)\sum_i f(x_1,\ldots,\delta(x_i),\ldots,x_n)\right]=$$

$$= pf(x_1,\ldots,x_n)pf(x_1,\ldots,x_n) + pf(x_1,\ldots,x_n)f^{\delta}(x_1,\ldots,x_n) +$$

$$+ pf(x_1,\ldots,x_n)\sum_i f(x_1,\ldots,\delta(x_i),\ldots,x_n) + f^{\delta}(x_1,\ldots,x_n)pf(x_1,\ldots,x_n) +$$

$$+ \sum_i f(x_1,\ldots,\delta(x_i),\ldots,x_n)pf(x_1,\ldots,x_n) +$$

$$+ \left(f^{\delta}(x_1,\ldots,x_n) + \sum_i f(x_1,\ldots,\delta(x_i),\ldots,x_n)\right)^2.$$

Since δ is outer, by Kharchenko's theorem (see Fact 2.4), we replace $\delta(x_i)$ by y_i in above expression, we get

$$apf(x_1,\ldots,x_n)^2 + af^{\delta}(x_1,\ldots,x_n)f(x_1,\ldots,x_n) +$$

$$+ a\sum_i f(x_1,\ldots,y_i,\ldots,x_n)f(x_1,\ldots,x_n) + af(x_1,\ldots,x_n)f^{\delta}(x_1,\ldots,x_n) +$$

$$+ af(x_1,\ldots,x_n)\sum_i f(x_1,\ldots,y_i,\ldots,x_n) + [q,p]f(x_1,\ldots,x_n)^2 +$$

$$+ p\big[q, f(x_1,\ldots,x_n)\big]f(x_1,\ldots,x_n) + pf(x_1,\ldots,x_n)\big[q, f(x_1,\ldots,x_n)\big] +$$

$$+ \left[q, f^{\delta}(x_1,\ldots,x_n)f(x_1,\ldots,x_n) + \sum_i f(x_1,\ldots,y_i,\ldots,x_n)f(x_1,\ldots,x_n)\right] +$$

$$+ \left[q, f(x_1,\ldots,x_n)f^{\delta}(x_1,\ldots,x_n) + f(x_1,\ldots,x_n)\sum_i f(x_1,\ldots,y_i,\ldots,x_n)\right] =$$

$$= pf(x_1,\ldots,x_n)pf(x_1,\ldots,x_n) + pf(x_1,\ldots,x_n)f^{\delta}(x_1,\ldots,x_n) +$$

$$+ pf(x_1,\ldots,x_n)\sum_i f(x_1,\ldots,y_i,\ldots,x_n) + f^{\delta}(x_1,\ldots,x_n)pf(x_1,\ldots,x_n) +$$

$$+ \sum_i f(x_1,\ldots,y_i,\ldots,x_n)pf(x_1,\ldots,x_n) +$$

$$+ \left(f^{\delta}(x_1,\ldots,x_n) + \sum_i f(x_1,\ldots,y_i,\ldots,x_n)\right)^2 \qquad (8)$$

for all $x_i, y_i \in U$. In particular, for $x_1 = 0$ in relation (8), we obtain $f(y_1, x_2, \ldots, x_n)^2 = 0$, a contradiction.

Case II. Let d be an outer derivation on R and δ be an inner derivation on R. For some $q \in U$ such that $\delta(x) = [q, x]$ for all $x \in R$. Then (5) implies that

$$apf(x_1,\ldots,x_n)^2 + a\big[q, f(x_1,\ldots,x_n)^2\big] + d(p)f(x_1,\ldots,x_n)^2 +$$

$$+ pd(f(x_1,\ldots,x_n))f(x_1,\ldots,x_n) + pf(x_1,\ldots,x_n)d(f(x_1,\ldots,x_n)) +$$

$$+ \big[d(q), f(x_1, \ldots, x_n)^2\big] + \big[q, d(f(x_1, \ldots, x_n)^2)\big] =$$
$$= pf(x_1, \ldots, x_n)pf(x_1, \ldots, x_n) + \big[q, f(x_1, \ldots, x_n)\big]pf(x_1, \ldots, x_n) +$$
$$+ pf(x_1, \ldots, x_n)\big[q, f(x_1, \ldots, x_n)\big] + \Big(\big[q, f(x_1, \ldots, x_n)\big]\Big)^2. \tag{9}$$

This can be rewritten as

$$apf(x_1, \ldots, x_n)^2 + a\big[q, f(x_1, \ldots, x_n)^2\big] + d(p)f(x_1, \ldots, x_n)^2 +$$
$$+ pd(f(x_1, \ldots, x_n))f(x_1, \ldots, x_n) + pf(x_1, \ldots, x_n)d(f(x_1, \ldots, x_n)) +$$
$$+ \big[d(q), f(x_1, \ldots, x_n)^2\big] + \big[q, d(f(x_1, \ldots, x_n))f(x_1, \ldots, x_n)\big] +$$
$$+ \big[q, f(x_1, \ldots, x_n)d(f(x_1, \ldots, x_n))\big] =$$
$$= pf(x_1, \ldots, x_n)pf(x_1, \ldots, x_n) + \big[q, f(x_1, \ldots, x_n)\big]pf(x_1, \ldots, x_n) +$$
$$+pf(x_1, \ldots, x_n)\big[q, f(x_1, \ldots, x_n)\big] + \Big(\big[q, f(x_1, \ldots, x_n)\big]\Big)^2. \tag{10}$$

Since d is an outer derivation on R, in (10) replace $d(f(x_1, \ldots, x_n))$ with $f^d(x_1, \ldots, x_n) + + \sum_i f(x_1, \ldots, y_i, \ldots, x_n)$, where $d(x_i) = y_i$, we obtain

$$apf(x_1, \ldots, x_n)^2 + a\big[q, f(x_1, \ldots, x_n)^2\big] + d(p)f(x_1, \ldots, x_n)^2 +$$
$$+ pf^d(x_1, \ldots, x_n)f(x_1, \ldots, x_n) + p\sum_i f(x_1, \ldots, y_i, \ldots, x_n)f(x_1, \ldots, x_n) +$$
$$+ pf(x_1, \ldots, x_n)f^d(x_1, \ldots, x_n) + pf(x_1, \ldots, x_n)\sum_i f(x_1, \ldots, y_i, \ldots, x_n) +$$
$$+ \Big[d(q), f(x_1, \ldots, x_n)^2\Big] + \Bigg[q, f^d(x_1, \ldots, x_n)f(x_1, \ldots, x_n) +$$
$$+ \sum_i f(x_1, \ldots, y_i, \ldots, x_n)f(x_1, \ldots, x_n)\Bigg] +$$
$$+ \Bigg[q, f(x_1, \ldots, x_n)f^d(x_1, \ldots, x_n) + f(x_1, \ldots, x_n)\sum_i f(x_1, \ldots, y_i, \ldots, x_n)\Bigg] =$$
$$= pf(x_1, \ldots, x_n)pf(x_1, \ldots, x_n) + \big[q, f(x_1, \ldots, x_n)\big]pf(x_1, \ldots, x_n) +$$
$$+ pf(x_1, \ldots, x_n)\big[q, f(x_1, \ldots, x_n)\big] + \Big(\big[q, f(x_1, \ldots, x_n)\big]\Big)^2.$$

Hence, U satisfies the blended component

$$p\sum_i f(x_1, \ldots, y_i, \ldots, x_n)f(x_1, \ldots, x_n) + pf(x_1, \ldots, x_n)\sum_i f(x_1, \ldots, y_i, \ldots, x_n) +$$

$$+\left[q,\sum_i f(x_1,\ldots,y_i,\ldots,x_n)f(x_1,\ldots,x_n)\right]+$$

$$+\left[q,f(x_1,\ldots,x_n)\sum_i f(x_1,\ldots,y_i,\ldots,x_n)\right]=0.$$

In particular, for $y_1 = x_1$ and $y_2 = \ldots = y_n = 0$, we obtain $2pf(x_1,\ldots,x_n)^2 + 2\big[q, f(x_1,\ldots$ $\ldots,x_n)^2\big] = 0$. Since $\mathrm{char}(R) \neq 2$, it implies that $pf(x_1,\ldots,x_n)^2 + \big[q, f(x_1,\ldots,x_n)^2\big] = 0$. This gives that

$$(p+q)f(x_1,\ldots,x_n)^2 - f(x_1,\ldots,x_n)^2 q = 0.$$

By Fact 2.5, we have one of the following:

$q \in C$ and $p = 0$, which implies that $H = 0$, a contradiction.

$p + q \in C$ and $p = 0$, which gives that $q \in C$. In this case $H = 0$, a contradiction.

$p = 0$ and $f(x_1,\ldots,x_n)^2$ is a central valued on R. By using the fact that if $z \in Z(R)$, then $d(z) \in Z(R)$, where d is a derivation on R, the equation (9) implies that $[q, f(x_1,\ldots,x_n)]^2 = 0$. By [9] (Theorem 1.1), we get $q \in C$ which implies that $H = 0$, a contradiction.

Case III. Let none of d and δ be inner derivations on R. We have the following two subcases.

Subcase I. Suppose that d and δ are C-dependent modulo inner derivation of U, that is, $\alpha d +$ $+ \beta\delta = ad_q$, where $\alpha, \beta \in C$, $q \in U$ and $ad_q(x) = [q, x]$ for all $x \in U$. If $\alpha = 0$, then δ is inner derivation on R, a contradiction. If $\beta = 0$, then d is inner derivation on R, a contradiction. Hence, α and β both can not be zero. This gives that $d(x) = \beta_1\delta(x) + [q', x]$ for all $x \in R$, where $\beta_1 = -\alpha^{-1}\beta$ and $q' = \alpha^{-1}q$. Thus, from (5), we have

$$apf(x_1,\ldots,x_n)^2 + a\delta(f(x_1,\ldots,x_n))f(x_1,\ldots,x_n) +$$
$$+ af(x_1,\ldots,x_n)\delta(f(x_1,\ldots,x_n)) + d(p)f(x_1,\ldots,x_n)^2 + p\Big(\beta_1\delta(f(x_1,\ldots,x_n)) +$$
$$+ \big[q', f(x_1,\ldots,x_n)\big]\Big)f(x_1,\ldots,x_n) + pf(x_1,\ldots,x_n)\Big(\beta_1\delta(f(x_1,\ldots,x_n)) +$$
$$+ \big[q', f(x_1,\ldots,x_n)\big]\Big) + \beta_1\delta^2(f(x_1,\ldots,x_n))f(x_1,\ldots,x_n) +$$
$$+ \beta_1 f(x_1,\ldots,x_n)\delta^2(f(x_1,\ldots,x_n)) + 2\beta_1\delta(f(x_1,\ldots,x_n))^2 +$$
$$+ \Big[q', \delta(f(x_1,\ldots,x_n))f(x_1,\ldots,x_n) + f(x_1,\ldots,x_n)\delta(f(x_1,\ldots,x_n))\Big] =$$
$$= pf(x_1,\ldots,x_n)pf(x_1,\ldots,x_n) + \delta(f(x_1,\ldots,x_n))pf(x_1,\ldots,x_n) +$$
$$+ pf(x_1,\ldots,x_n)\delta(f(x_1,\ldots,x_n)) + (\delta(f(x_1,\ldots,x_n)))^2. \tag{11}$$

First, we can replace $\delta(f(x_1,\ldots,x_n))$ with $f^\delta(x_1,\ldots,x_n) + \sum_i f(x_1,\ldots,y_i,\ldots,x_n)$ and $\delta^2(f(x_1,\ldots,x_n))$ with

$$f^{\delta^2}(x_1,\ldots,x_n) + 2\sum_i f^\delta(x_1,\ldots,y_i,\ldots,x_n) +$$

$$+ \sum_i f(x_1, \ldots, w_i, \ldots, x_n) + \sum_{i \neq j} f(x_1, \ldots, y_i, \ldots, y_j, \ldots, x_n),$$

where $\delta(x_i) = y_i$ and $\delta^2(x_i) = w_i$ in (11) and then U satisfies the blended component

$$\beta_1 \left(\sum_i f(x_1, \ldots, w_i, \ldots, x_n) \right) f(x_1 \ldots, x_n) +$$

$$+ \beta_1 f(x_1, \ldots, x_n) \left(\sum_i f(x_1, \ldots, w_i, \ldots, x_n) \right) = 0 \tag{12}$$

for all $x_1, \ldots, x_n \in R$ and $w_i \in R$. In particular, for $w_1 = x_1$ and $w_2 = \ldots = w_n = 0$, we obtain $2\beta_1 f(x_1, \ldots, x_n)^2 = 0$. Since $\mathrm{char}(R) \neq 2$, it implies that $\beta_1 = 0$. Then d is an inner derivation, a contradiction.

Subcase II. Suppose that d and δ are C-independent modulo inner derivation of U. By using Kharchenko's theorem (see Fact 2.4), we can replace $d(f(x_1, \ldots, x_n))$ with $f^d(x_1, \ldots, x_n) + \sum_i f(x_1, \ldots, y_i, \ldots, x_n)$, $\delta(f(x_1, \ldots, x_n))$ with $f^\delta(x_1, \ldots, x_n) + \sum_i f(x_1, \ldots, z_i, \ldots, x_n)$ and $d\delta(f(x_1, \ldots, x_n))$ with $f^{d\delta}(x_1, \ldots, x_n) + \sum_i f^d(x_1, \ldots, z_i, \ldots, x_n) + \sum_i f^\delta(x_1, \ldots, y_i, \ldots, x_n) + \sum_i f(x_1, \ldots, w_i, \ldots, x_n) + \sum_{i \neq j} f(x_1, \ldots, y_i, \ldots, z_j, \ldots, x_n)$, where $d(x_i) = y_i$, $\delta(x_i) = z_i$ and $d\delta(x_i) = w_i$ in equation (6) and then U satisfies the blended component

$$\left(\sum_i f(x_1, \ldots, w_i, \ldots, x_n) \right) f(x_1, \ldots, x_n) +$$

$$+ f(x_1, \ldots, x_n) \left(\sum_i f(x_1, \ldots, w_i, \ldots, x_n \right) = 0. \tag{13}$$

Equation (13) is similar to equation (12), we get a contradiction.

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